Today

• Spreadsheets
• Differentiation rules
• Tangent lines
• Reminders:
  • PL4.2 Wednesday 7am,
  • Assignment 3 Thursday 7am,
  • OSH 2 Friday 11:59 pm.
  • Sign up for midterm time/room.
How to graph $f'(x)$ using a spreadsheet

- Sketch $f'(x)$ for the following functions:

  $$f(x) = |\sin(x)|$$

  $$f(x) = e^{-x^2} \sin(5x)$$

- Zooming in on a specific region...

Demo in Google sheets or Excel.
A comment on derivative notation

Leibniz: \[ y = f(x) \]
\[ \frac{dy}{dx} = f'(x) \]

Newton: \[ \frac{dy}{dx} \bigg|_{x=2} = f'(2) \]
Power rule

\[ f(x) = x^2 \]

Find \( f' \) at \( x=2 \) (using the definition of the derivative).

\[
\begin{align*}
  f'(2) &= \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} \\
  &= \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \\
  &= \lim_{h \to 0} \frac{4h + h^2}{h} \\
  &= \lim_{h \to 0} (4 + h) = 4
\end{align*}
\]

Do this on doc cam. Emphasize binomial expansion, Pascal's triangle and how it generalizes.
Power rule

\[ f(x) = x^2 \]

Find \( f'(x) \) at all points \( x \) at the same time

\[
f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2hx + h^2}{h} = 2x
\]
Power rule

\[ f(x) = x^3 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^3 - x^3}{h} \]

\[ = \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{h} \]

\[ = 3x^2 \]

Do this on doc cam. Emphasize binomial expansion, Pascal’s triangle and how it generalizes.
Power rule

\[ f(x) = x^n \]

\[ f'(x) = nx^{n-1} \]

So far, we have justified this rule ONLY when \( n \) is positive integer!
Rules for differentiation

• Addition rule
• Product rule
• Chain rule (for composition of functions)
• Quotient rule
Suppose \( f(x) = g(x) + k(x) \) and that
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

- What is \( f'(2) \)?

(A) 4

(B) 7

(C) 10

(D) 11
Suppose \( f(x) = g(x) + k(x) \) and that
\[ g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5. \]

- What is \( f'(2) \)?

- (A) 4
- (B) 7
- (C) 10
- (D) 11
Suppose \( f(x) = g(x)k(x) \) and that
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

• What is \( f'(2) \)?

(A) 3

(B) 10

(C) 11

(D) 17
Suppose \( f(x) = g(x)k(x) \) and that
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

• What is \( f'(2) \)?

(A) 3

Try \( g(x) = x \) and \( k(x) = x^2 \). (if many choose B)

(B) 10

If \( f'(x) = g'(x)k'(x) \) then
\[
f(x) = (x) (x^2) \text{ so } f'(x) = (1) (2x) = 2x.
\]

(C) 11

But \( f(x) = x^3 \) and power rule says
\[
f'(x) = 3x^2.
\]

(D) 17

So \( g'(x)k'(x) \) can’t be right.
What is the correct derivative for $f(x) = g(x)k(x)$?

$$A(t) = d(t)w(t)$$

$$A(t + h) = A(t) + (d(t + h) - d(t)) \cdot w(t)$$

$$+ d(t) \cdot (w(t + h) - w(t)) + \text{small corner}$$

$$A(t + h) - A(t) \approx \frac{(d(t + h) - d(t)) \cdot w(t)}{h}$$

$$+ \frac{d(t) \cdot (w(t + h) - w(t))}{h}$$

$$d(t) \cdot (w(t + h) - w(t))$$

$$A'(t) = d'(t)w(t) + d(t)w'(t)$$