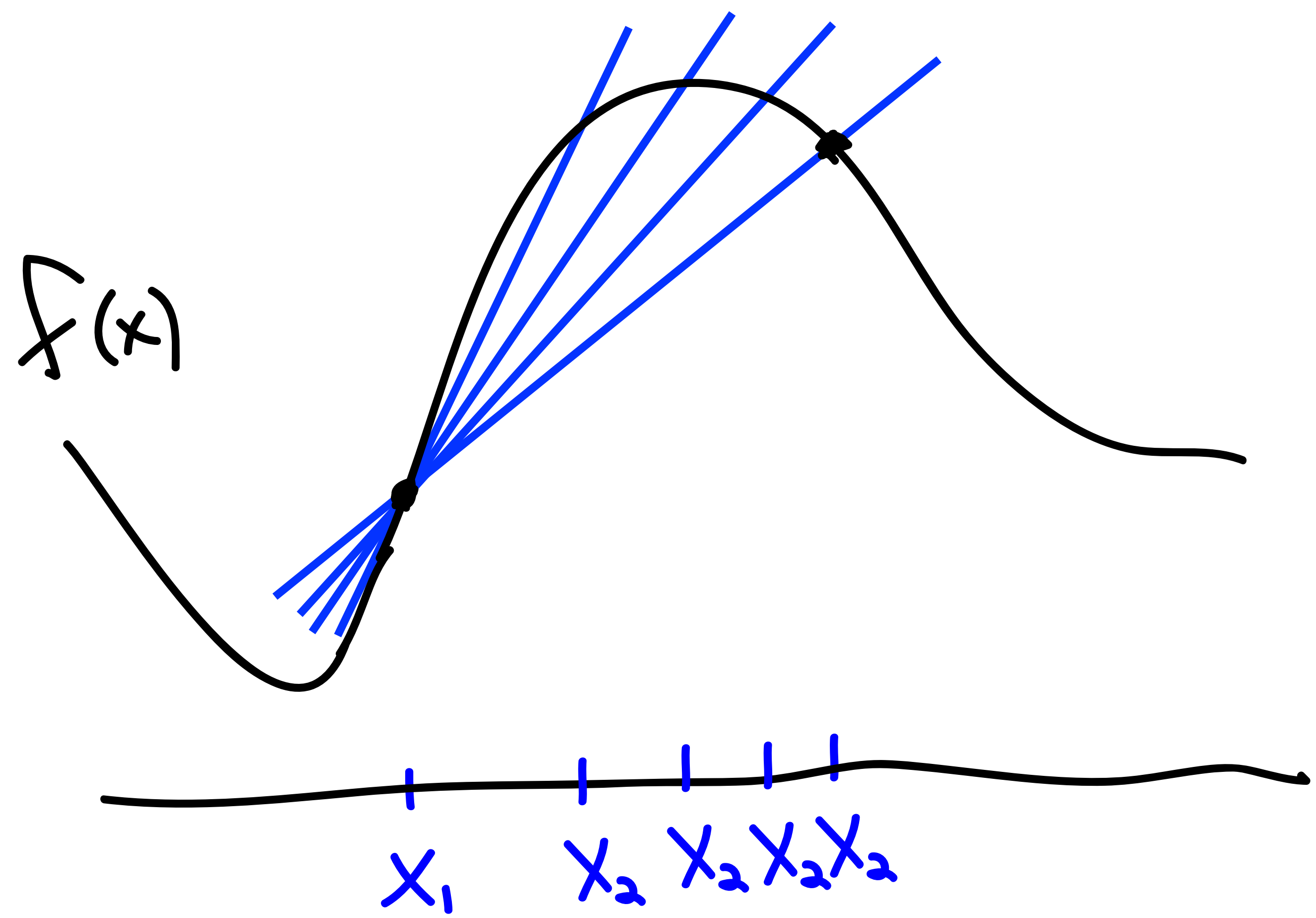
Today...

- From secant line to tangent line.
- Definition of the derivative.
- Limits, left limit, right limit.
- Continuity.
- Types of limits we'll see this semester.

What if you want the rate of change AT x_1?

(instantaneous instead of average)

Take a point x_2 so that the secant line is closer to the "secant line" AT x_1.



Alternate notation: let $x_2 = x_1 + h$ so that

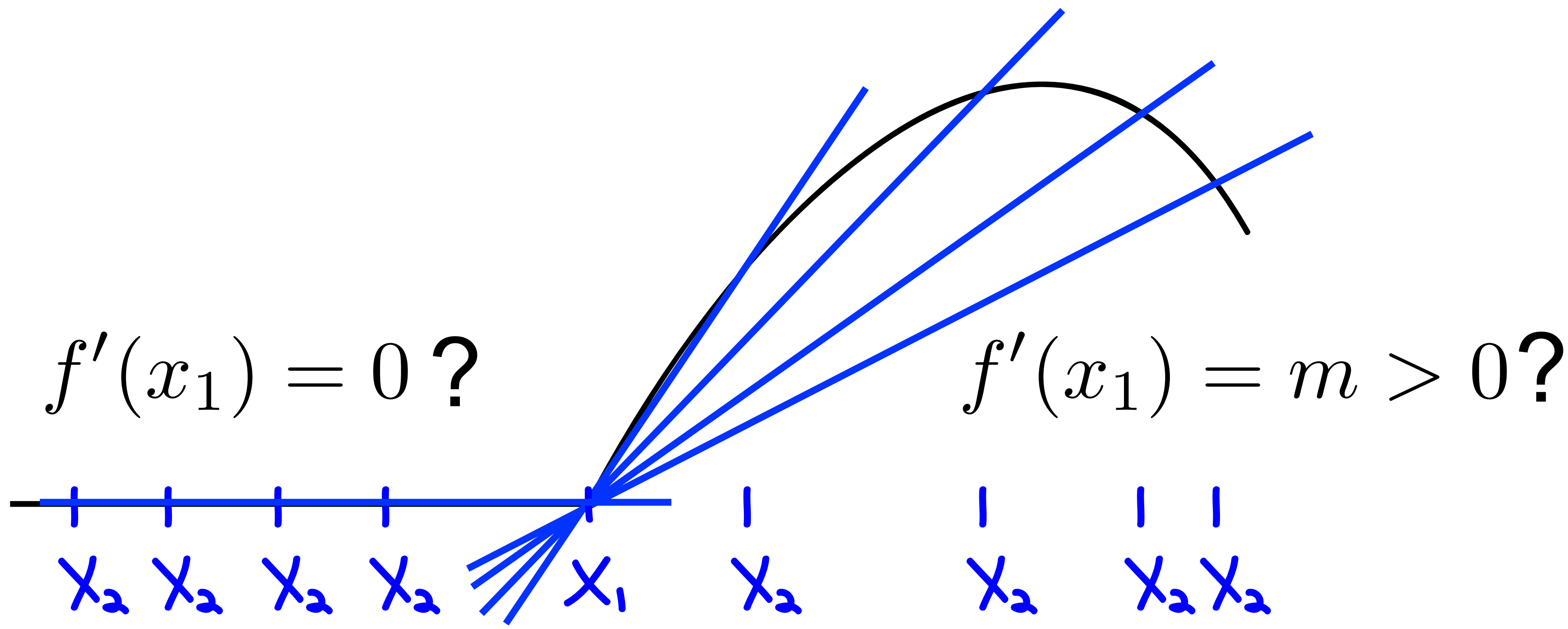
$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{x_1 + h - x_1}$$

If we take h values closer and closer to 0...

- The secant line approaches the tangent line.
- The slope of the secant line approaches the slope of the tangent line.
- We call the resulting slope the derivative at x_1.
- We now have to learn how to take limits!

$$\text{slope at } x_1 = f'(x_1) = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$

Another example



(A) $\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = m > 0$

(B) $\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = 0$

(C) Both (A) and (B)

(D) The limit does not exist.

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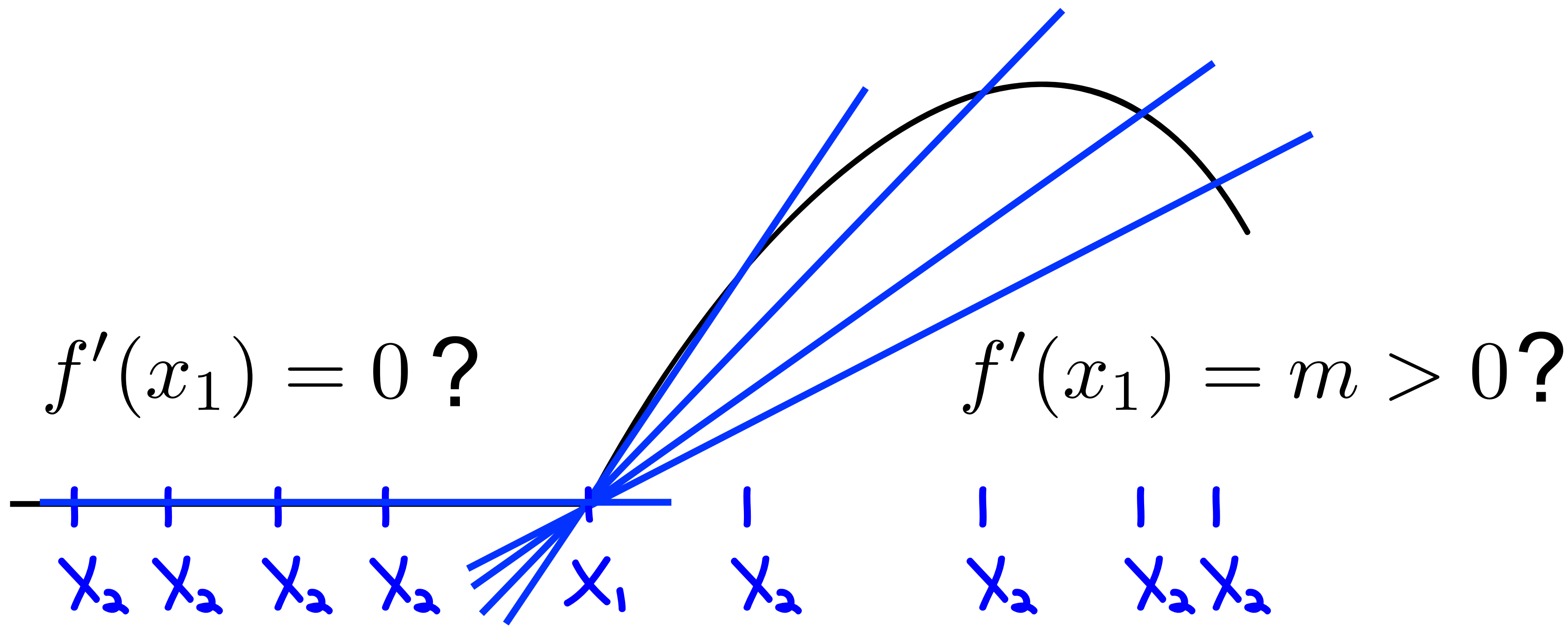

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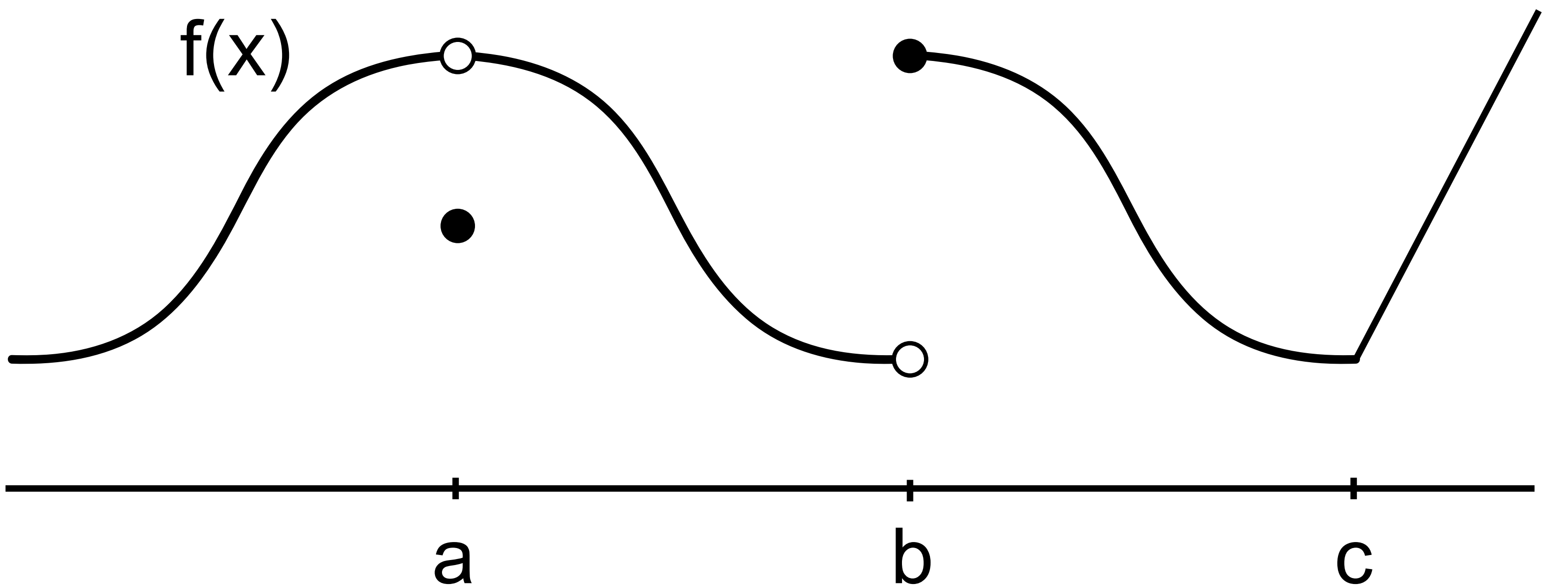
To evaluate a limit

To evaluate $\lim_{x \to a} f(x)$, plug in values closer and closer to a but you never get to a. In fact, f(a) may not even be defined. If you always get the same number no matter how you approach a, then the limit exists.

Limits

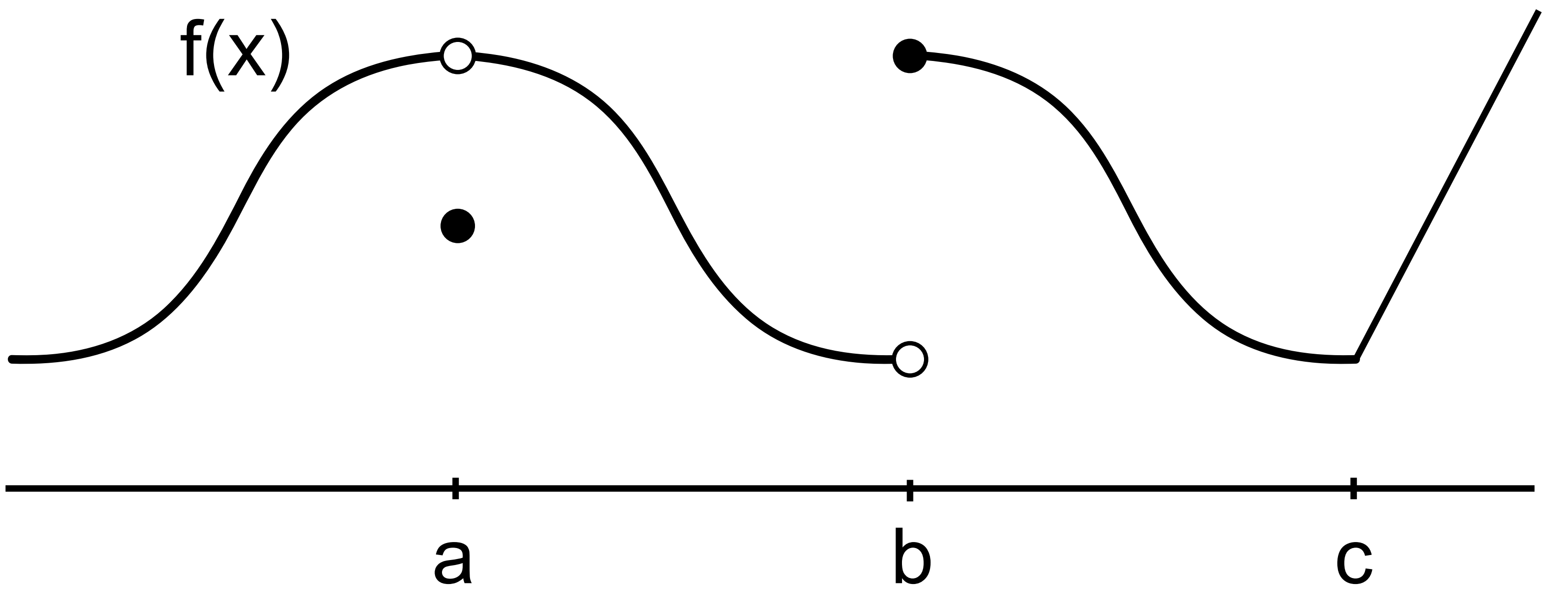


Which of the following are true?

1. $\lim_{x \to a} f(x) = f(a)$
2. $\lim_{x \to b} f(x) = f(b)$
3. $\lim_{x \to c} f(x)$ does not exist.
4. $\lim_{x \to a} f(x)$ exists.
5. $\lim_{x \to b} f(x)$ exists.

(A) 1, 4
(B) 2, 5
(C) 3
(D) 4
(E) 5

Limits



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5. $\lim_{x \to b} f(x)$ exists.

(A) 1, 4

(B) 2, 5

(C) 3

(D) 4

(E) 5

Left and right limits

- The right limit at a - plug in x values approaching a from above (x>a):

$$\lim_{x \longrightarrow a^+} f(x)$$

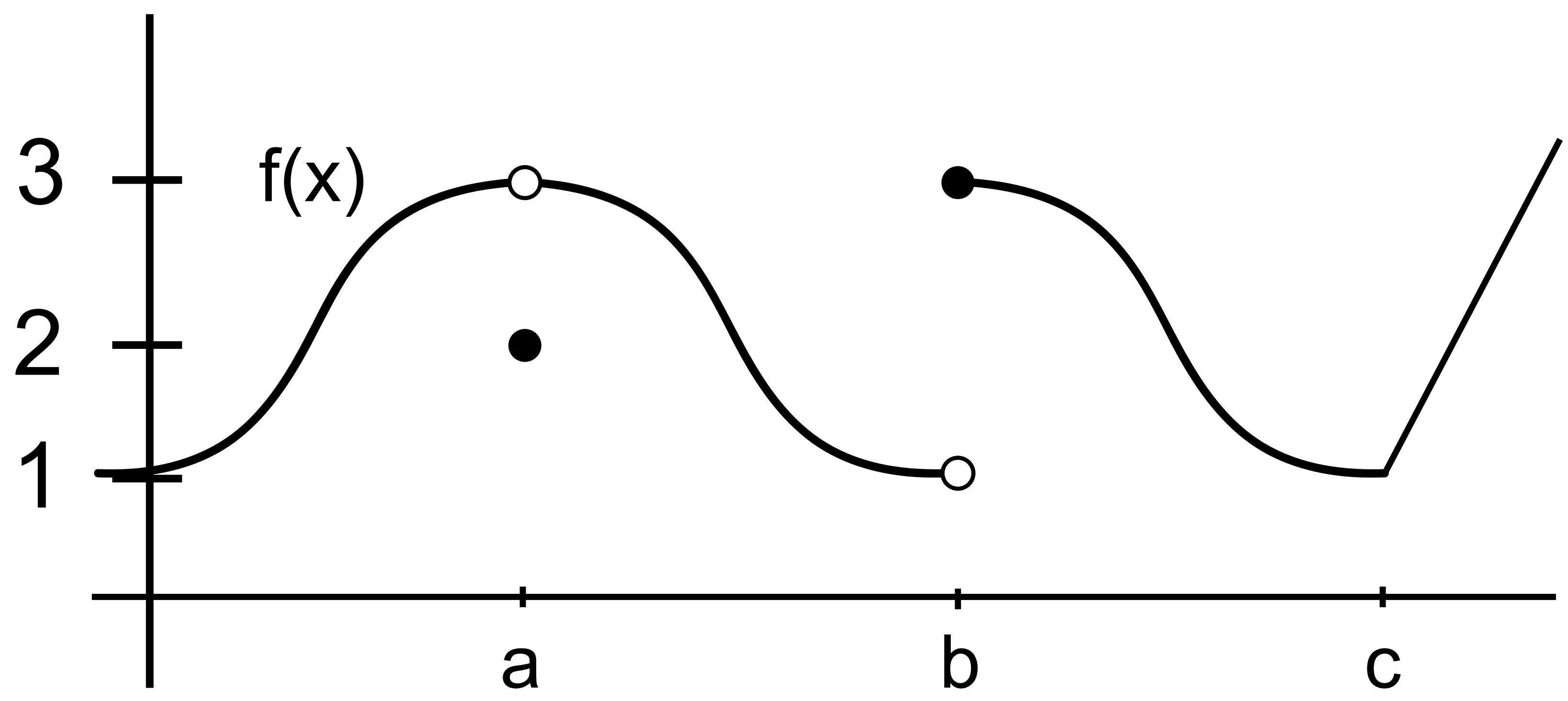
- The left limit at a - plug in x values approaching a from below (x<a):

$$\lim_{x \longrightarrow a^-} f(x)$$

- When these exist and are equal, $\lim_{x \longrightarrow a} f(x)$ exists

$$\lim_{x \longrightarrow a} f(x) = \lim_{x \longrightarrow a^+} f(x) = \lim_{x \longrightarrow a^-} f(x).$$

Limits



(A) $\lim_{x \to a} f(x)$ = 2

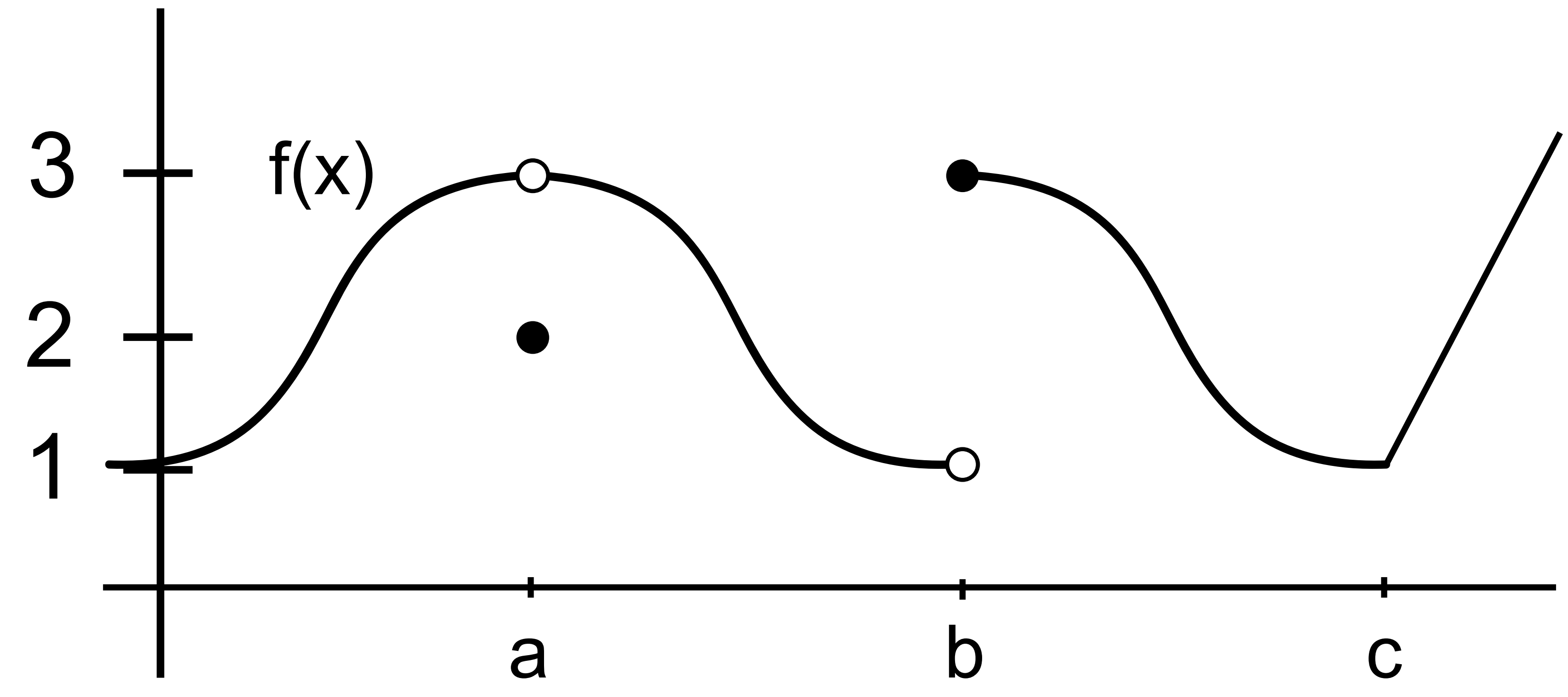
(B) $\lim_{x \to b^-} f(x)$ = 3

(C) $\lim_{x \to a} f(x)$ = 3

(D) $\lim_{x \to b} f(x)$ = 3

(E) $\lim_{x \to b^+} f(x)$ does not exist

Limits



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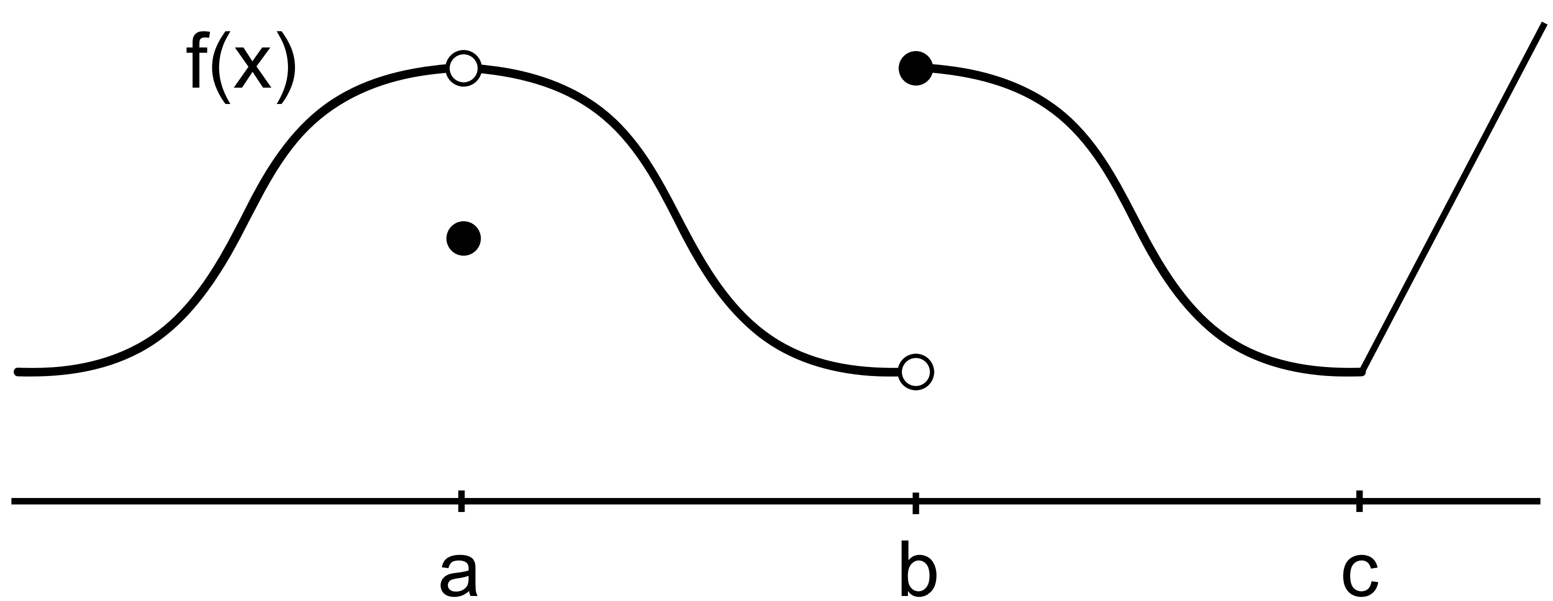
Continuity

When $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$

we say that f(x) is continuous at x=a.



f(x) is continuous at all x except at x=a and x=b.

Types of limits we'll talk about

- Points of continuity: $\lim_{x \to a} f(x) = f(a)$
- Hole-in-the-graph (like derivative limit)
- Limits at $\pm\infty$ (asymptotes)
- Left/right, jumps
- Vertical asymptotes

Hole-in-the-graph and derivatives

Suppose you have a function f(x).

Define the function $g(h) = \dfrac{f(2+h) - f(2)}{h}$.

It just so happens that

$$f'(2) = \lim_{h \to 0} g(h) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

We can't evaluate g(0) because of a divide by 0 issue.

But for many fs, the limit exists and gives f'(2).

On the board: draw g(h) in "good" case.

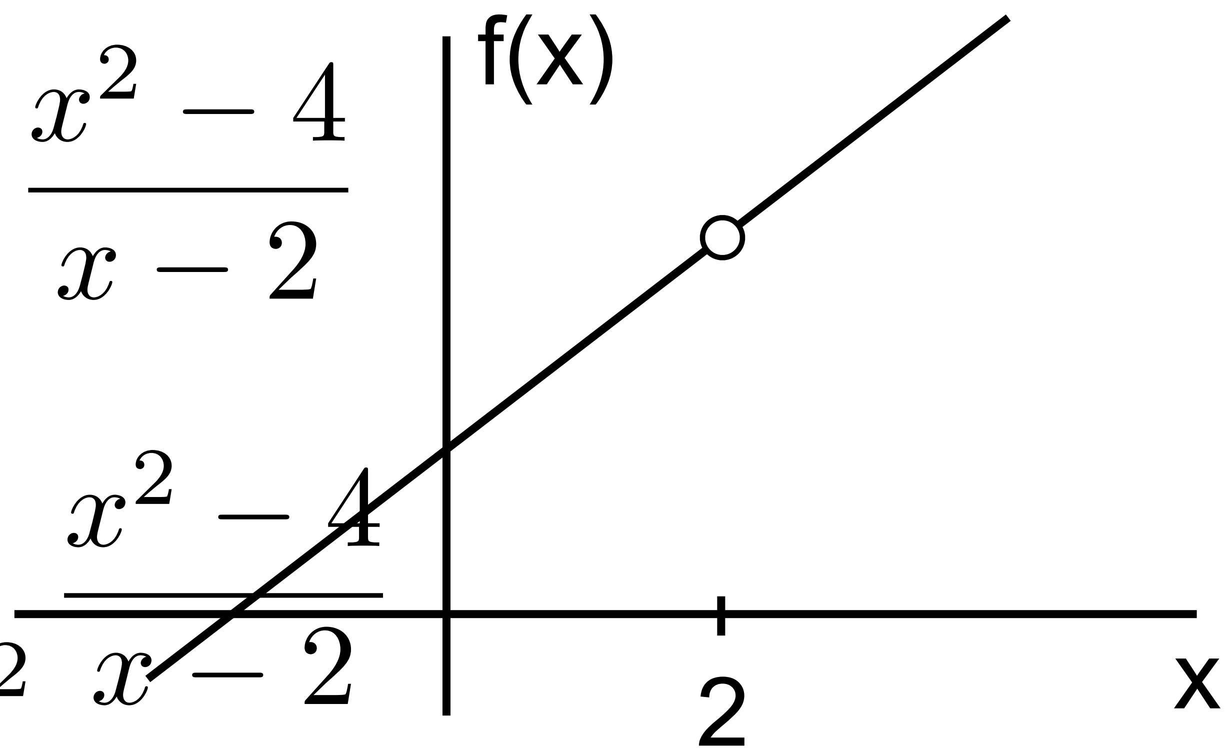
Which of the following is a "point-of-continuity" limit?

$$f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x-2)(x+2)}{x-2}$$

(A) $\lim_{x \to \infty} \frac{x^2 - 4}{x - 2}$

(B) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(C) $\lim_{x \to -2} \frac{x^2 - 4}{x - 2}$



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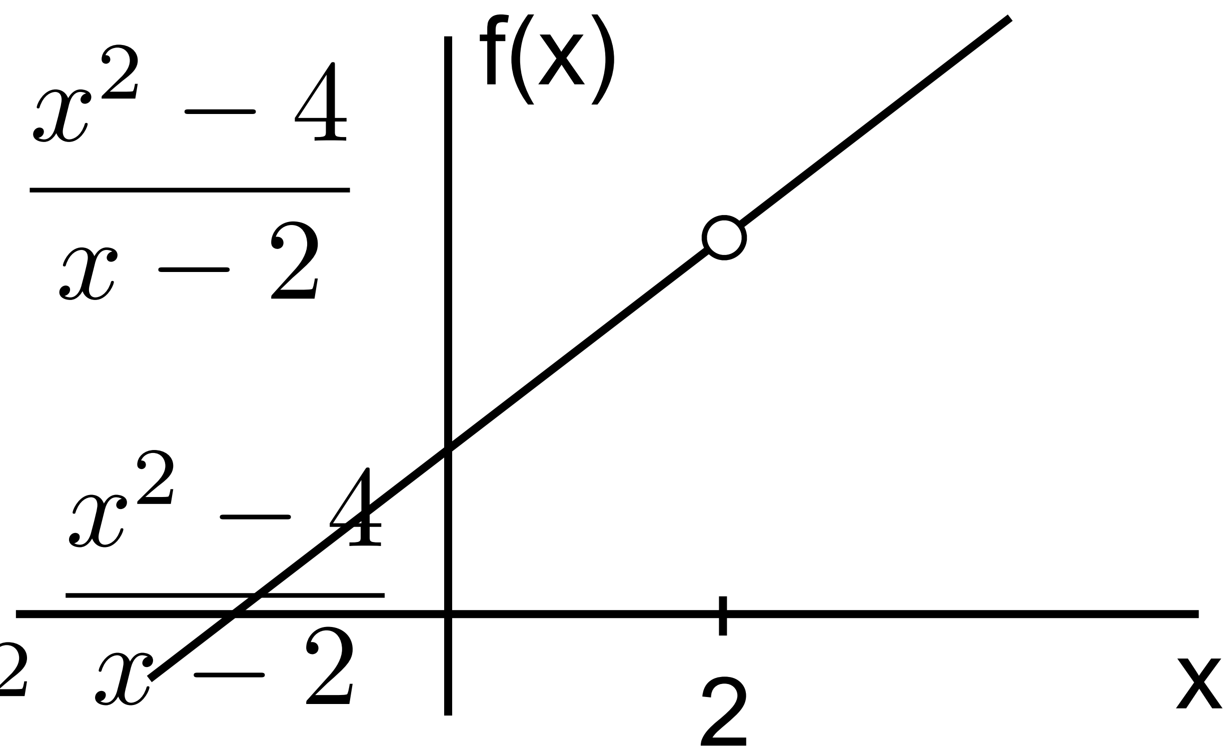

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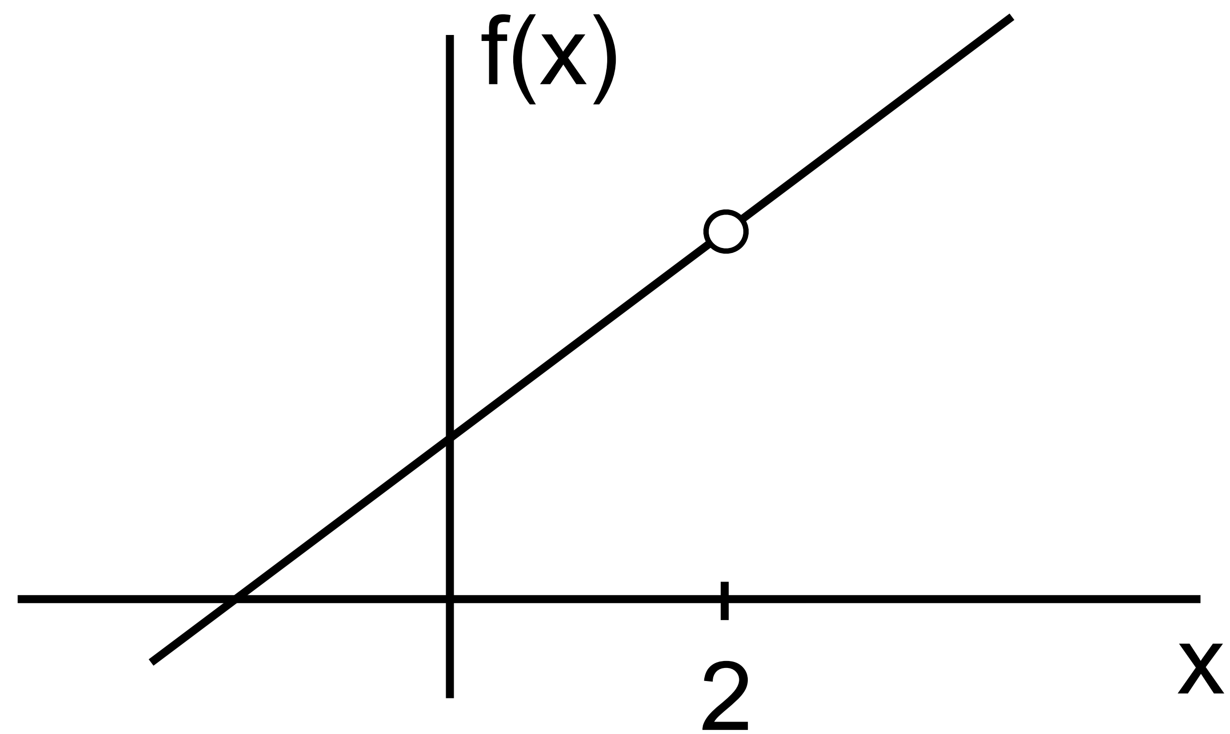

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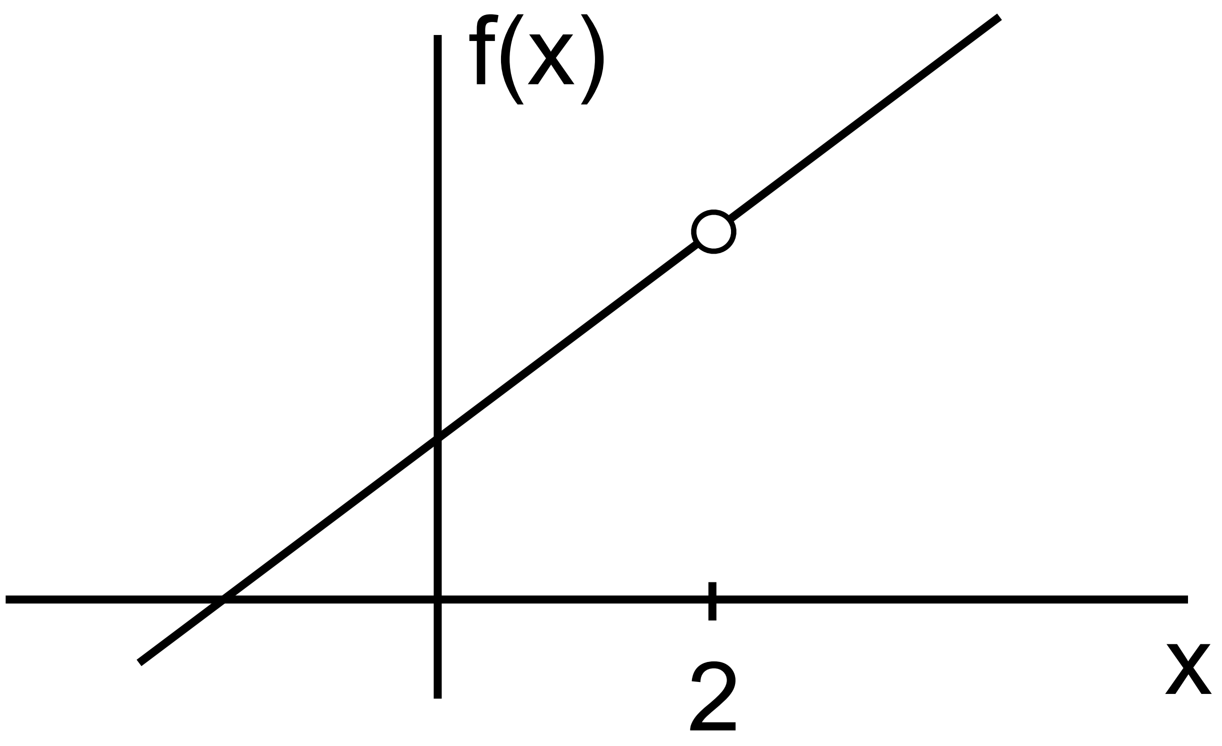

Which of the following is a "limit-at-infinity" limit?

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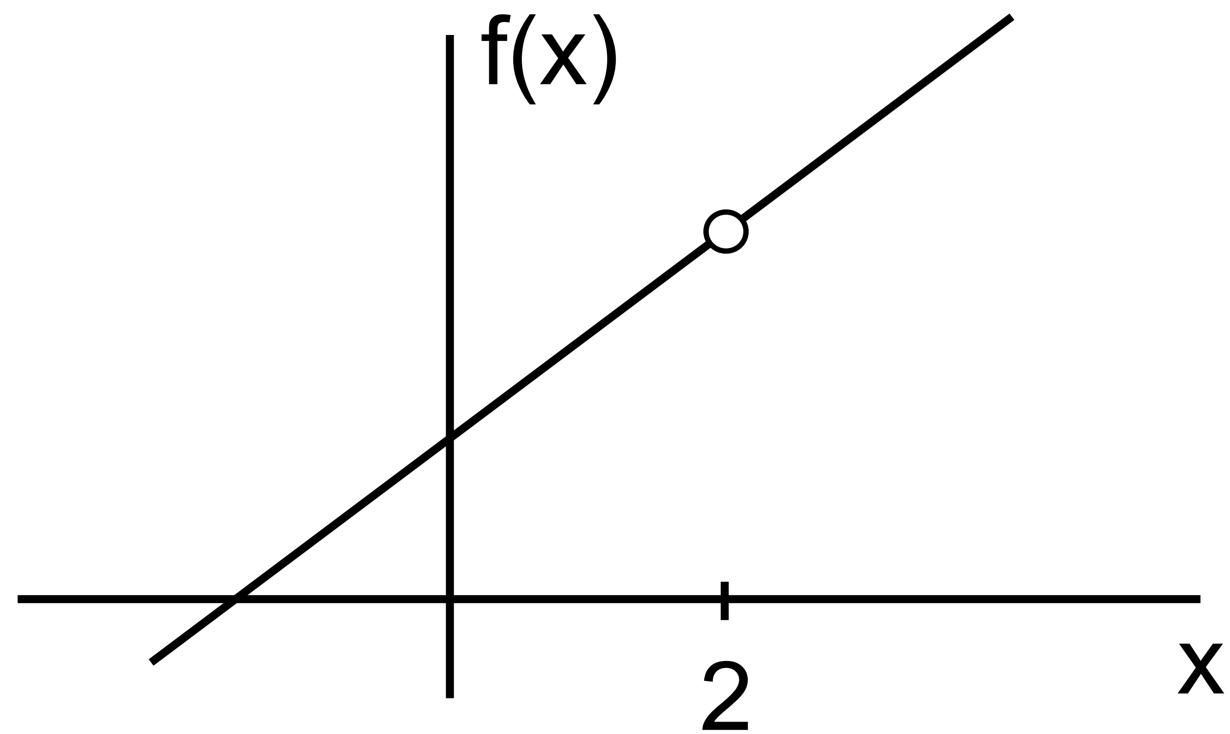

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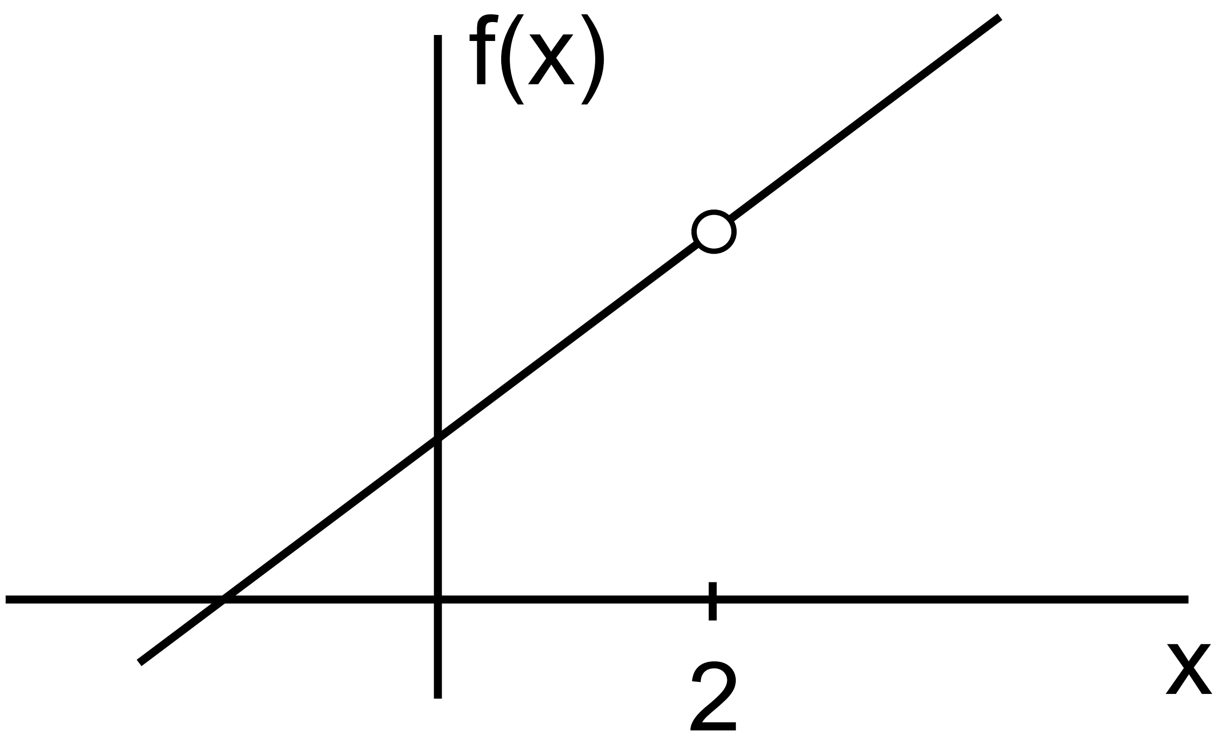

Examples in which f'(a) does not exist

On the board...