Today

- Midterm discussion
- Inflection points
- Putting it all together - using f, f′ and f″ to sketch a graph.

Today, I'd like to ...

(A) ...talk about the midterm.

(B) ...talk about the BIOL 112 midterm.

(C) ...go for coffee.

(D) ...learn more math so I can ace Midterm 2.

I thought the midterm was...

(A) ...easier than I expected.

(B) ...pretty much what I expected.

(C) ...harder than I expected.

The hardest part of the midterm was...

(A) ...the multiple choice section.

(B) ...the short answer section.

(C) ...long-answer #1 (tangent line || to y=-x).

(D) ...long-answer #2 (Find a,b so f′ exists).

(E) ...long-answer #3 (All-you-can-eat).

The most useful thing I did to study was...

(A) ...doing/reviewing WeBWorK assignments.

(B) ...doing/reviewing OSH.

(C) ...doing practice problems from the course notes.

(D) ...reading the course notes.

(E) ...reviewing the lecture slides.

Potential IPs

- A potential IP is a point **a** at which **$f''(a)=0$** because that MIGHT be a min/max of $f'(x)$.
- If $f''(x)$ changes sign at a potential IP of $f(x)$, then it is an IP of $f(x)$ because it's an extrema of $f'(x)$.
- If $f''(x)$ does not change sign at a potential IP of $f(x)$, then the potential IP is not an IP of $f(x)$!

Summary

- Use f′(x) to determine intervals of increase/decrease of f(x).
- Solve f′(x)=0 to find potential extrema (x=a). Check that f′(x) changes sign at a (FDT) or that f″(a) <> 0 (SDT) to make sure.
- Use f″(x) to determine intervals of concave up/down.
- Solve f″(x)=0 to find potential inflection points (x=a). Check that f″(x) changes sign at a ("FDT" or that f‴(a) <> 0 ("SDT") to make sure.

Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

(E) Don't know.

Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no. ← "Second DT" applied to $f'(x)$ - fails so no conclusion.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

$f''(x) = 12x^2$

(E) Don't know.

Not sure about (C)? Try this for $f(x)=x^5$.

$g(x) = 12x^3 - 12x^2$ has...

(A) a maximum at $x=0$ and a minimum at $x=1/3$.

(B) a minimum at $x=0$ and a maximum at $x=1/3$.

(C) a maximum at $x=0$ and an inflection pt at $x=1/3$.

(D) an inflection pt at $x=0$ and a minimum at $x=1/3$.

$g(x) = 12x^3 - 12x^2$ has...

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(C) a maximum at x=0 and an inflection pt at x=1/3.

(D) an inflection pt at x=0 and a minimum at x=1/3.

$f(x) = 3x^4 - 4x^3$ has...

(A) a maximum at $x=0$ and a minimum at $x=1$.

(B) a minimum at $x=0$ and a maximum at $x=1$.

(C) a maximum at $x=0$ and an inflection pt at $x=1$.

(D) an inflection pt at $x=0$ and a minimum at $x=1$.

$f(x) = 3x^4 - 4x^3$ has...

(A) a maximum at $x=0$ and a minimum at $x=1$.

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(D) an inflection pt at $x=0$ and a minimum at $x=1$.

How do you know? Next few slides will explain...

$f(x) = 3x^4-4x^3$

- $f'(x) = 12\ (x^3 - x^2) = 0$ --> $x=0$, $x=1$.
- $f''(x) = 12\ (3x^2 - 2x)$.
- SDT: $f''(1) = 1 > 0$

Could also do FDT: $f'(0^{+/-})$

--> $f'(x)$ is increasing near $x=1$.

--> $f'(x)$ goes from - to 0 to + near $x=1$.

--> $f(x)$ has a minimum at $x=1$.

- SDT: $f''(0) = 0$ --> Min/max? Inflection point?

Is x=0 an inflection point of $f(x) = 3x^4 - 4x^3$?

(A) Yes because $f''(0) = 0$.

(B) Yes because $f''(0) = 0$ and $f'''(0) < 0$.

(C) No because $f''(-1) = 60$ and $f''(1) = 12$.

(D) Yes because $f''(-1) = 60$ and $f''(1/2) = -3$.

Note: $f'(x) = g(x) = 12x^3 - 12x^2$ from earlier and we agreed that g(x) had a max at x=0!

Is x=0 an inflection point of $f(x) = 3x^4 - 4x^3$?

(A) Yes because $f''(0) = 0$.

"SDT" applied to $f'(x)$.

(B) Yes because $f''(0) = 0$ and $f'''(0) < 0$.

Jumped over a zero of $f''(x)$

(C) No because $f''(-1) = 60$ and $f''(1) = 12$.

"FDT" applied to $f'(x)$.

(D) Yes because $f''(-1) = 60$ and $f''(1/2) = -3$.

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,\infty)$
$f''(x)$	+	0	-	0	+

$f'''(0)<0$