



**Math 256 – Final examination**  
**University of British Columbia**  
*April 28, 2015, 3:30 pm to 6:00 pm*

Last name (print):

First name:

ID number:

*This exam is “closed book” with the exception of a single 8.5”x11” formula sheet. Calculators or other electronic aids are not allowed.*

Page	Score	Max
2		8
3		8
4		8
5		12
6		8
7		6
8		10
Total		60

**Rules governing examinations**

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  - (a) speaking or communicating with other candidates, unless otherwise authorized;
  - (b) purposely exposing written papers to the view of other candidates or imaging devices;
  - (c) purposely viewing the written papers of other candidates;
  - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)-(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

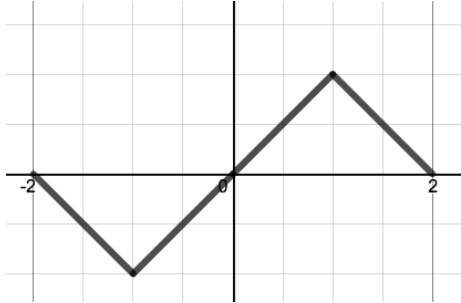
## Multiple choice

1. [2 pts] How many distinct possibilities are there for  $\lim_{t \rightarrow \infty} x(t)$  where  $x(t)$  solves  $x' - x = e^{-t}$ ?  
 (a) 1            (b) 2            (c) 3            (d) 4

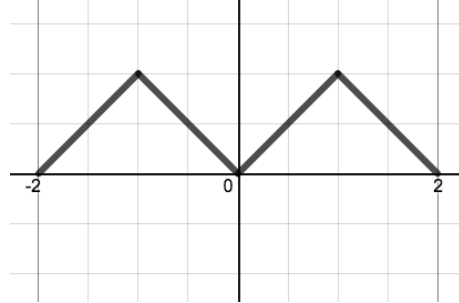
2. [2 pts] Which of the following pairs of functions are not independent?  
 (a)  $\cos(2t), \sin(2t)$   
 (b)  $e^t, e^{3t}$   
 (c)  $t, \frac{1}{t}$   
 (d)  $\sin(t), \cos\left(t + \frac{\pi}{2}\right)$

3. [2 pts] Which of the following functions has a Fourier series with only sine terms of the form  $b_n \sin(n\pi x/2)$  with  $b_n = 0$  for all  $n$  even?

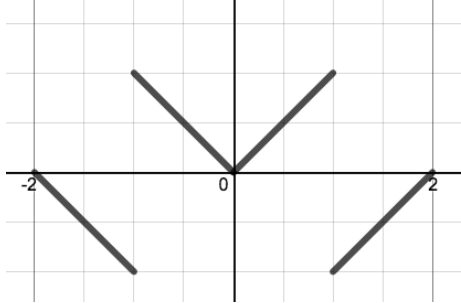
(a)



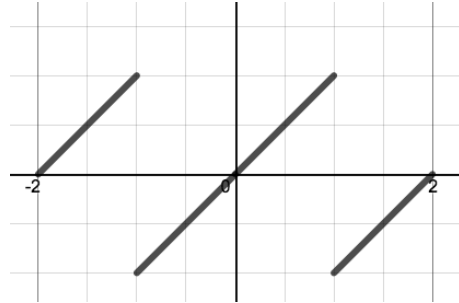
(b)



(c)



(d)



4. [2 pts] The solution to the equation  $y'' + 16\pi^2 y = f(t)$ , where  $f(t)$  is a piecewise constant and periodic function with period  $T$  and  $2T$  is a positive integer, ...  
 (a) ...can be found by representing both  $f(t)$  and the particular solution  $y_p(t)$  as a Fourier series and using the Method of Undetermined Coefficients.  
 (b) ...can be written as an infinite sum of sine functions.  
 (c) ...will eventually (for  $t$  large) look like a sine function with period  $1/2$ .  
 (d) ...is described by all of the options above.  
 (e) ...is not described by any of the options above.



3. [8 pts]

(a) Find the general solution to the equation  $\mathbf{x}' = A\mathbf{x}$  where

$$A = \begin{pmatrix} a & a-2 \\ a-2 & a \end{pmatrix}.$$

Note: one of the eigenvalues is  $\lambda = 2$ .

(b) For what values of  $a$  is the steady state a saddle? Unstable node?

4. [8 pts] Consider the function  $f(t) = u_2(t)(t - 2) - u_3(t)(t - 3) + u_3(t)(t - 2)^2$ .

(a) Sketch the graph of  $f(t)$ .

(b) Calculate the Laplace transform of  $f(t)$ .

5. [4 pts] Omar owns a restaurant and would like to estimate how many people will be in the restaurant at any time during an evening so he knows how many tables he'll need. Every time a train stops of the Skytrain station next door, 6 people enter the restaurant. Between trains, nobody comes in. The train comes through every 8 minutes with the first one arriving at 5 pm when the restaurant opens and last one arriving at 11 pm (46 trains). People finish eating and leave the restaurant at a rate proportional to the number of people currently in the restaurant with rate constant  $1/120 \text{ min}^{-1}$ . Write down a model (ODE and IC) for the number of people  $P(t)$  in the restaurant at time  $t$  measured in minutes after 5 pm. You do not need to solve the equation.

6. [8 pts] The Laplace transform of the solution to a differential equation is given by

$$Y(s) = \frac{5}{(s+1)(s^2+4)}.$$

(a) Find the solution  $y(t)$  by inverting  $Y(s)$ .

(b) Provide a differential equation and initial values that have a transformed solution  $Y(s)$ .

7. [6 pts]

- (a) What is the form of the particular solution that you would use to solve the equation below using method of undetermined coefficients? You do not need to determine the coefficients.

$$y'' + 4y' + 4y = 2te^{-2t}$$

- (b) Solve the equation  $y'' + 2y' + 3y = e^{-t}$

8. [5 pts] Calculate the Fourier series for  $f(x) = u_{-1}(x) - u_1(x)$  where  $u_c(x)$  is the Heaviside function at  $x = c$  and the domain of the function is  $[-2, 2]$ .

9. [5 pts] Solve the Diffusion equation  $u_t = 7u_{xx}$  with boundary conditions  $u(0, t) = 0$  and  $u_x(4, t) = 5$  and initial condition  $u(x, 0) = 5x + \sin(3\pi x/8)$ .



This page is for rough work and doodles. It will not be marked.