How to extend an IC to satisfy BCs

Basic rules:

(1) If you need to satisfy a Dirichlet condition at $x=a$, extend the IC as an odd function about $x=a$.

(2) If you need to satisfy a Neumann condition at $x=a$, extend the IC as an even function about $x=a$.

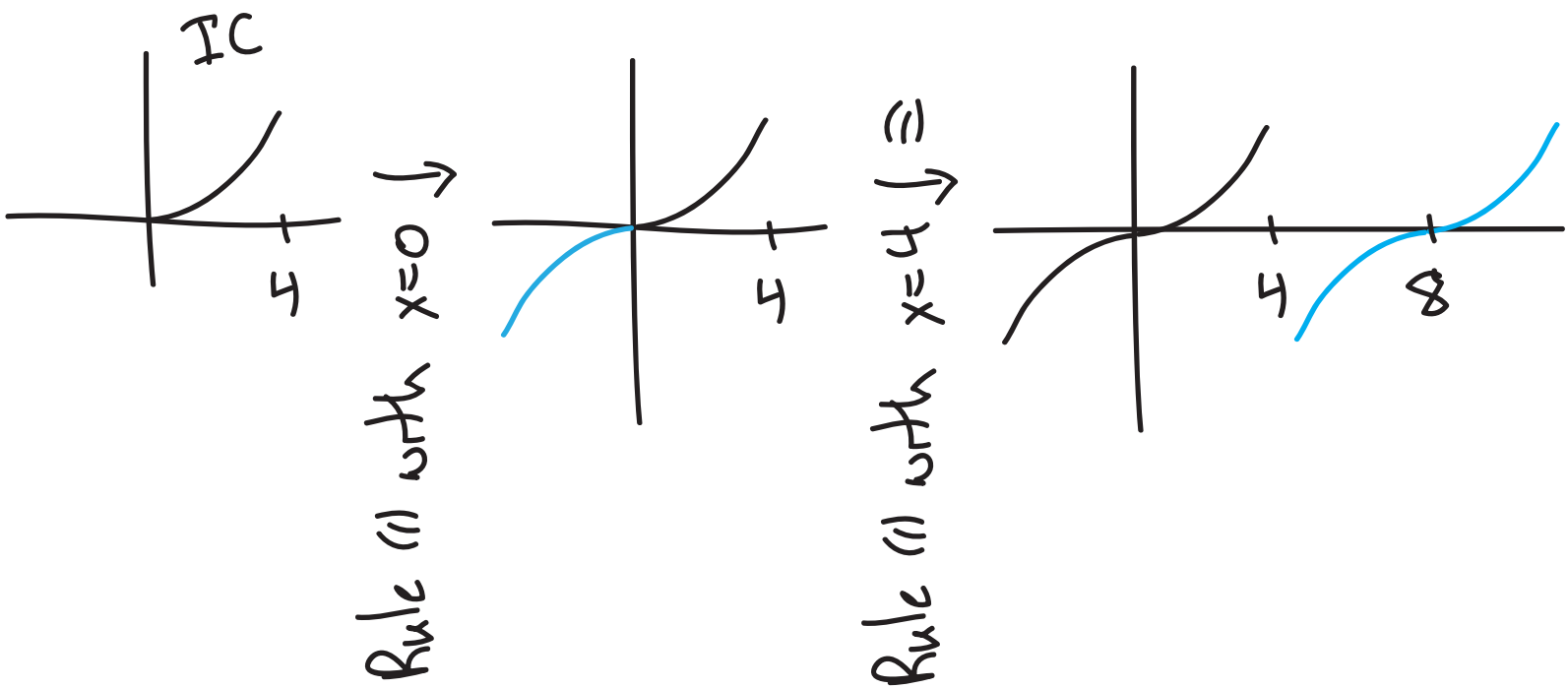
(3) All extensions should be done sequentially so that earlier extensions get included in later ones.

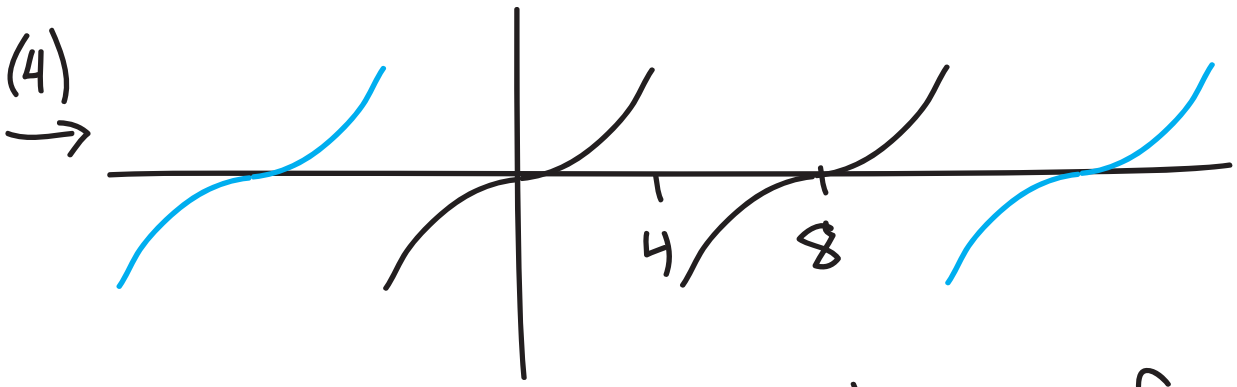
(4) Extend the final result of the first three rules periodically with the smallest period possible.

Note: The IC itself often does not satisfy the BC and that's ok.

ex. $u_t = 9u_{xx}$, $u(x,0) = x^2$ on $[0,4]$,

$u(0,t) = 0$, $u(4,t) = 0$.

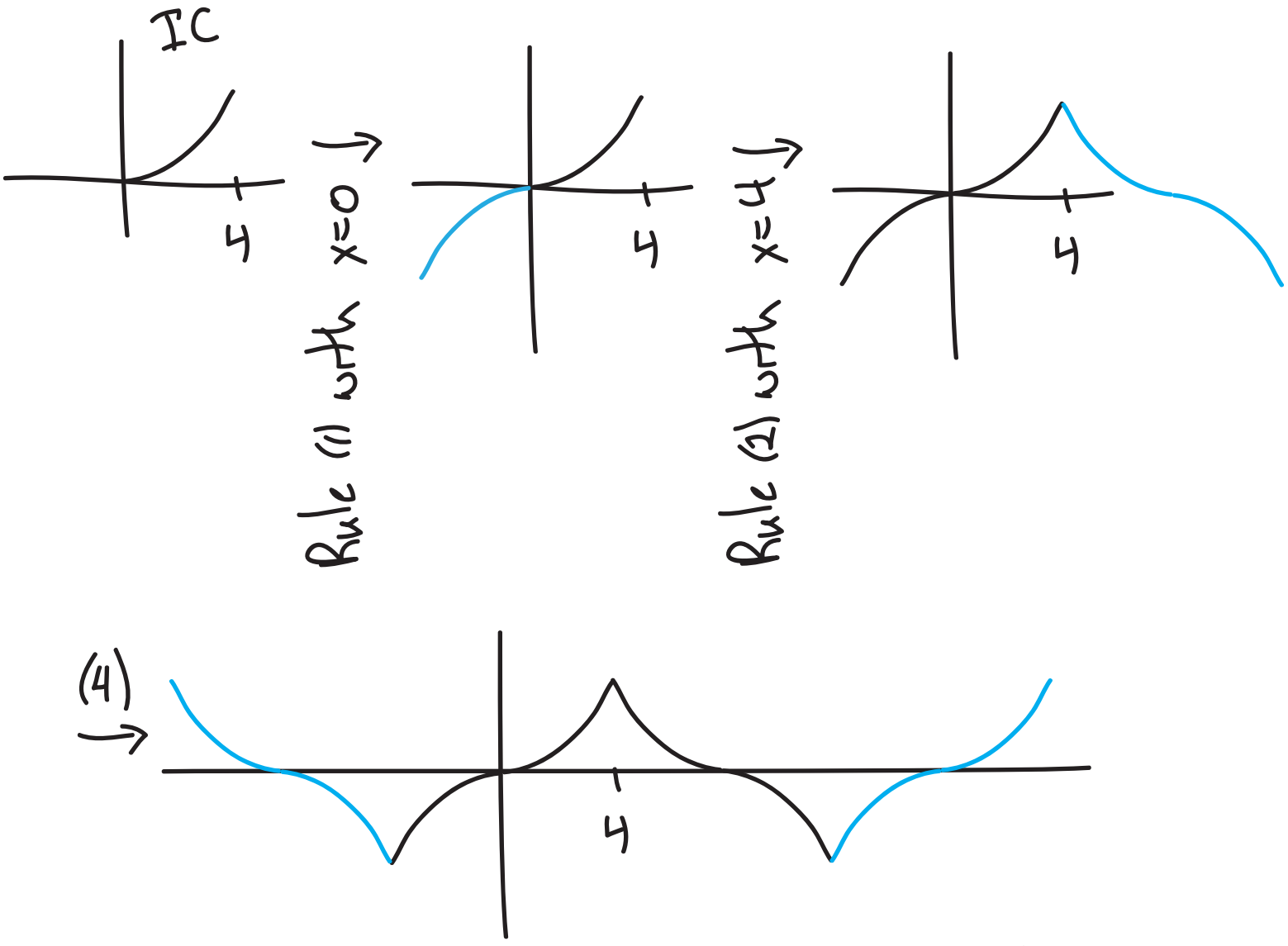




Because we extended twice for BCs, you might be tempted to say the period is 16 (doubling the domain size each time) but the minimum period is 8 so use $\sin \frac{n\pi x}{4}$ for the F.S. This happens for both Dirichet and Neumann BCs (not mixed).

ex. $u_t = 9u_{xx}$, $u(x,0) = x^2$ on $[0,4]$,

$u(0,t) = 0$, $u_x(4,t) = 0$.



Because we extended twice for BCs, you might be tempted to say the period is 16 (doubling the domain size each time) and in this case you would be right so use $\sin \frac{n\pi x}{8}$ for the FS.