Solving the Diffusion equation

$u_t = D u_{xx}$

$u(x,0) = f(x)$

Dirichlet, Neumann, mixed (x2)

$\begin{cases} u(0,t)=0 \\ u(L,t)=0 \end{cases}$ $\begin{cases} u_x(0,t)=0 \\ u_x(L,t)=0 \end{cases}$ $\begin{cases} u(0,t)=0 \\ u_x(L,t)=0 \end{cases}$ $\begin{cases} u_x(0,t)=0 \\ u(L,t)=0 \end{cases}$ Homogeneous

$\begin{cases} u(0,t)=a \\ u(L,t)=b \end{cases}$ $\begin{cases} u_x(0,t)=c \\ u_x(L,t)=d \end{cases}$ . . . Non-homogeneous

(1) Find the steady state solution. $u_{ss}(x) = C_1 x + C_2$

(2) Determine the eigenfunctions by considering BCs

$e^{\lambda t}\sin\frac{n\pi x}{L}$, $e^{\lambda t}\cos\frac{n\pi x}{L}$, $e^{\lambda t}\sin\frac{n\pi x}{2L}$, $e^{\lambda t}\cos\frac{n\pi x}{2L}$

(3) Subtract $u_{ss}(x)$ from $f(x)$: $g(x) = f(x) - u_{ss}(x)$.

(4) Using eigenfunctions, find the FS of $g(x)$.

(5) Write $u(x,t) = u_{ss}(x) + \sum c_n e^{\lambda_n t}$ eigenfunctions