MATH 102 - MIDTERM TEST

University of British Columbia

Last name: First name:

Student number:

Section number:

Date: *October 18, 2016.*
Number of pages: *6 (including cover page and final blank page)*
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For marking purposes only		
Page	Mark	Possible marks
2		8
3		5
4		11
5		9
Total		33

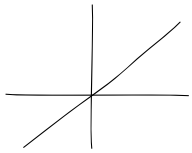
Multiple choice questions

Circle the letter corresponding to your choice.

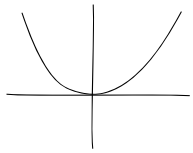
1. (2 pts) Which of the following represents the graph of a good approximation to $g(x)$ near the origin where

$$g(x) = \frac{x^3 + 3x^2}{3 + 2x^3}?$$

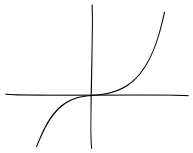
(a)



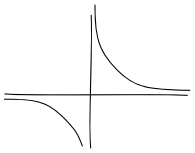
(b)



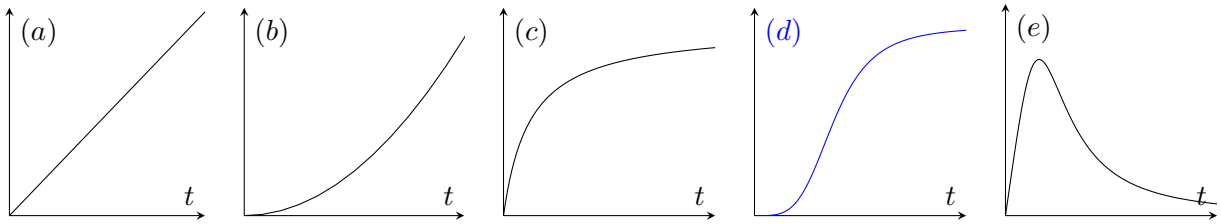
(c)



(d)



2. (2 pts) Wild pigs dig in the dirt to find insects to eat. When a pig starts digging in a new area, it has to spend some time digging before it finds a good spot. For a while after that, finding insects becomes easier. Eventually, it becomes more difficult as the pig depletes the area of insects. Which of the following graphs represents the amount of food found as a function of time spent digging in a particular area?



3. (2 pts) Which of the following functions would you use to approximate $\sqrt{\frac{2}{3}}$ if you were using Newton's method to do the calculation?

   (a) $f(x) = \sqrt{x}$.

   (b) $f(x) = 3\sqrt{x} - 2$.

   (c) $f(x) = 3x^2 - 2$.

   (d) $f(x) = x - \sqrt{\frac{2}{3}}$. ← This has a zero at the correct location but you need to already know $\sqrt{\frac{2}{3}}$ to use Newton's method with this one.

4. (2 pts) Let $f(x)$ be a continuous function at $x = 0$. Which of the following statements is correct? Recall that if there exists any function for which a statement is false then the statement is false.

   (a) If $f'(0)$ exists and $f'(0) = 0$ then $x = 0$ is an extremum.

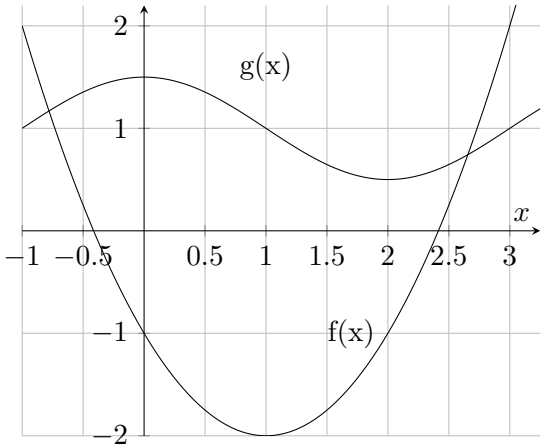
   (b) If $x = 0$ is an extremum then $f'(0)$ exists and $f'(0) = 0$.

   (c) If $f'(x)$ exists for all x and $f'(0) = 0$, $f'(1) > 0$, and $f'(-1) < 0$ then $x = 0$ is a minimum.

   (d) If $f'(0)$ and $f''(0)$ exist and $f'(0) = f''(0) = 0$ then $x = 0$ is not an extremum.

   (e) None of the above are true.

5. (2 pts) Consider the function $h(x) = f(g(x))$ where the graphs of f and g are shown in the figure below.



Circle the letter next to the true statement.

(a) $h'(0.5) > 0$.

(b) $h'(1.5) > 0$.

(c) $h'(2.5) = 0$.

(d) None of the above.

Written-answer problems

6. (3 pts) Calculate the limit (where $x_0 \neq 0$)

$$\lim_{x_1 \to x_0} \frac{\frac{1}{x_1} - \frac{1}{x_0}}{x_1 - x_0}.$$

$$\begin{aligned}
&= \lim_{x_1 \to x_0} \frac{\frac{x_0 - x_1}{x_1 x_0}}{x_1 - x_0} && 1 \text{ pt} \\
&= \lim_{x_1 \to x_0} \frac{x_0 - x_1}{x_1 x_0} \frac{1}{x_1 - x_0} \\
&= \lim_{x_1 \to x_0} \frac{-1}{x_1 x_0} && 1 \text{ pt} \\
&= \frac{-1}{x_0^2} && 1 \text{ pt}
\end{aligned}$$

Limit: $\boxed{-\frac{1}{x_0^2}}$

7. (6 pts) Give the equation of a tangent line to the function $f(x) = x^3 - 2x$ that is parallel to the line $y = x + 4$. How many such tangent lines are there?

Find a point at which $f'(x) = 1$:

$$f'(x) = 3x^2 - 2 \qquad 1 \text{ pt}$$
$$3x^2 - 2 = 1 \qquad 1 \text{ pt}$$
$$3x^2 = 3$$
$$x = \pm 1 \qquad 1 \text{ pt}$$

This means there are two such tangent lines (1 pt). I'll find the one at $x = 1$.

$$f(1) = -1$$
$$f'(1) = 1$$

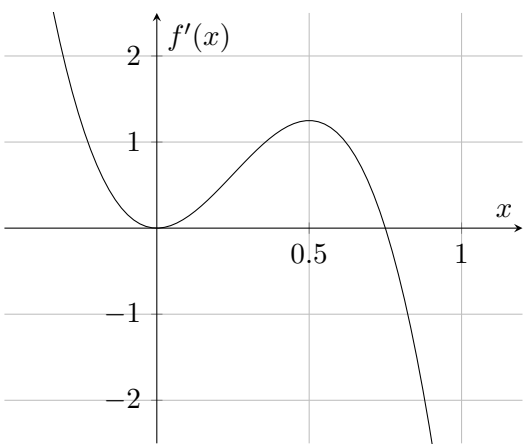
So

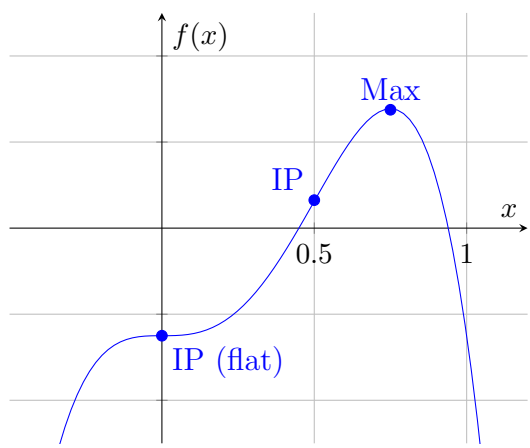
$$y = f(1) + f'(1)(x - 1) \qquad 1 \text{ pt}$$
$$= -1 + 1 \cdot (x - 1) = x - 2. \qquad 1 \text{ pt}$$

Tangent line: $y =$ y=x-2

There are 2 such tangent lines.

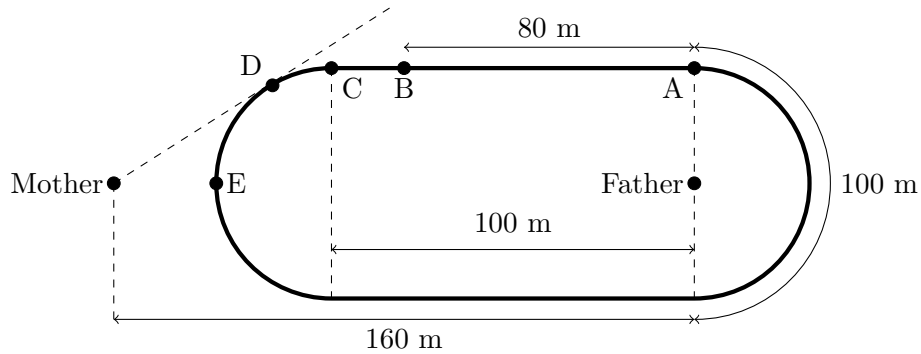
8. (5 pts) Use the axes on the right below to sketch the graph of an antiderivative of the function shown on the left below. You do not need to include a vertical scale on the axes. Label all extrema and inflection points with "min", "max" or "IP".





Points for increasing below $x = 0.75$, decreasing above $x = 0.75$ with max at $x = 0.75$, a horizontal IP at $x = 0$, an IP at $x = 0.5$. Up to 3 pts points were taken off for missing labels on the Max and IP, depending on how clear those features were.

9. (9 pts) Lucas is running a race on a track with two straight sections linked by two semi-circular sections. He runs at a constant rate of 2 m/s, starting at A and going counter-clockwise. His father stands at the centre of one of the semi-circular sections and his mother stands on the far side of the other semi-circular section as shown in the diagram below. Let $P(t)$ be the distance from Lucas to the nearest parent.



(a) As Lucas runs through each of the intervals between the labeled points listed below, determine whether $P(t)$ is increasing or decreasing, concave up or concave down. "Neither" is also a possibility.

Interval	inc./dec.	concave up/down
AB	inc. (1 pt)	CU (1 pt)
BC	dec. (1 pt)	CU (1 pt)
CD	dec. (1 pt)	CD (1 pt)
DE	dec. (1 pt)	CU (1 pt)

(b) Is $P(t)$ differentiable? If not, at which of the labeled points is it not differentiable?
$P(t)$ is differentiable at all points between A and E except at B. (1 pt)

Note that the marking scheme for this problem was altered slightly, from 10 pts to 9 pts. As a result, the midterm was out of 33 instead of 34 points.

This page may be used for rough work. It will not be marked.