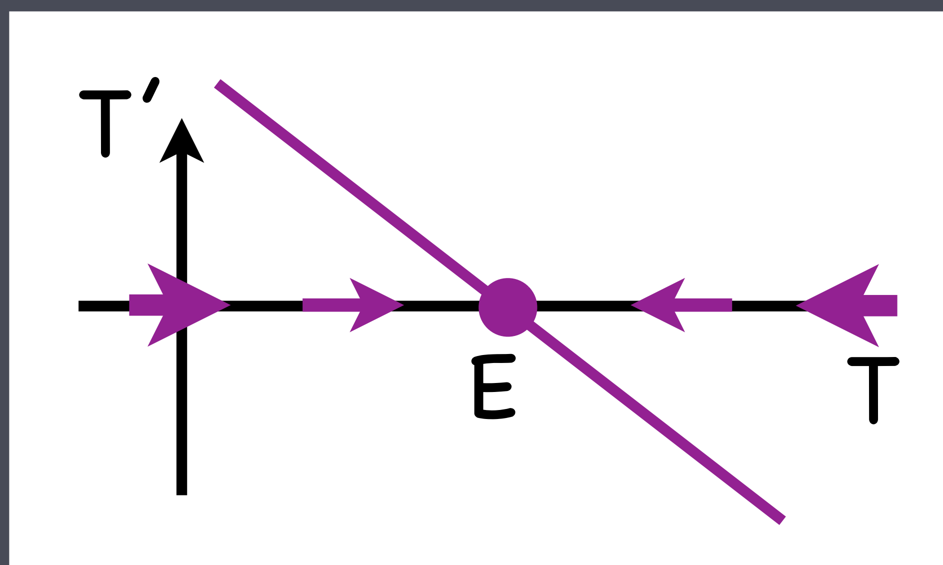
Today

- Qualitative analysis of DEs continued.
  - Drawing the phase line.
  - Determining long term behaviour.
  - Sketching solutions from the phase line.

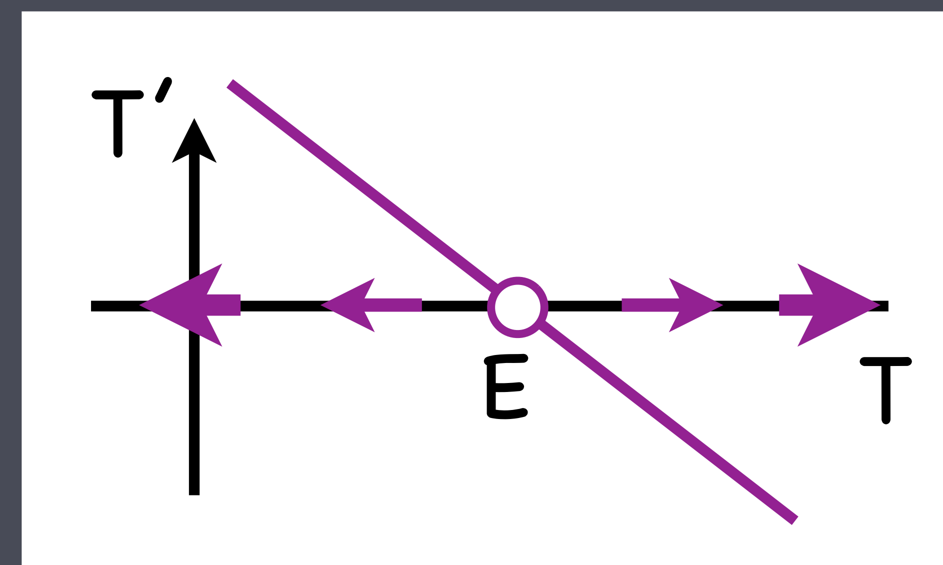
Phase line for NLC:

$$\frac{dT}{dt} = k(E - T)$$

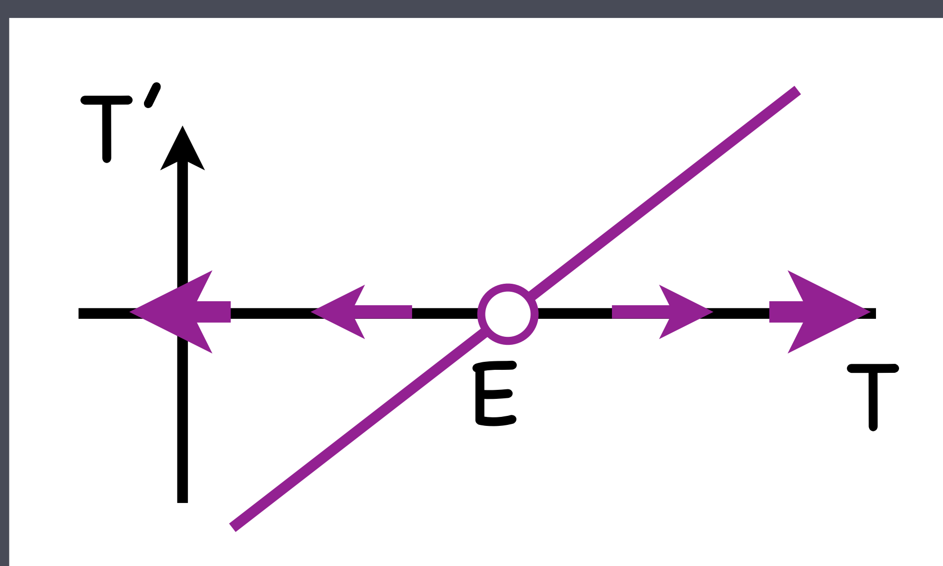
(A)



(B)



(C)



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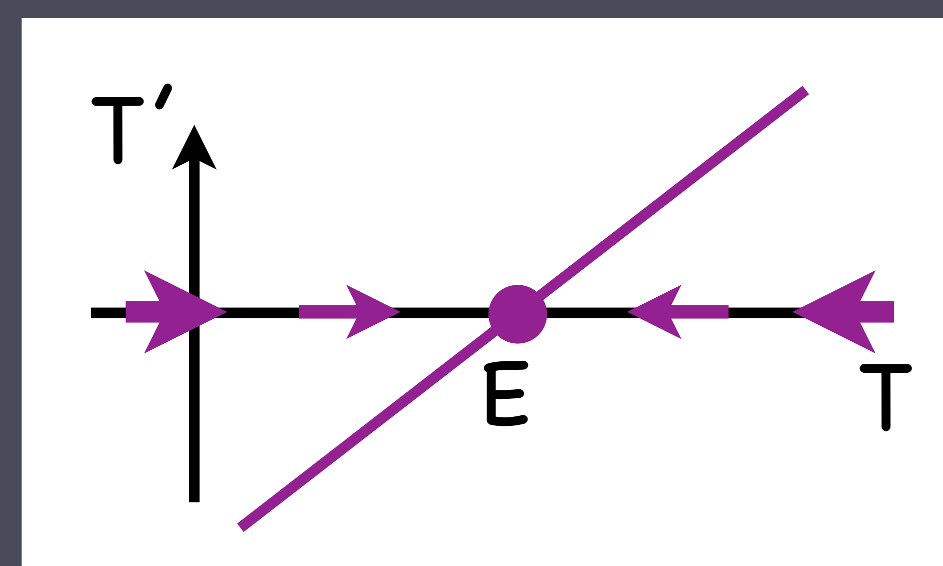

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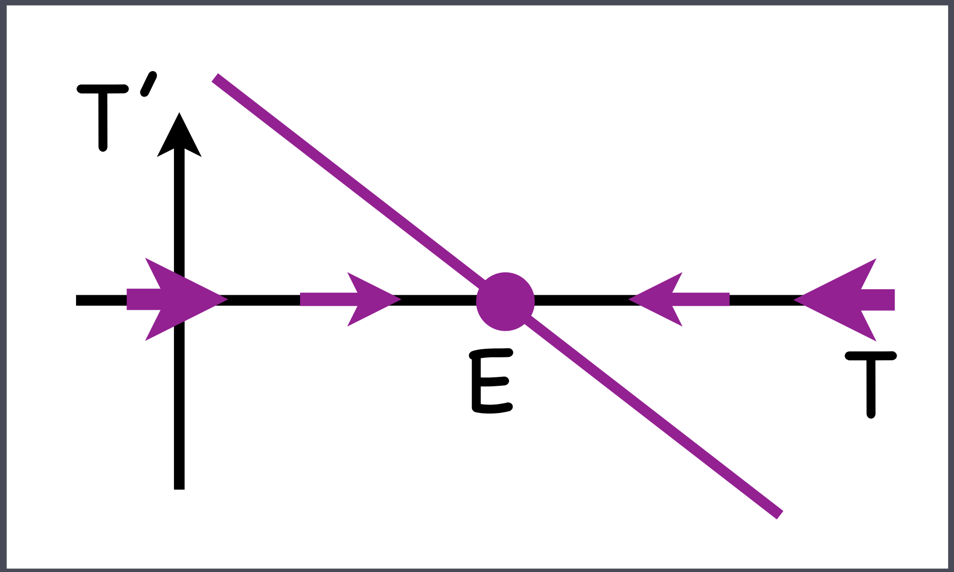

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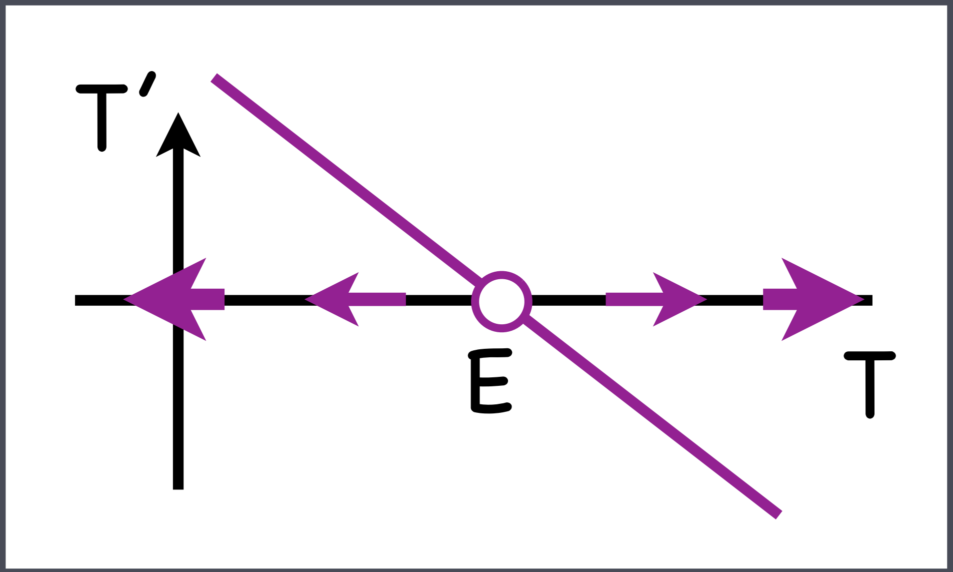

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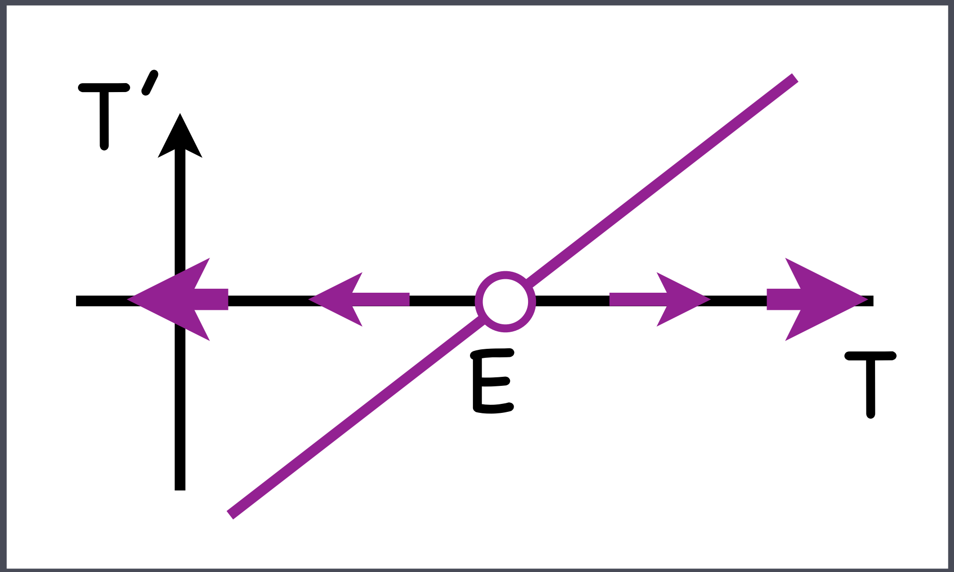

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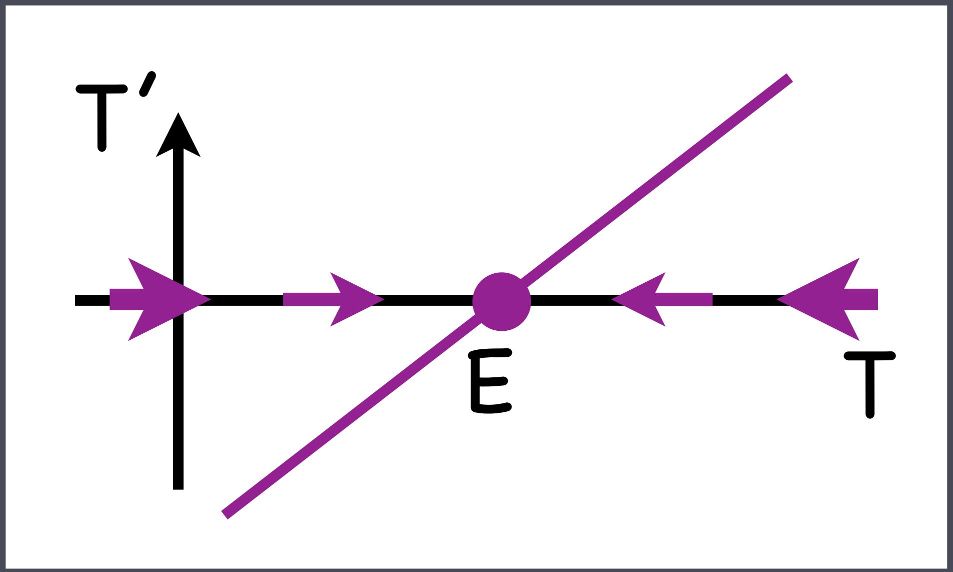

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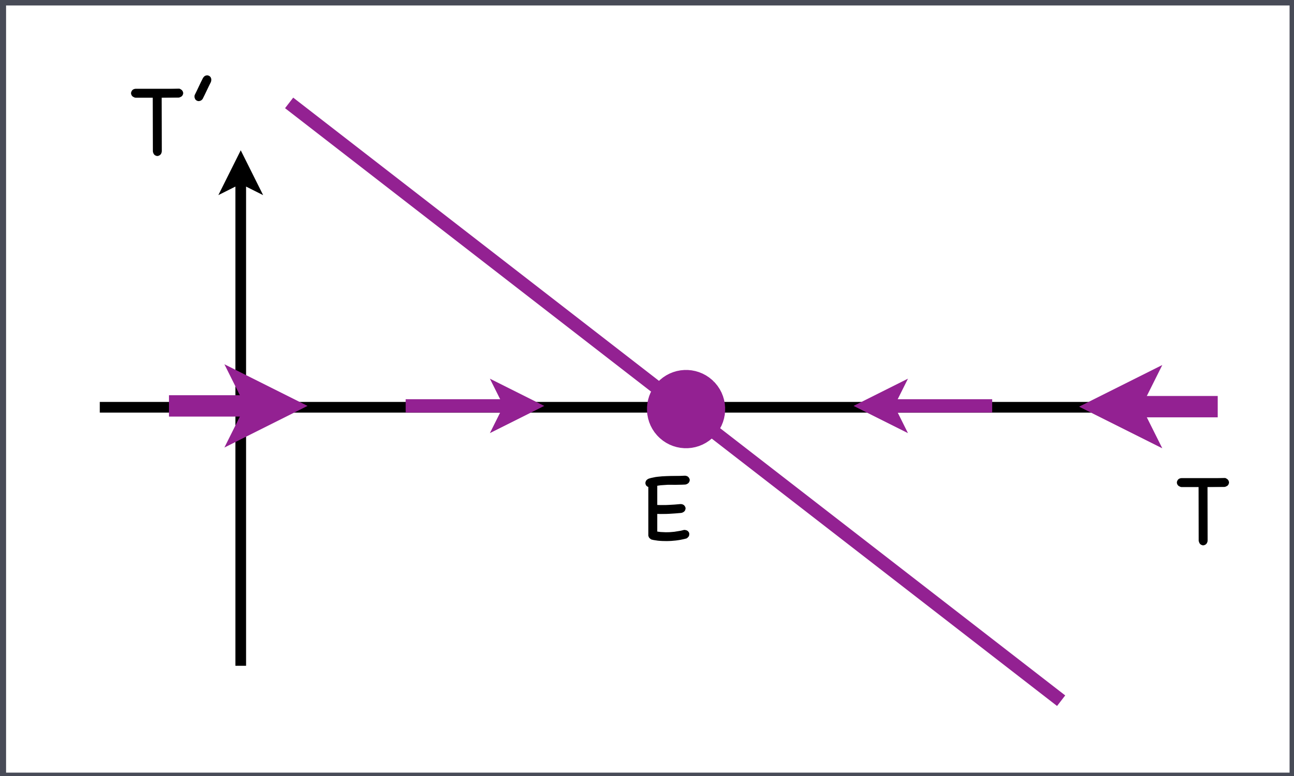

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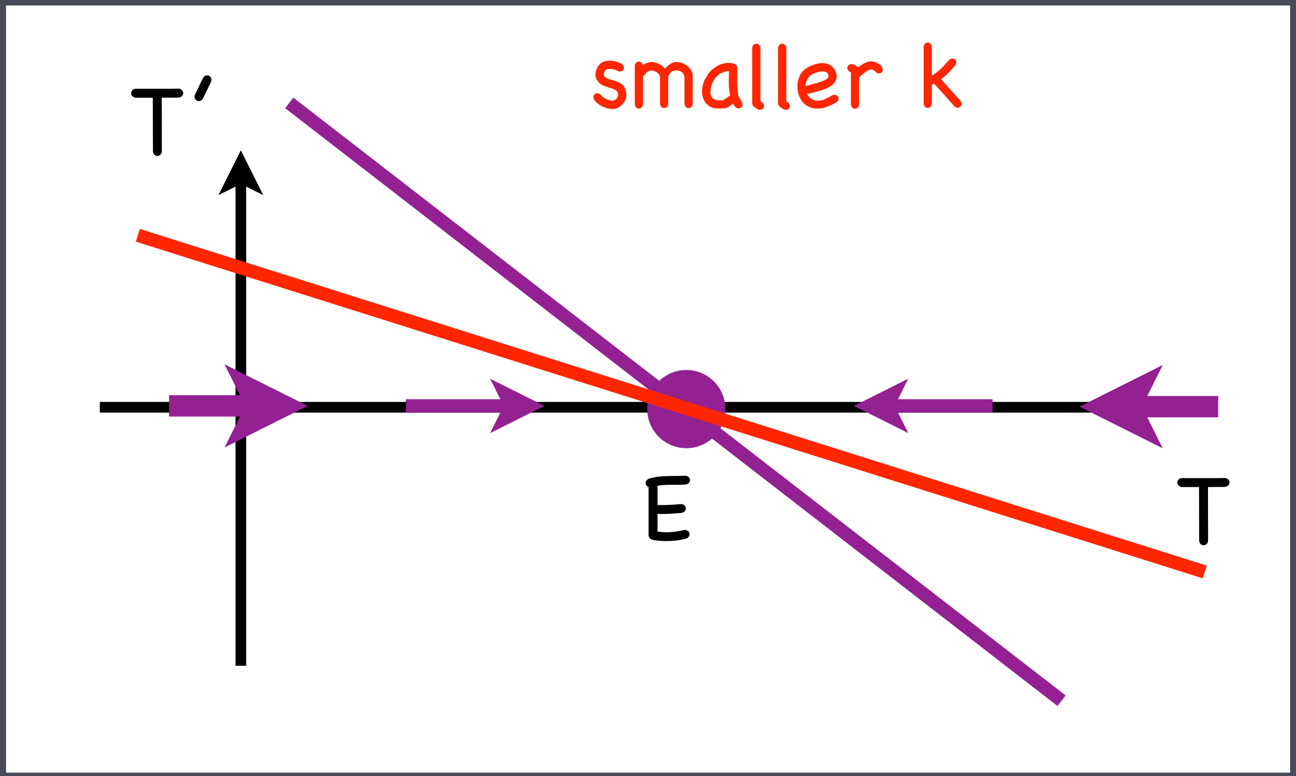

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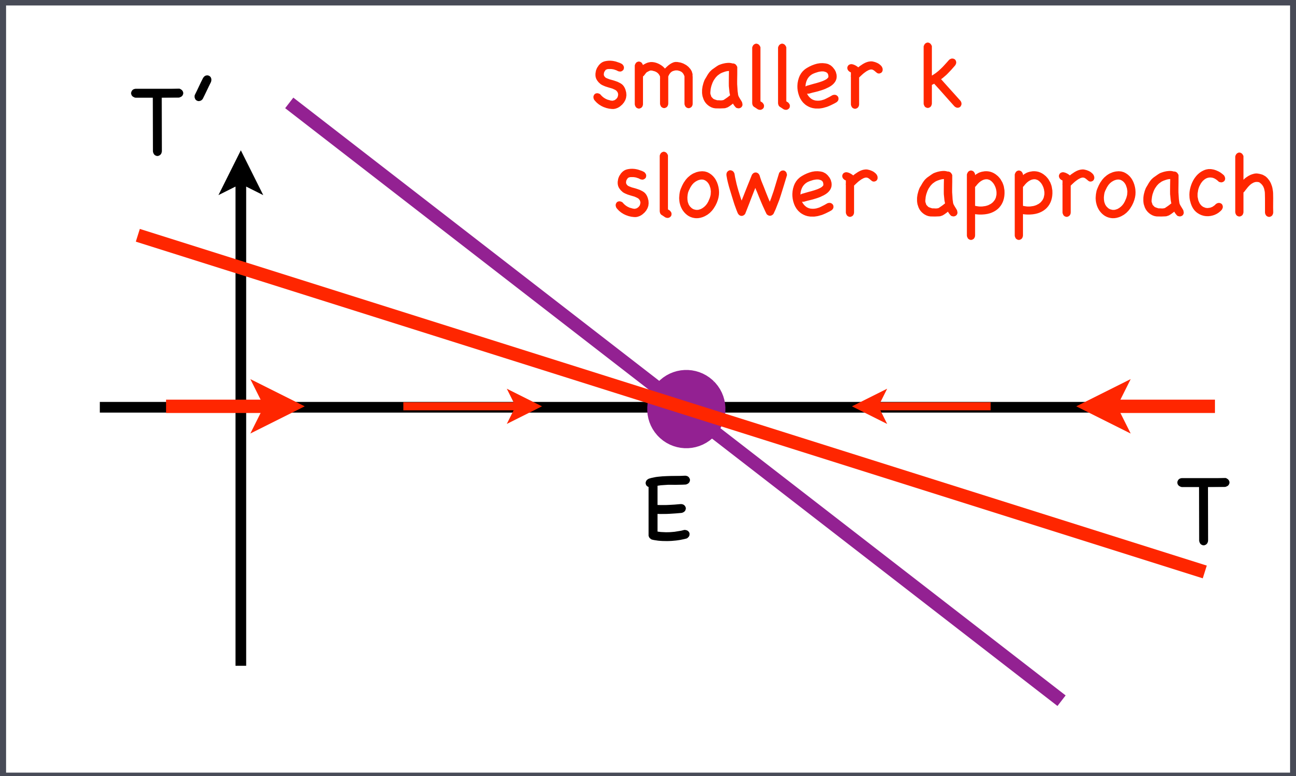

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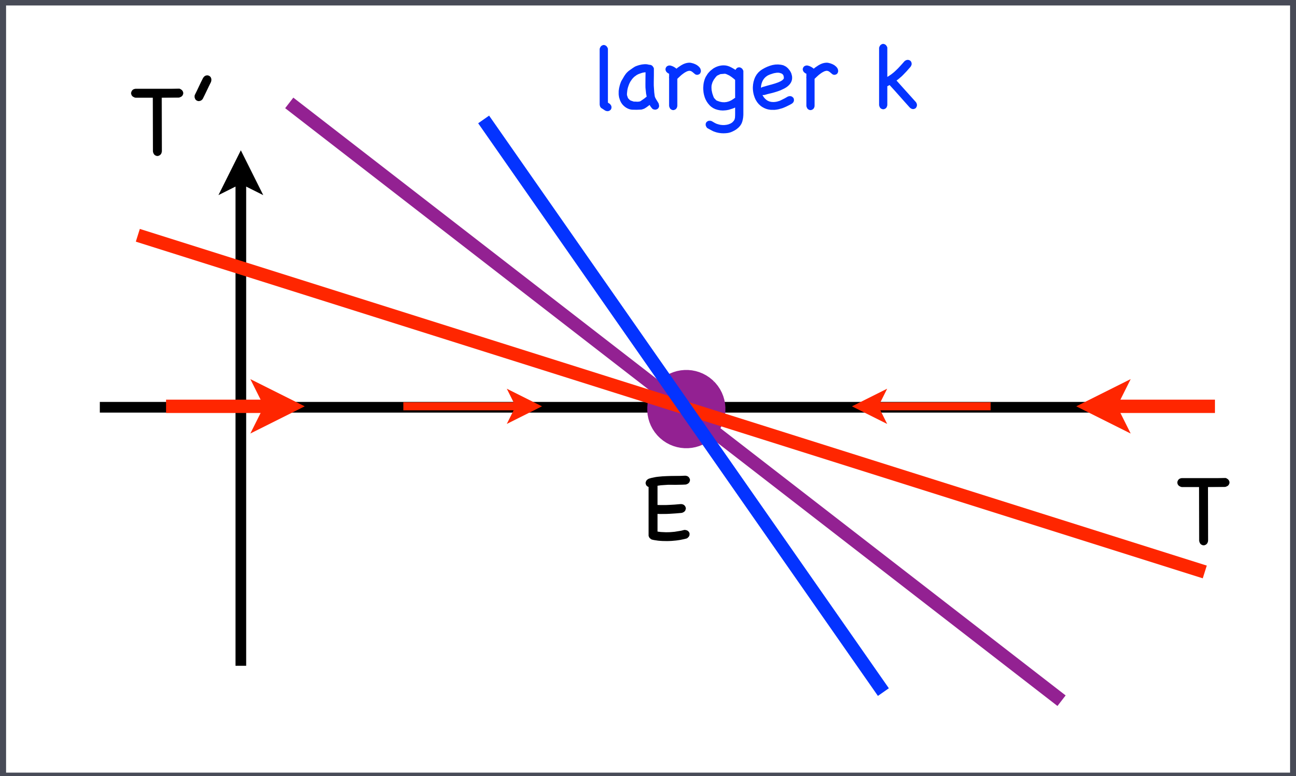

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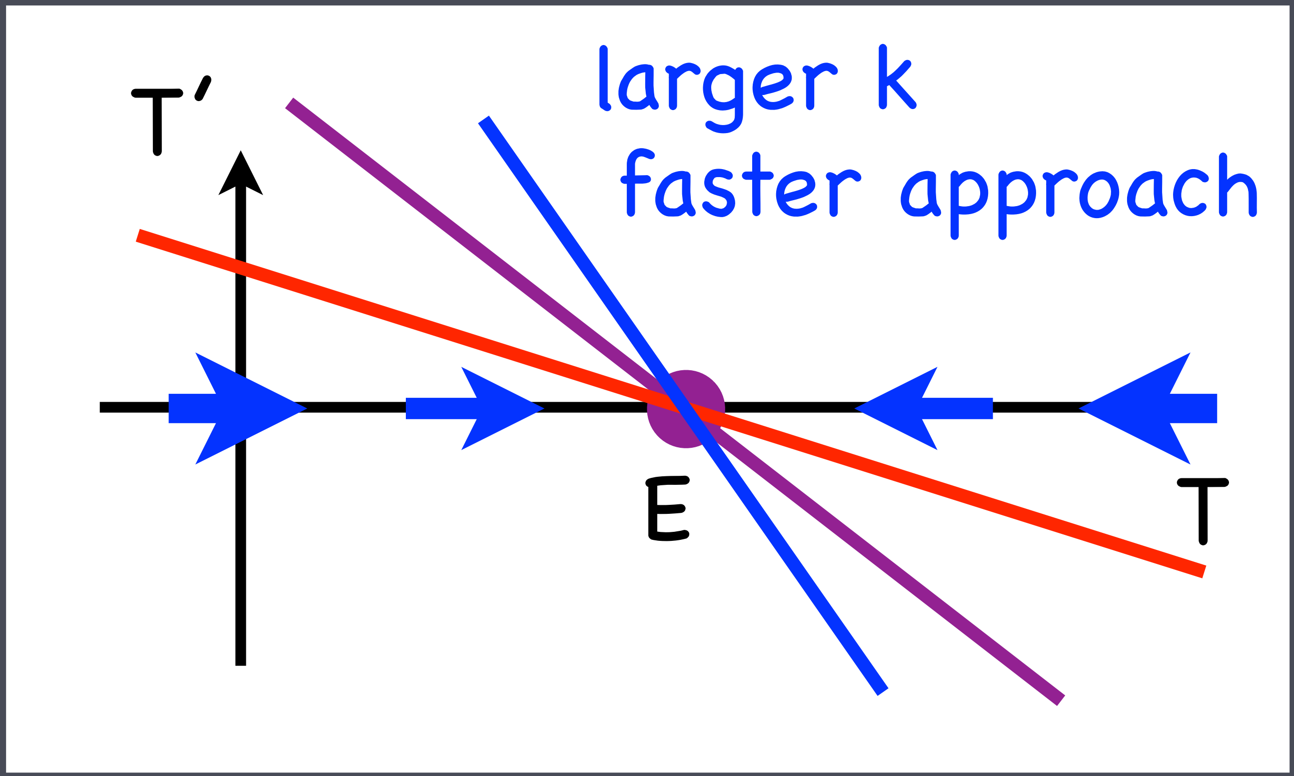

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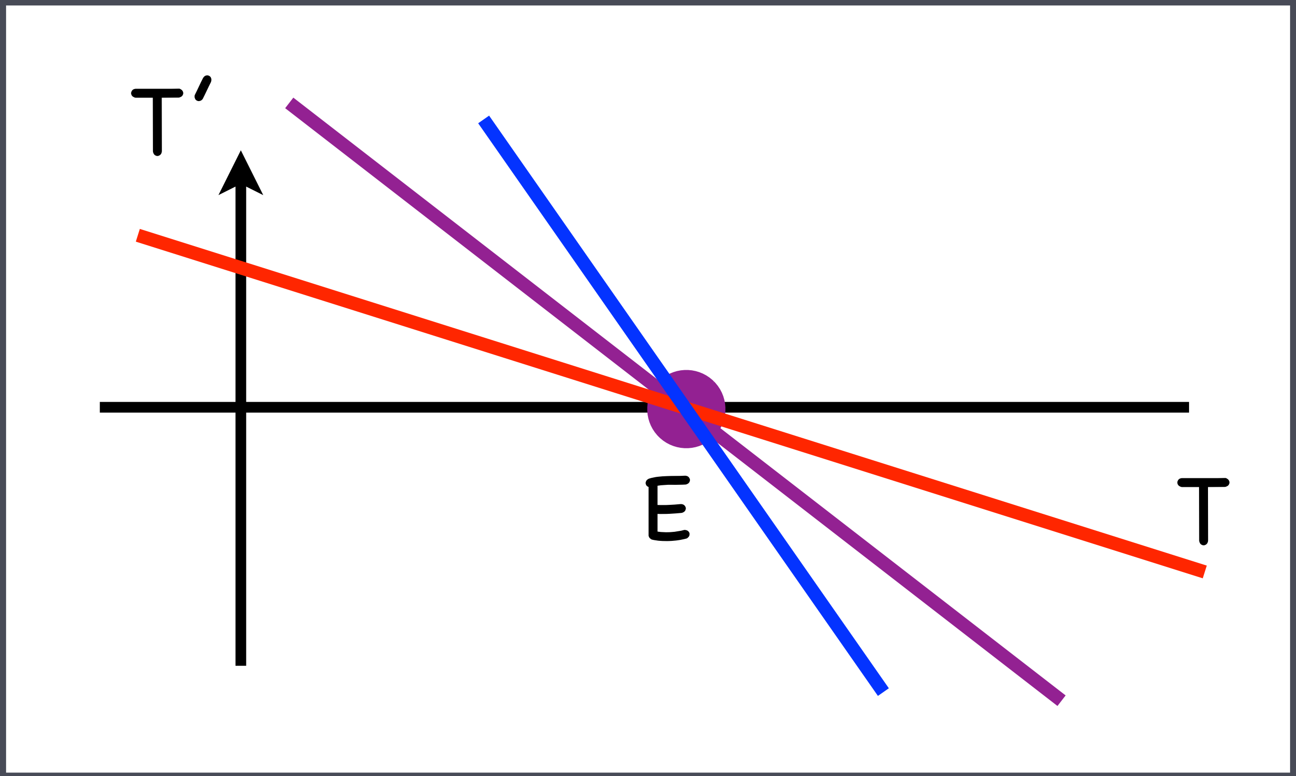

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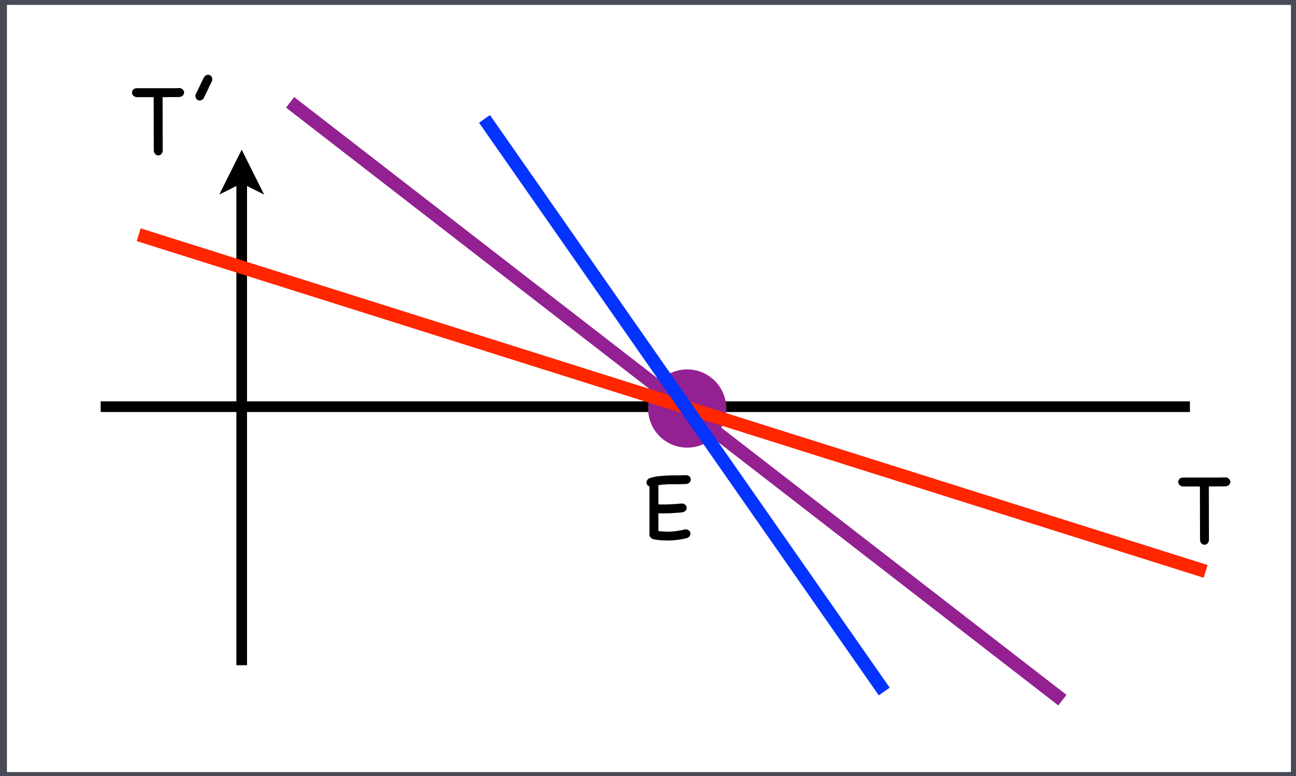

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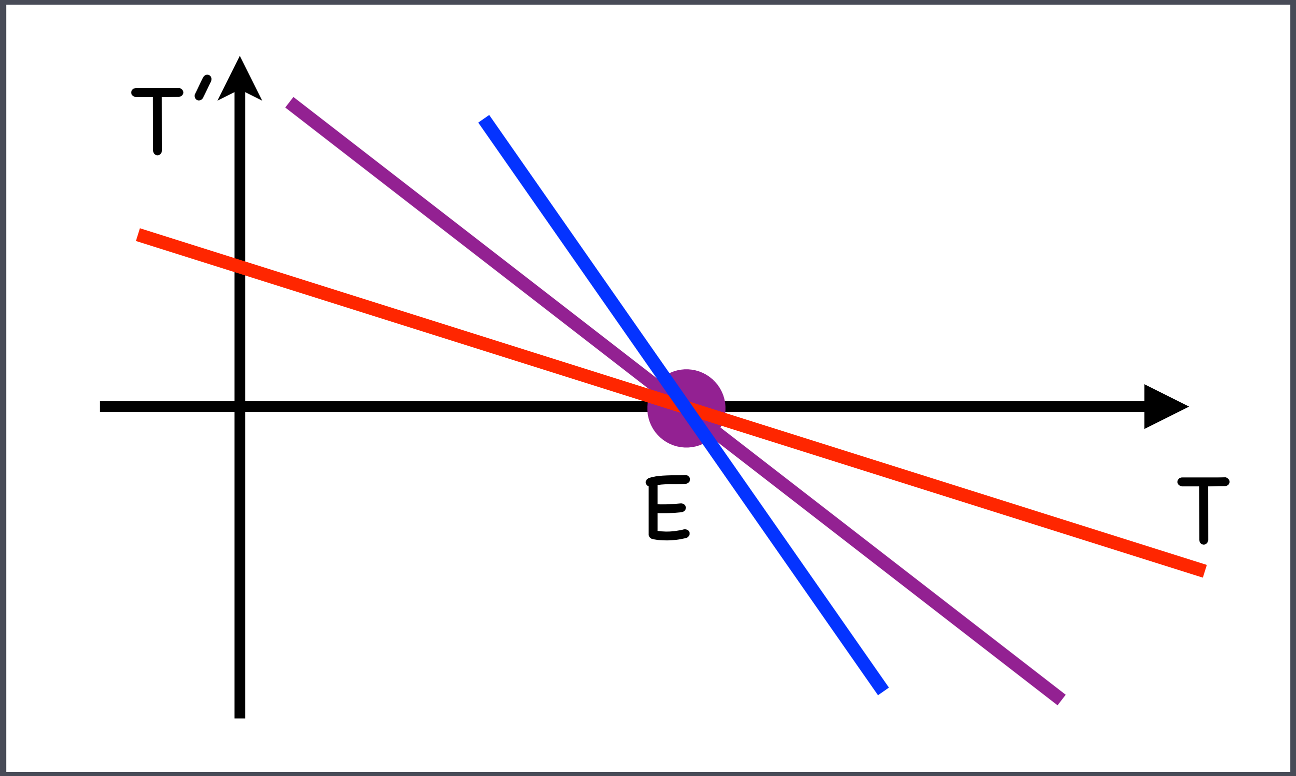

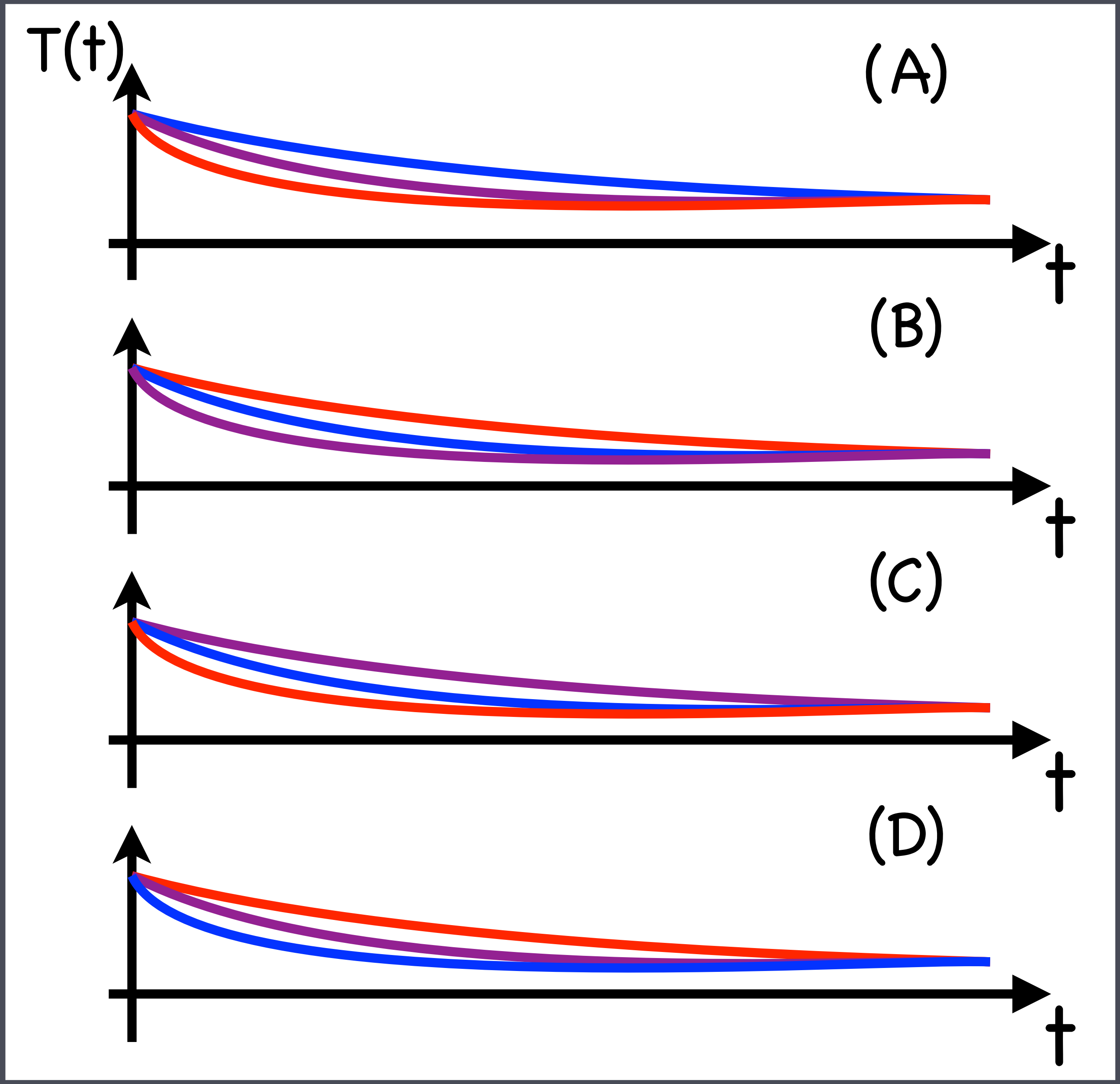

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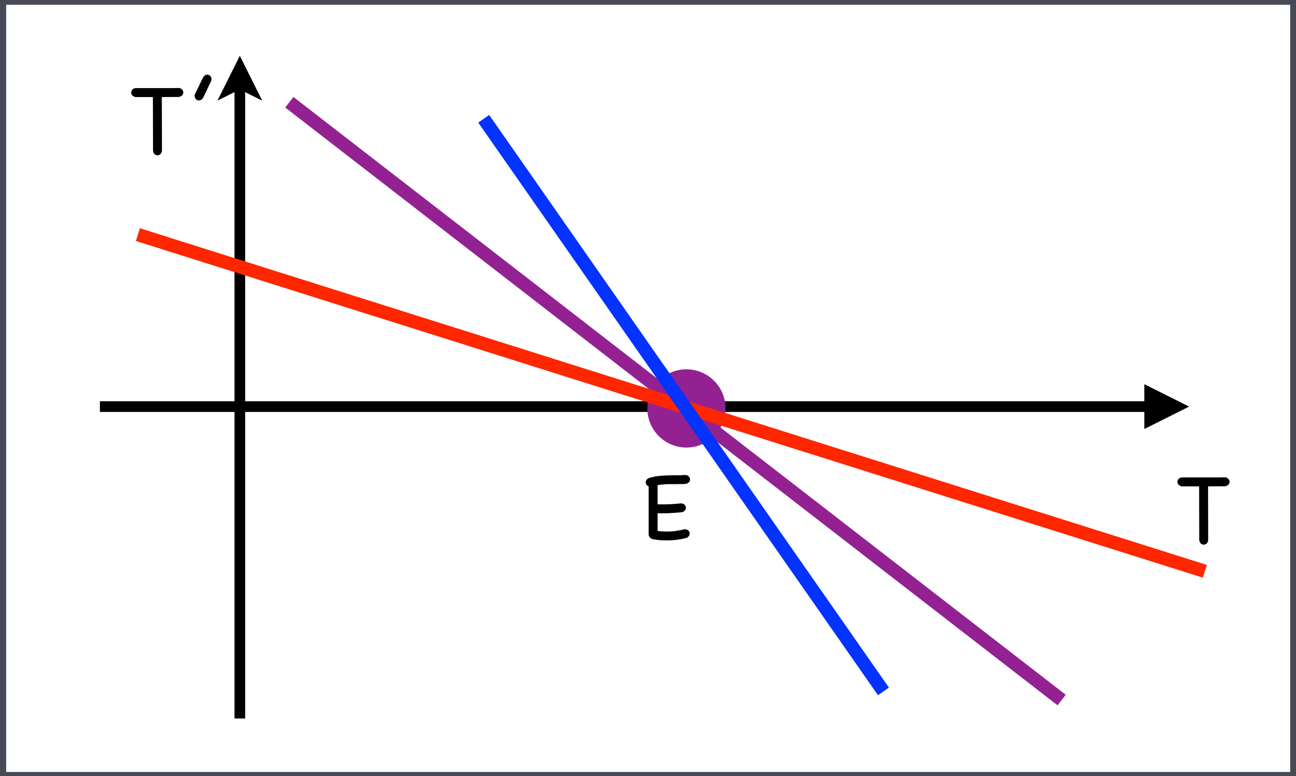

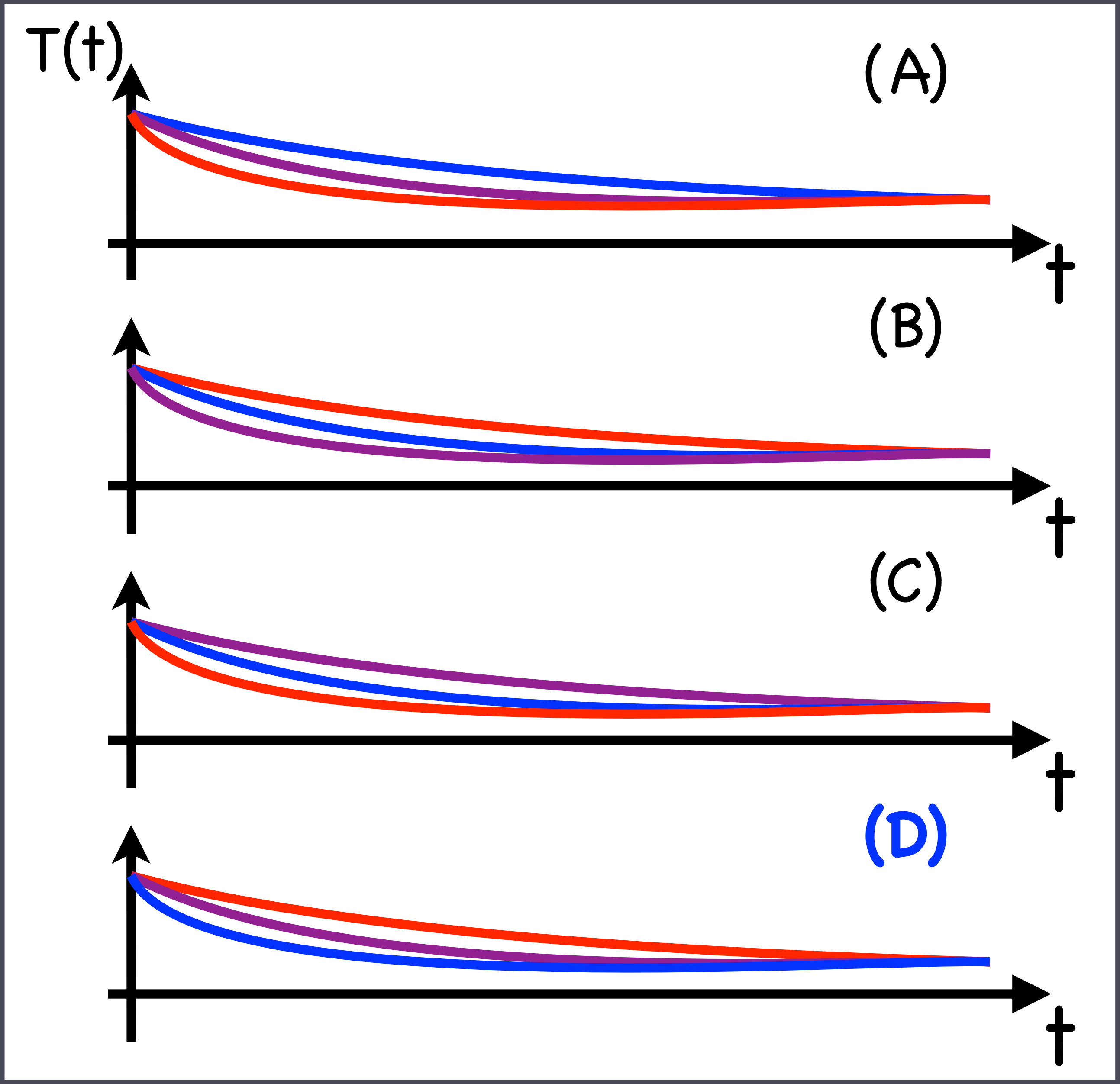

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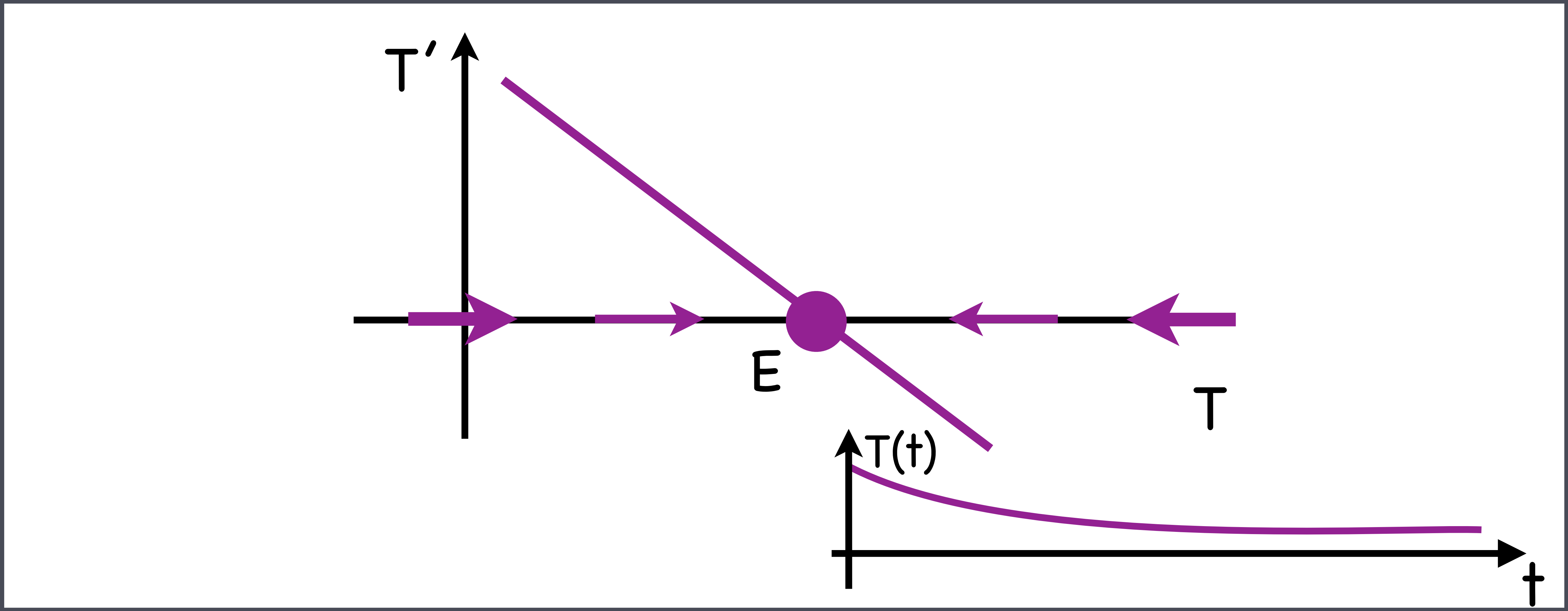

Notice that the arrows are always the same for any E, just shifted left or right.

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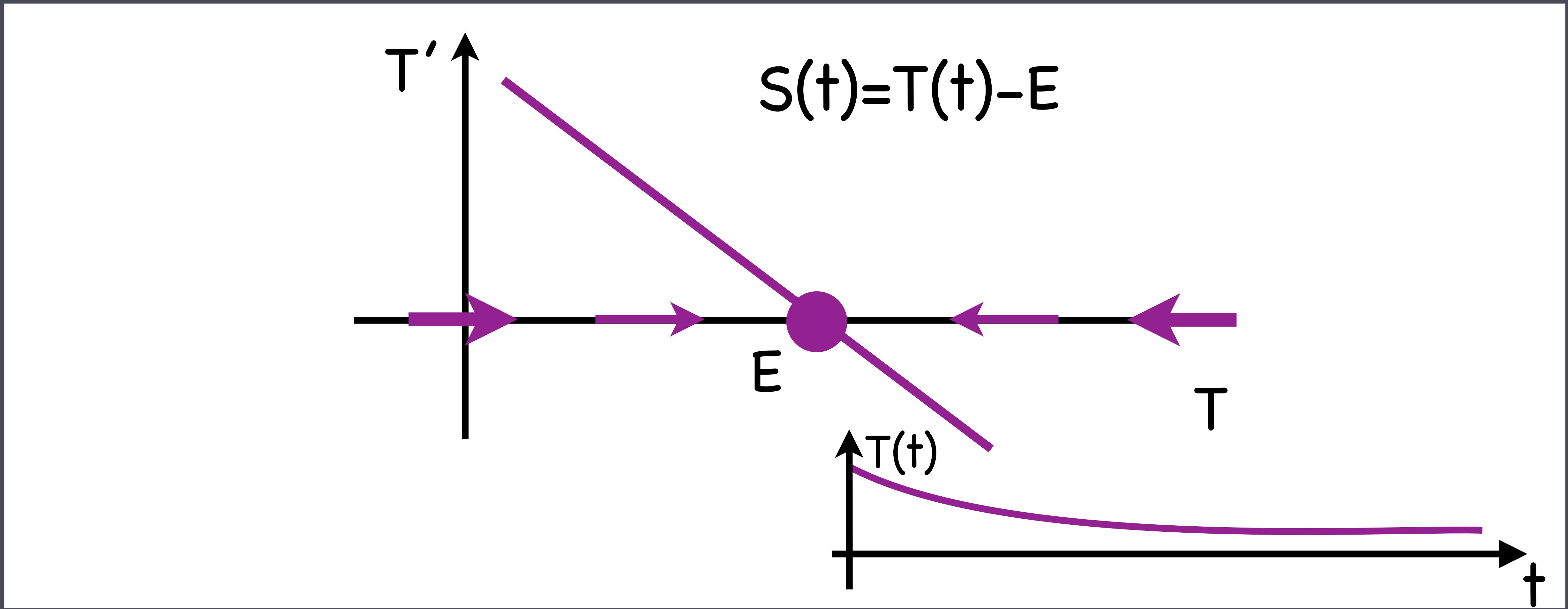

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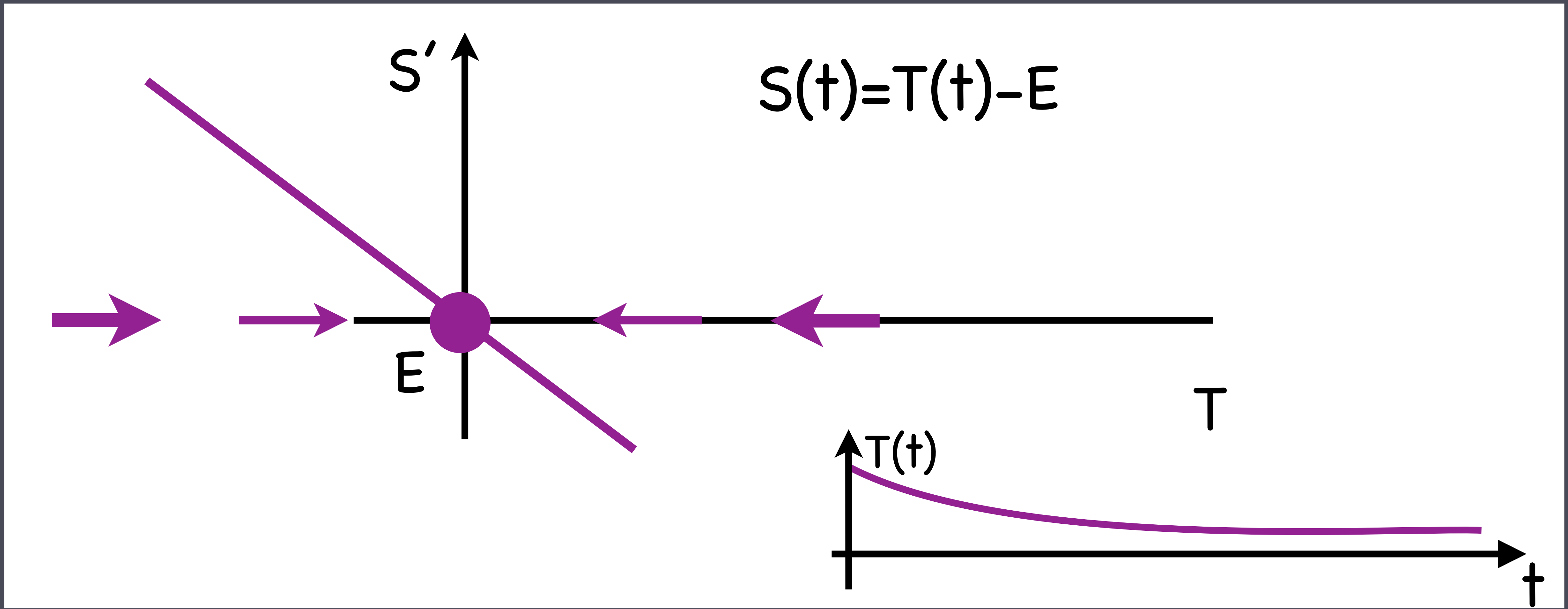

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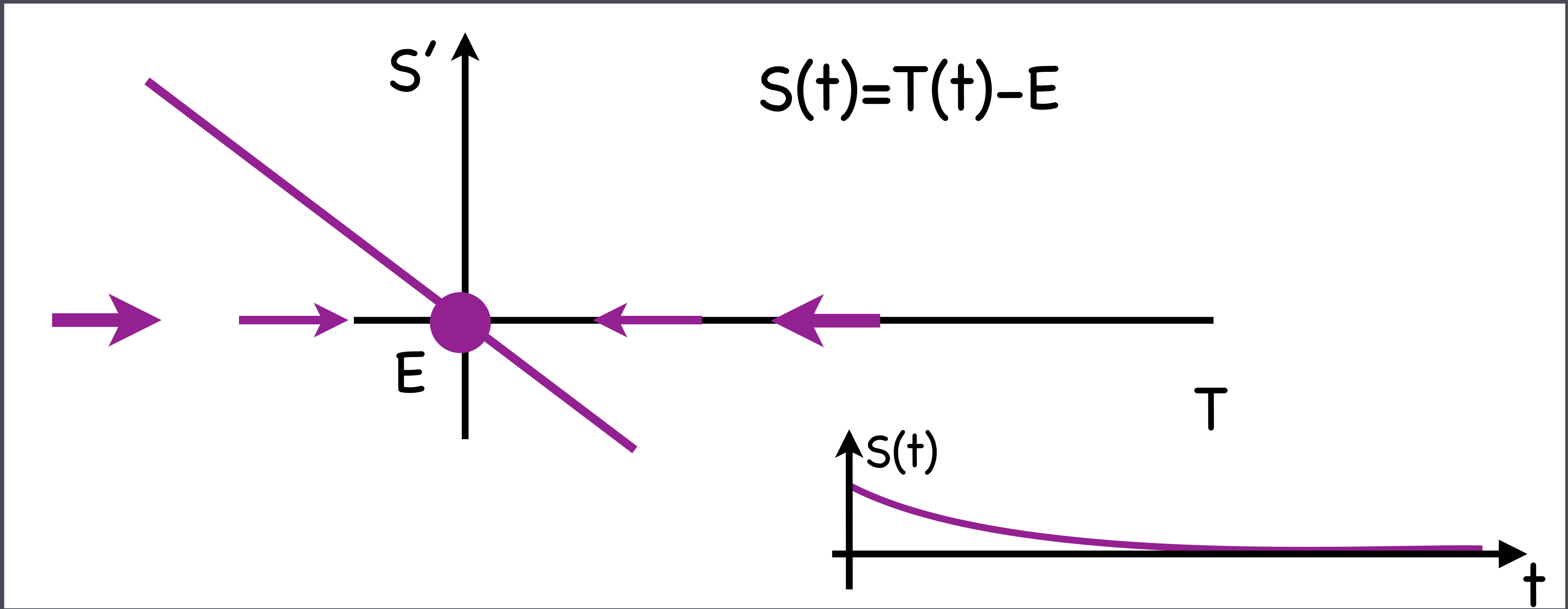

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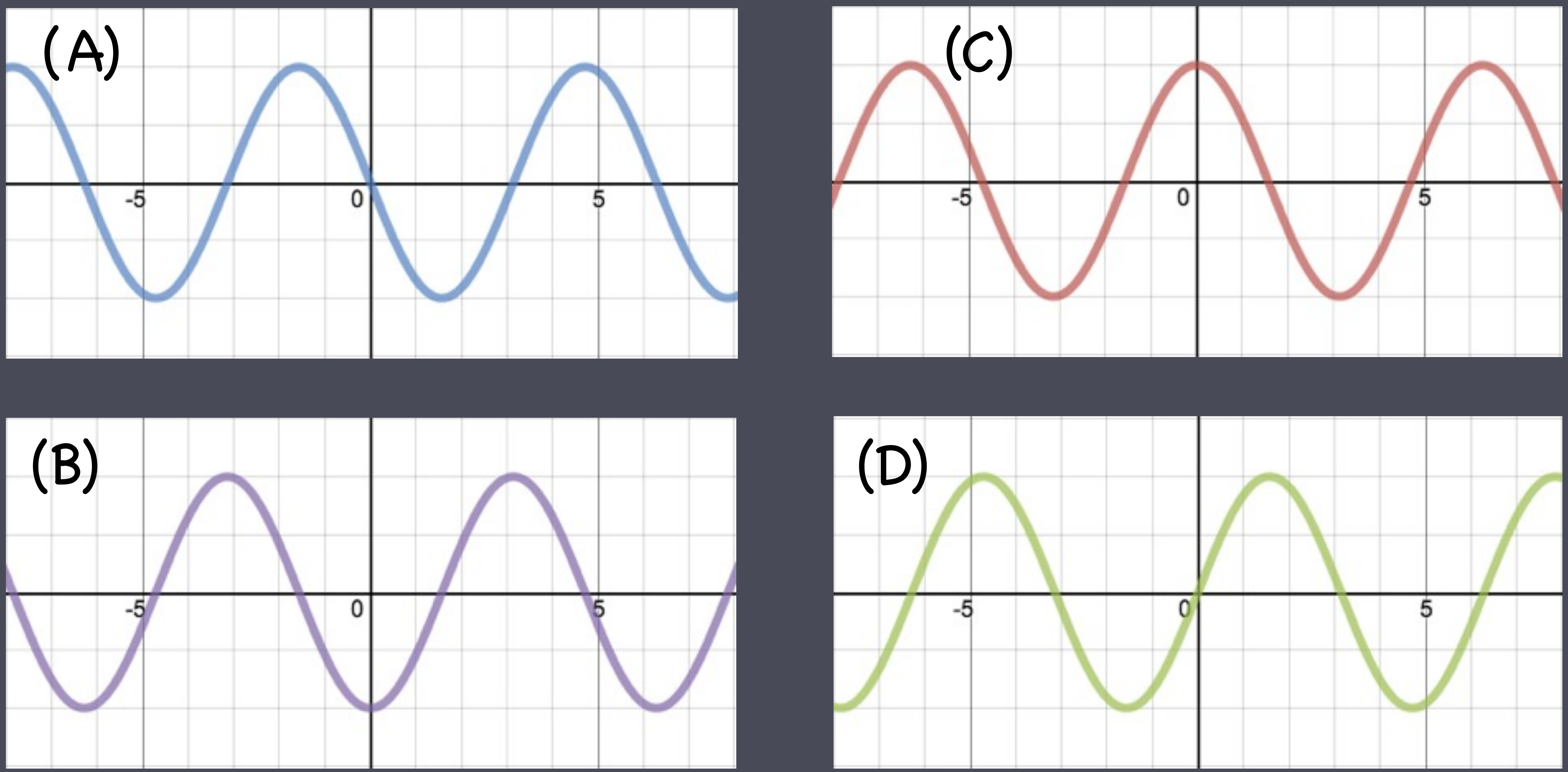
$f(y) = \cos(y)$



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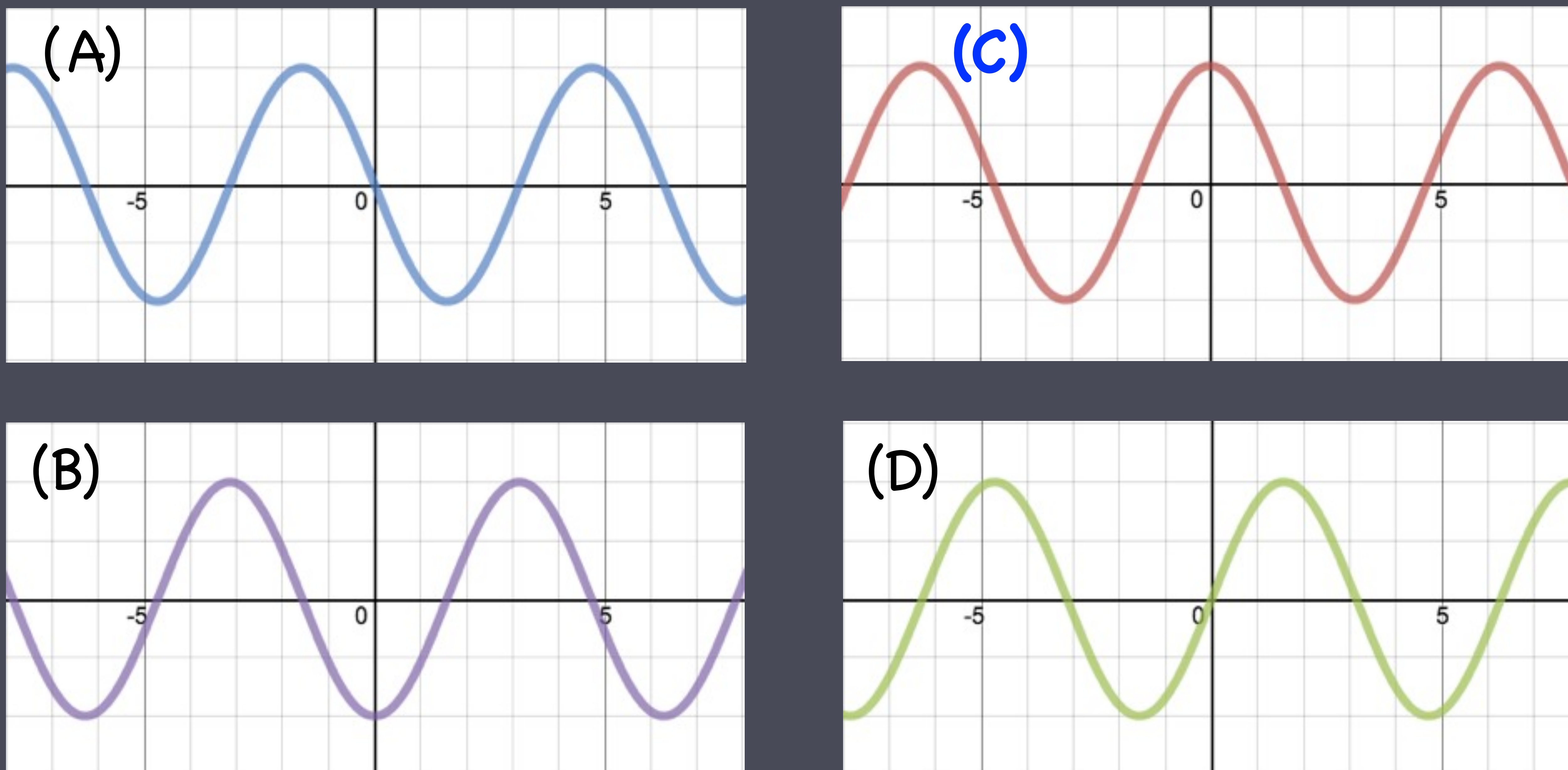
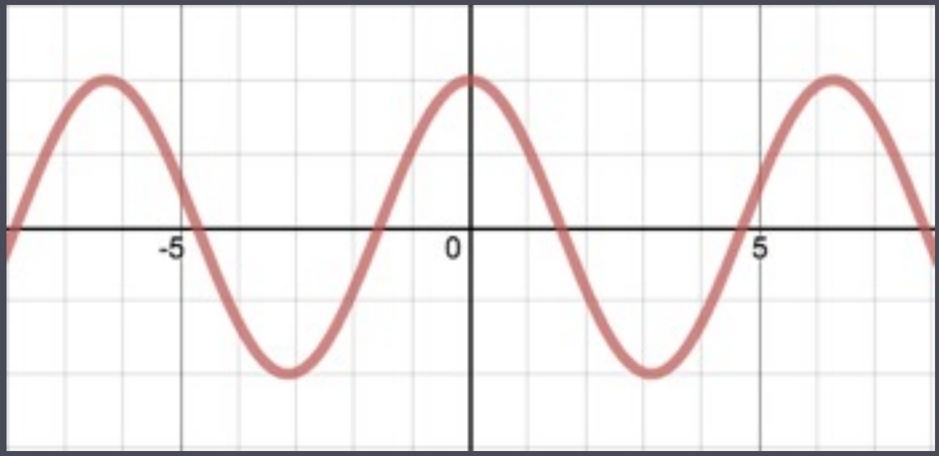


$y' = \cos(y)$



- A solution satisfying the initial condition $y(0)=y_0$ will approach y^* as $t \to \infty$. Which y_0 and y^* pair is correct?

(A) $y_0 = 0$, $y^* = \pi$.

(B) $y_0 = -\pi$, $y^* = -\pi/2$.

(C) $y_0 = 2\pi$, $y^* = 3\pi/2$.

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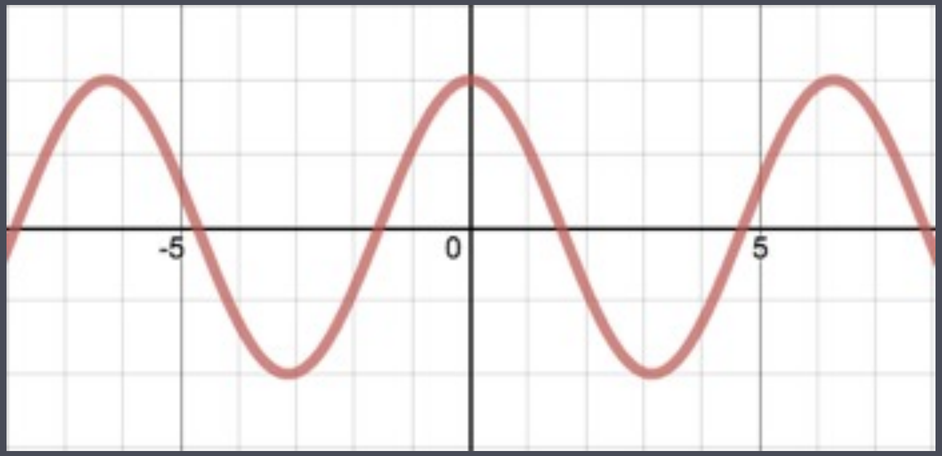

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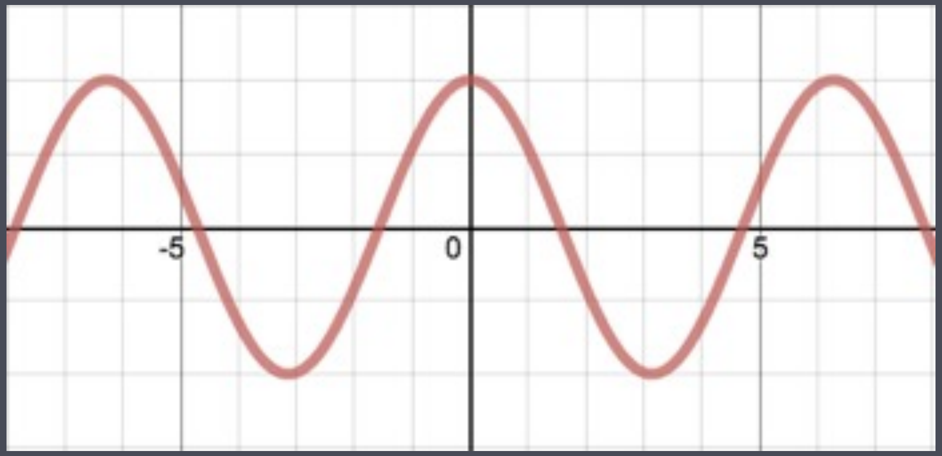

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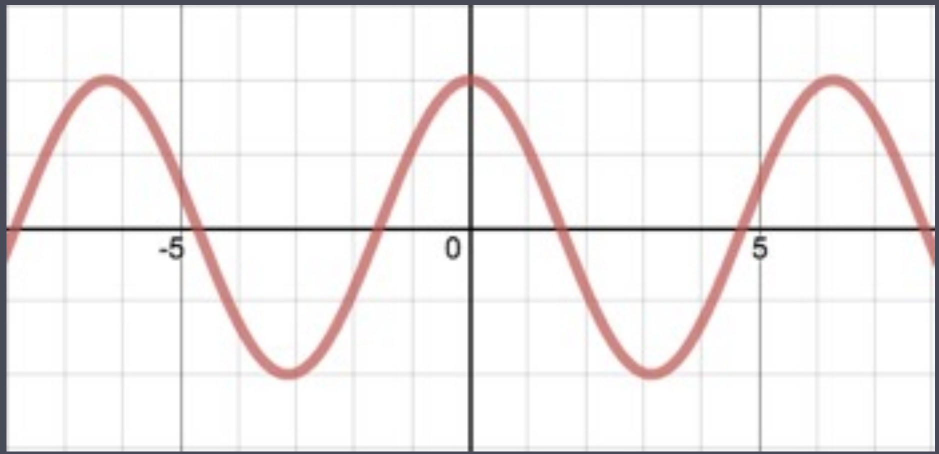

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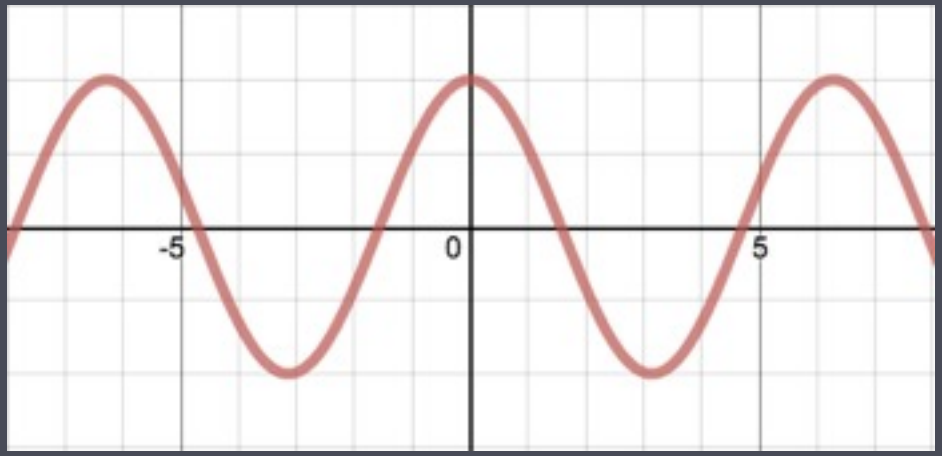

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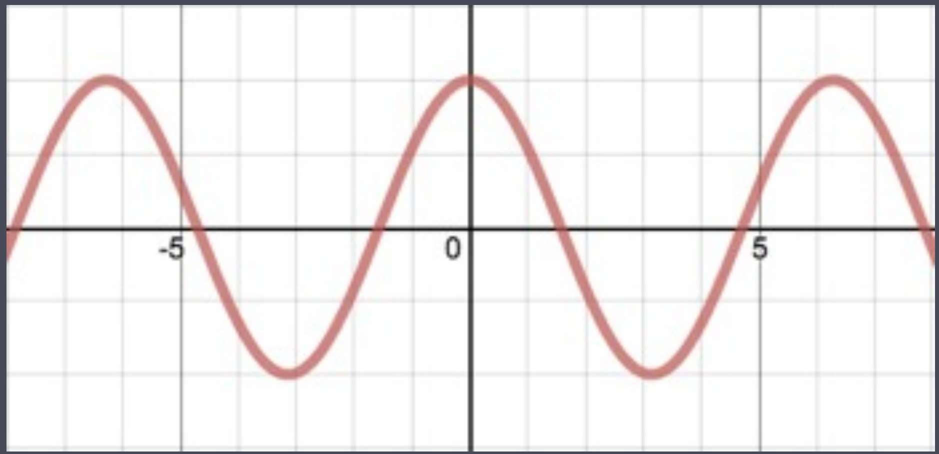

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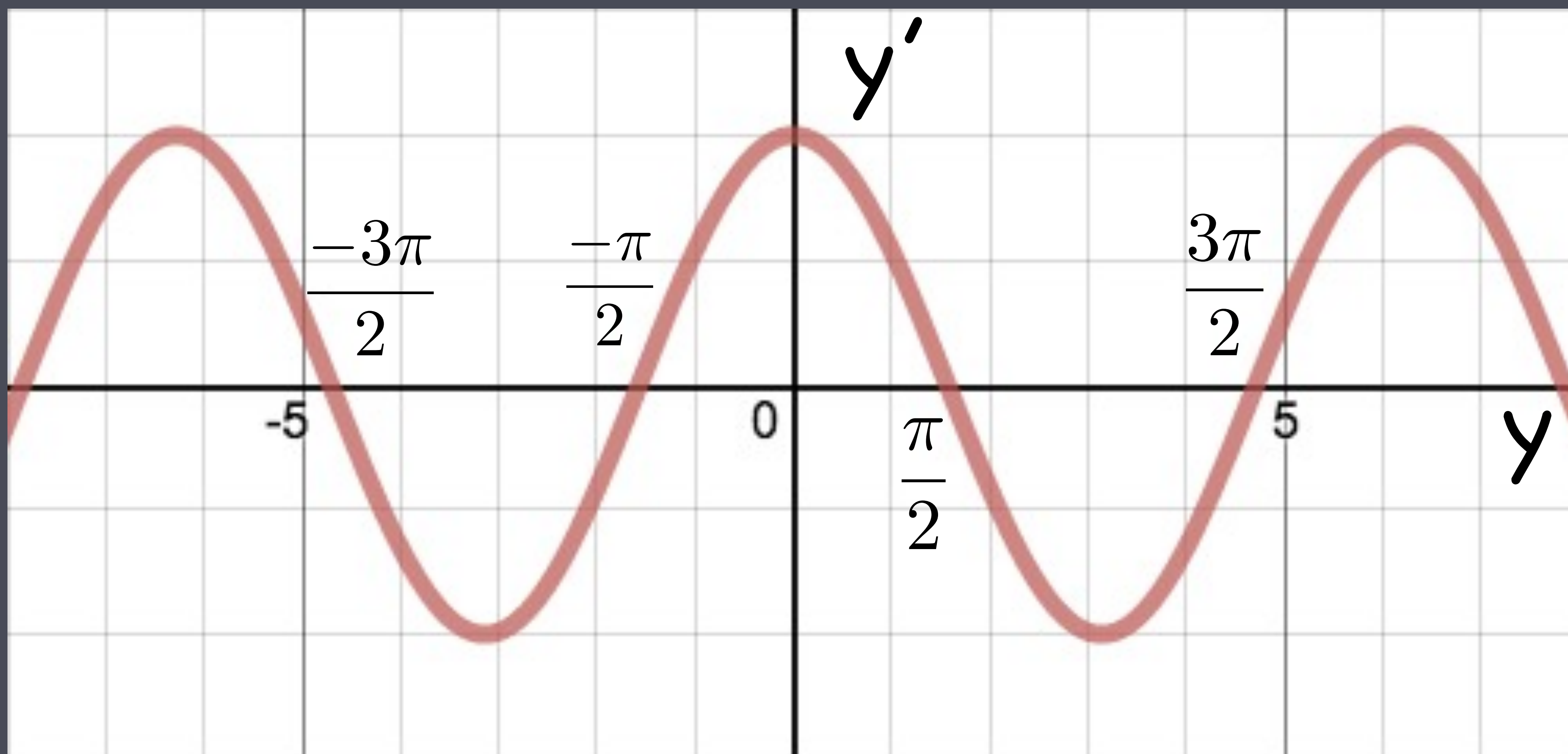
Fill in the arrows and steady states on the phase line.



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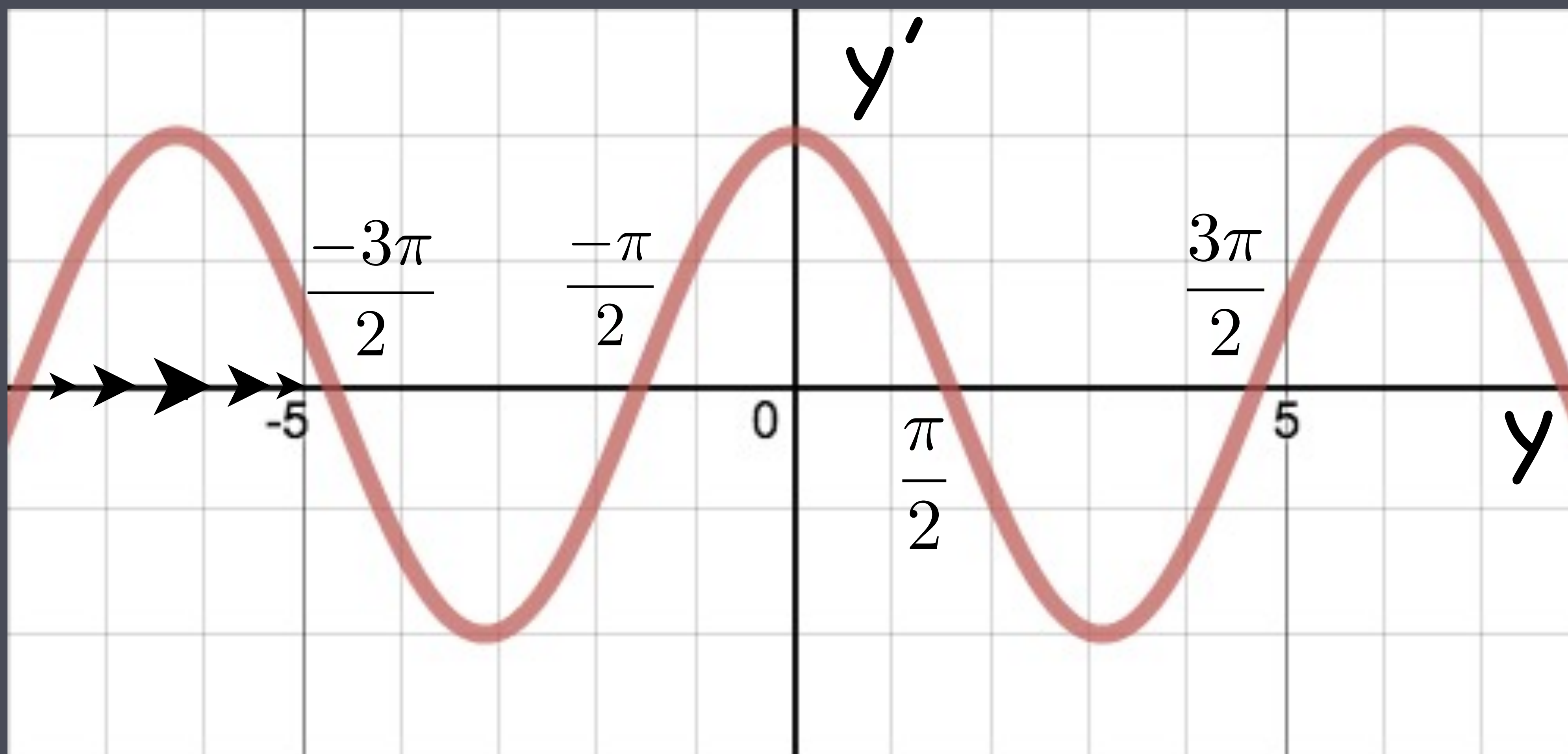

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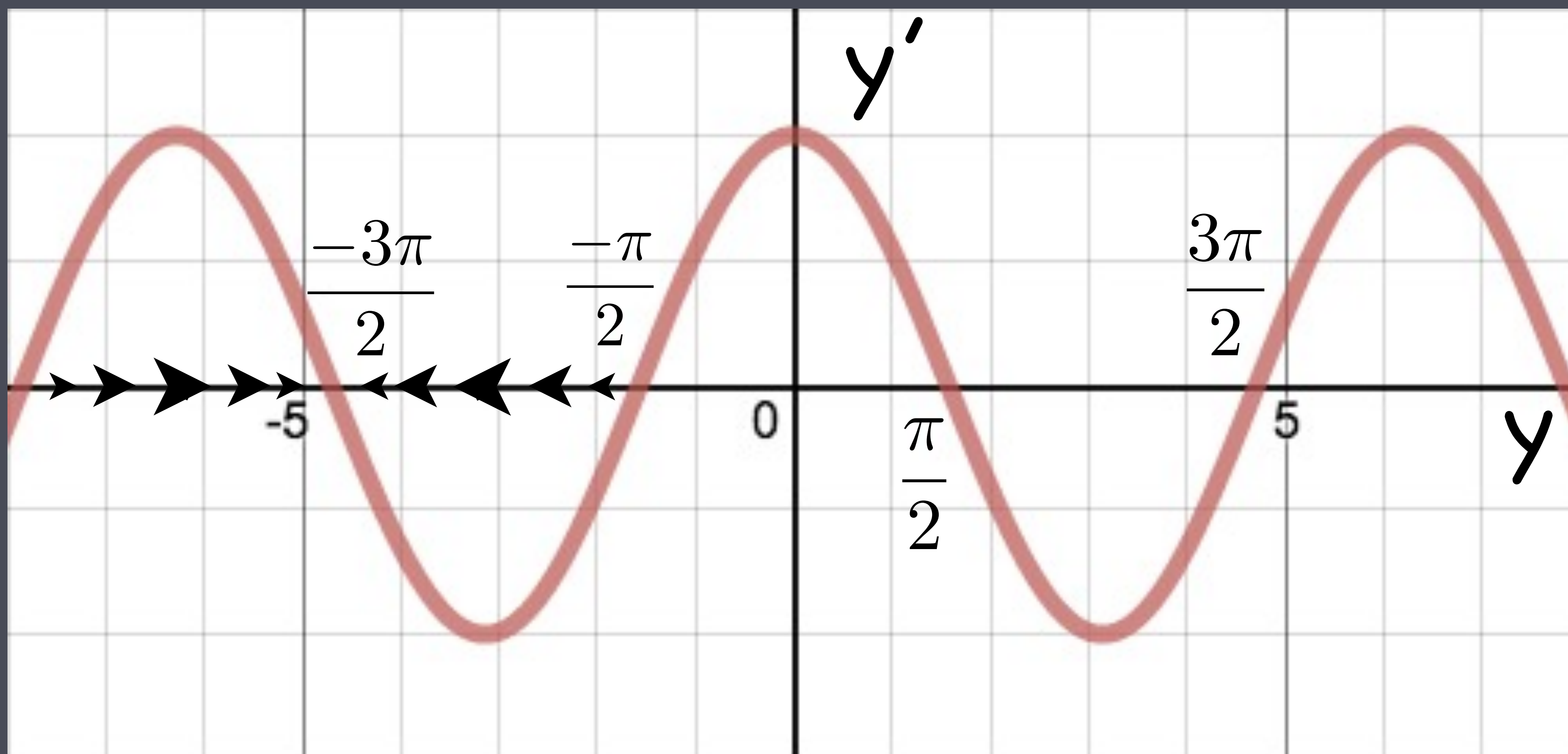

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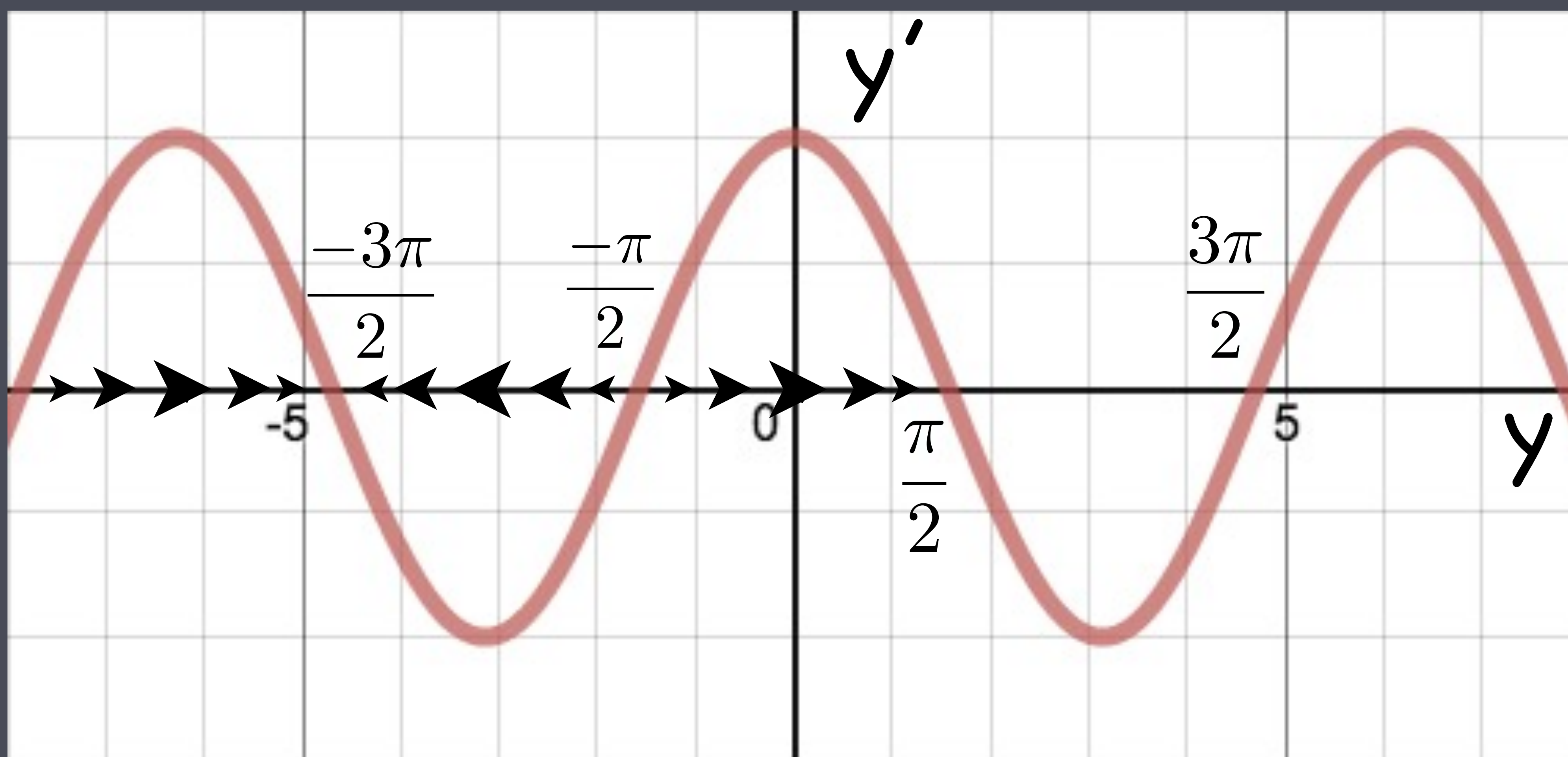

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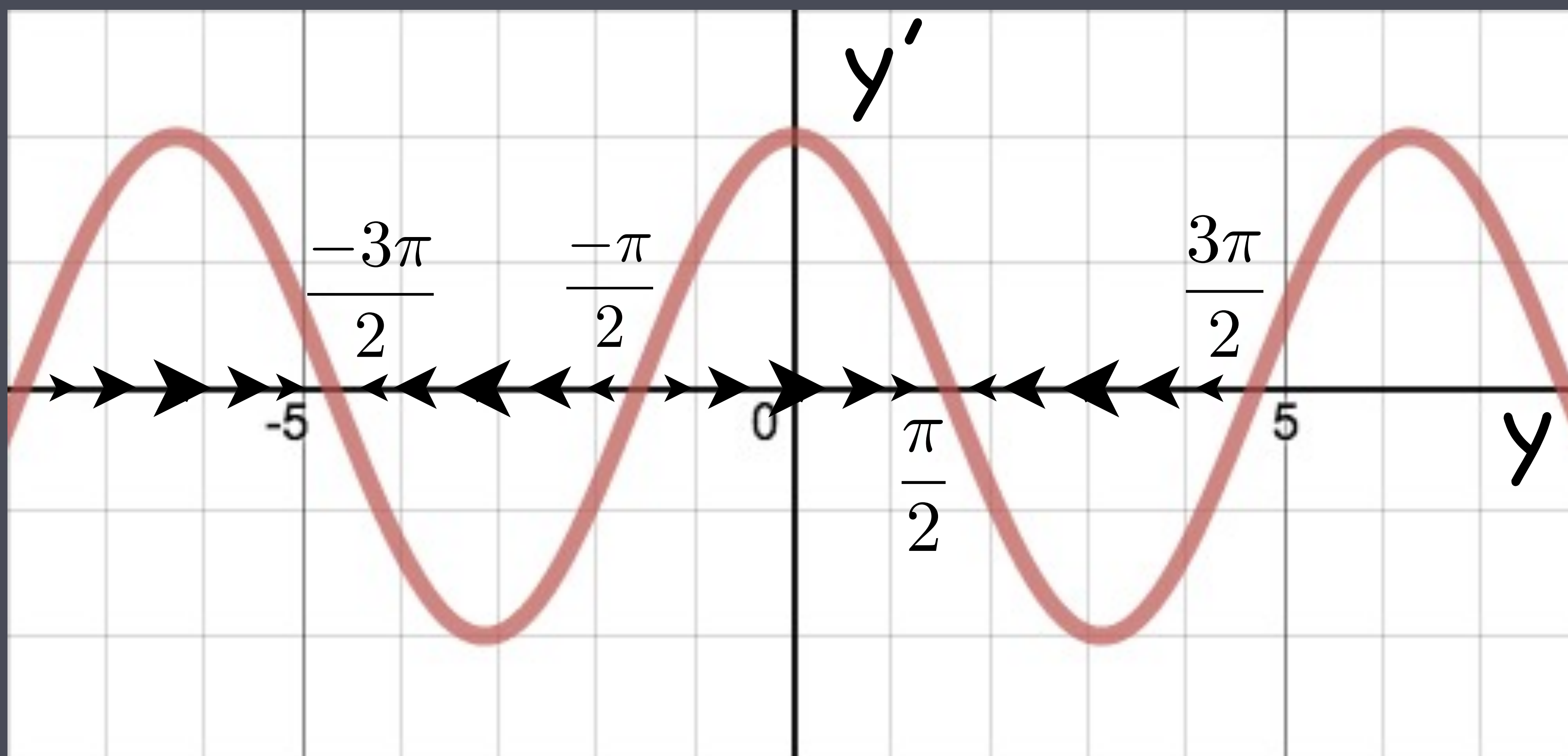

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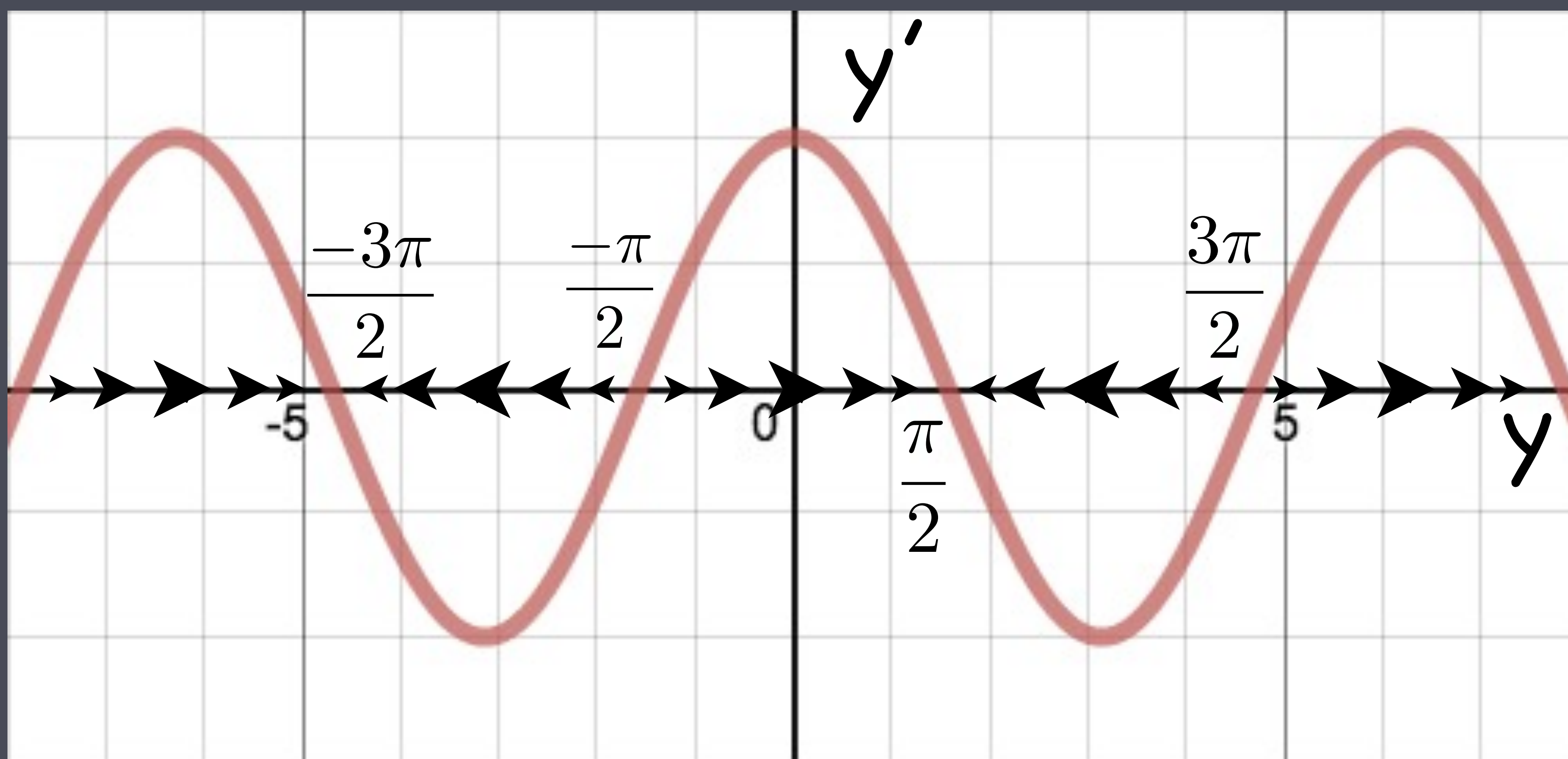

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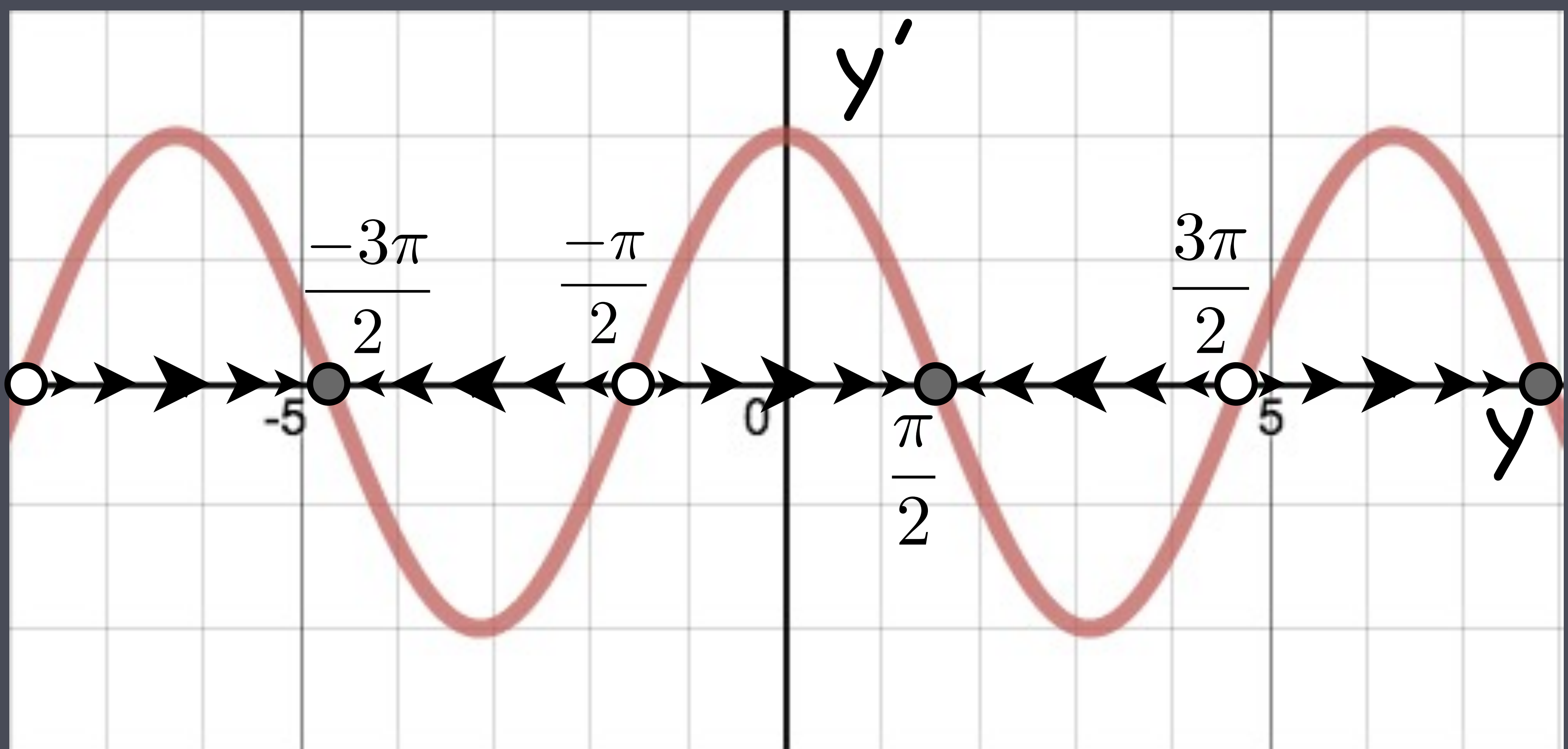
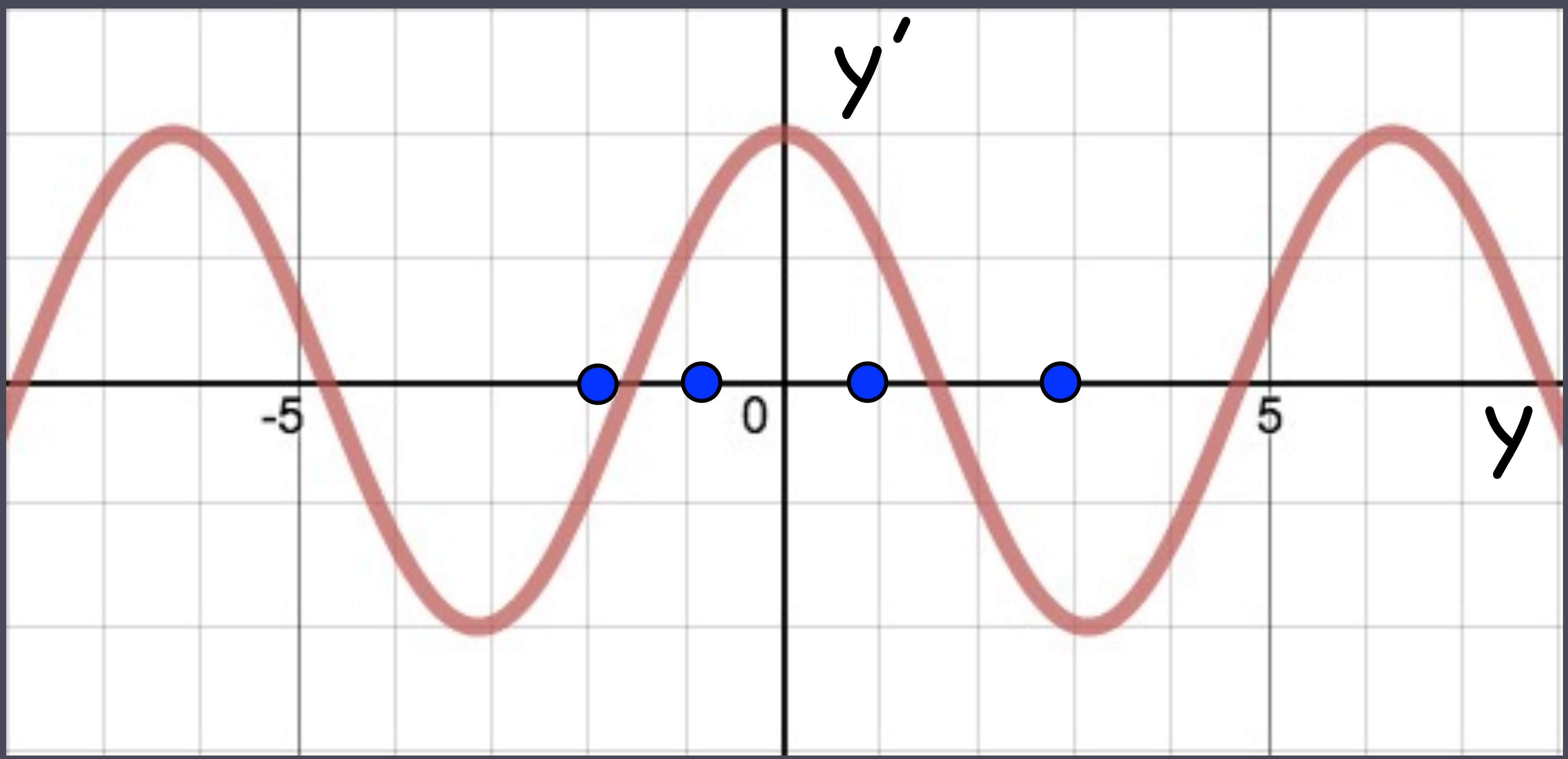


Filled circle ● – stable steady state

Empty circle ○ – unstable steady state

$y' = \cos(y)$

Sketch a few solutions y(t).



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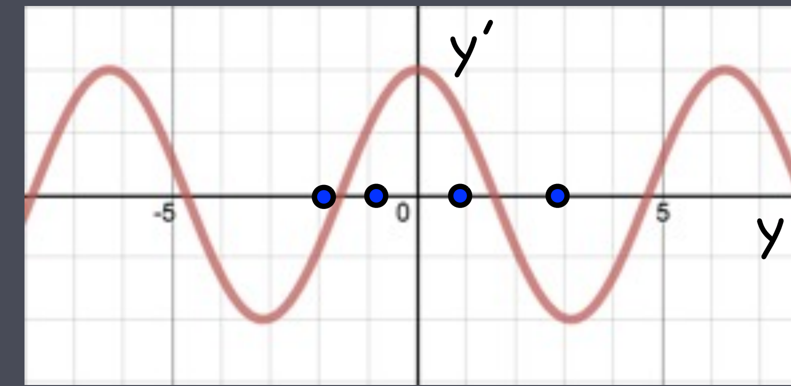

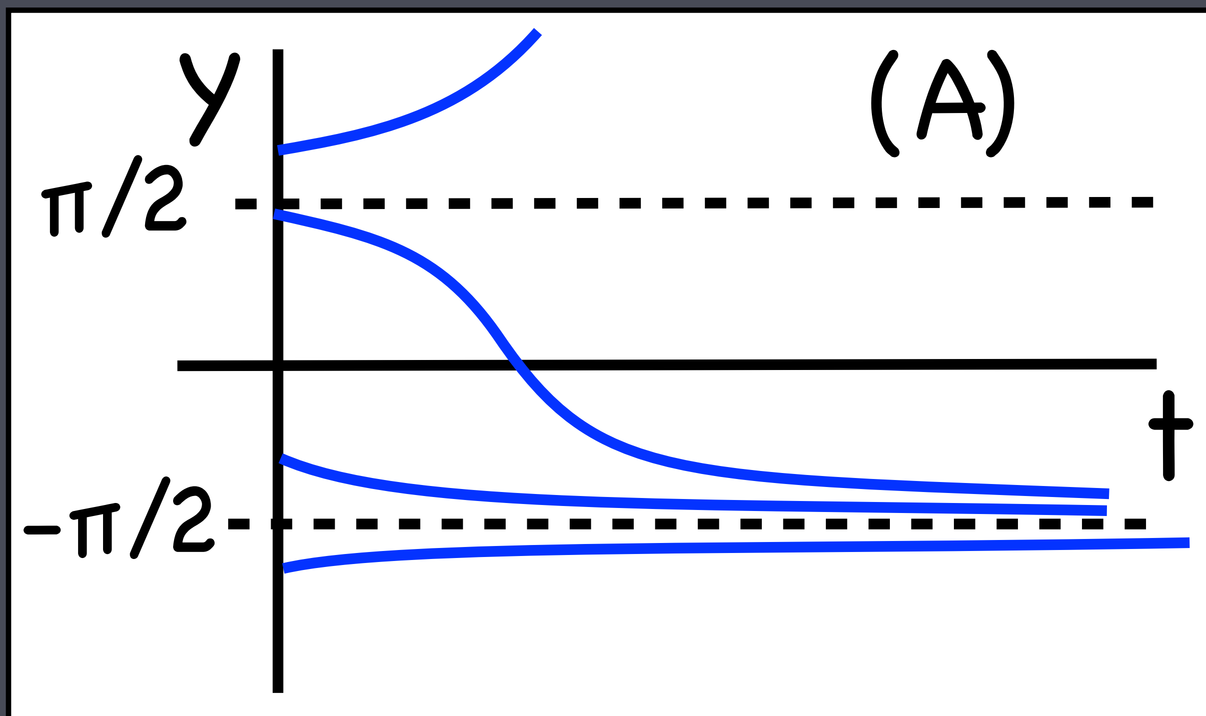

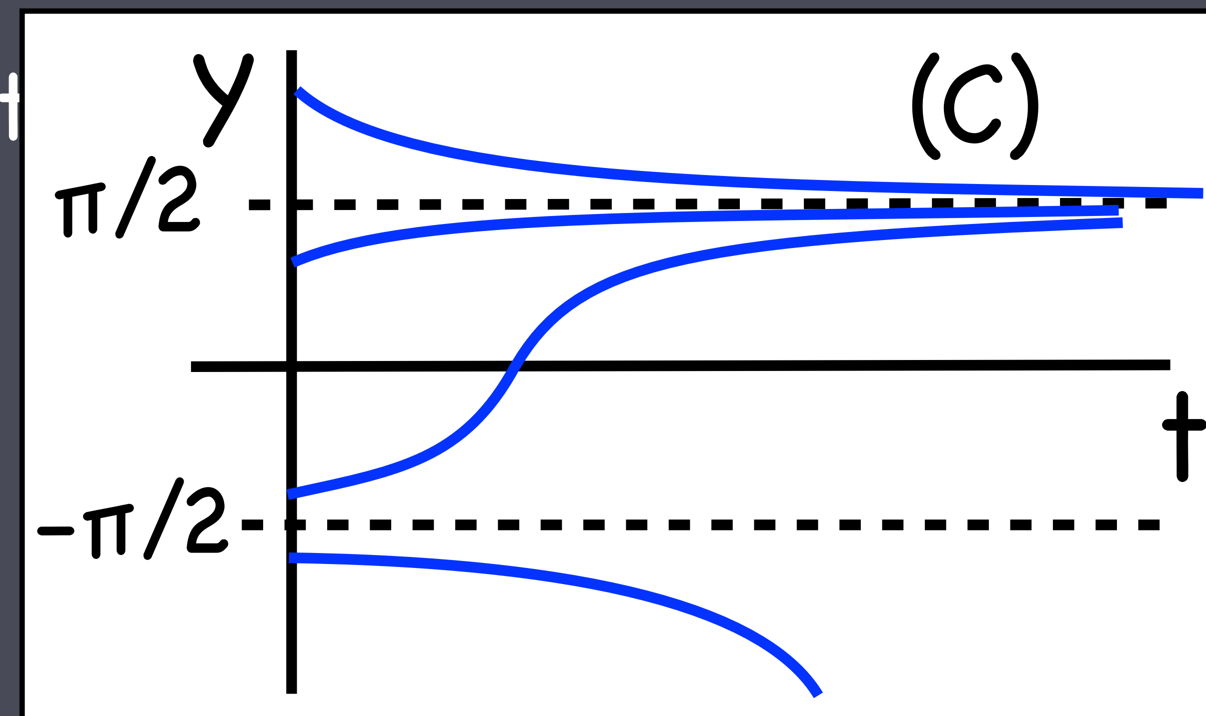

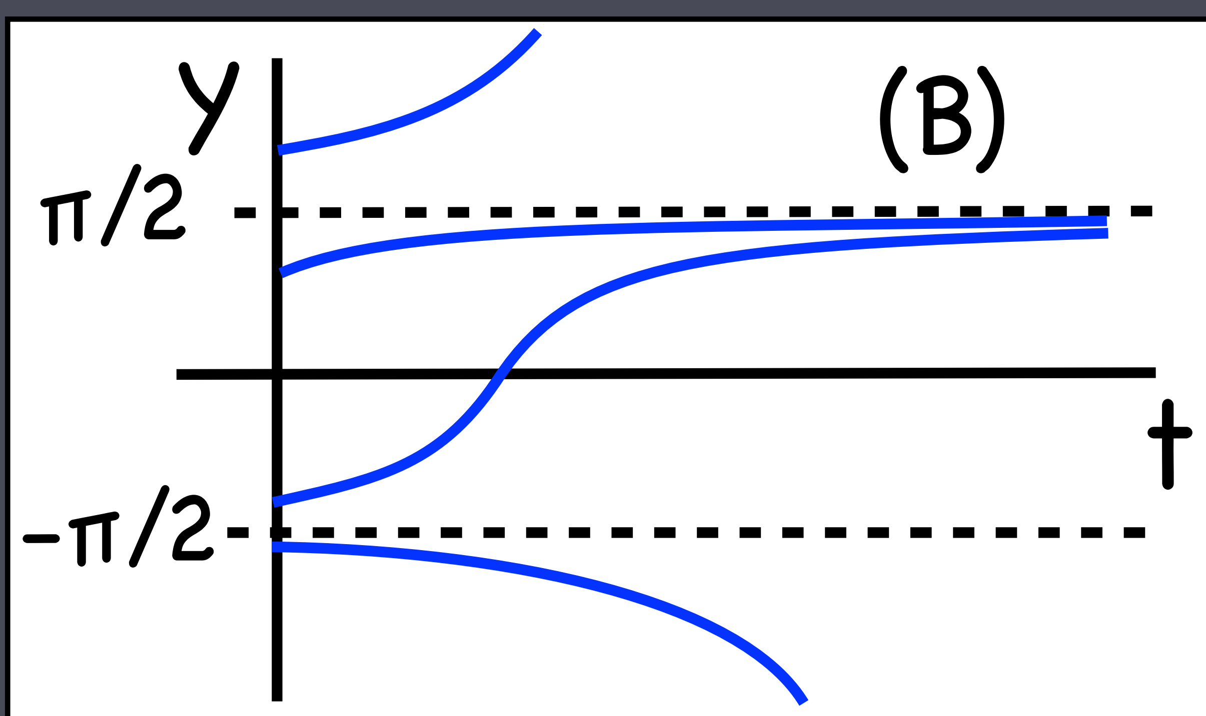

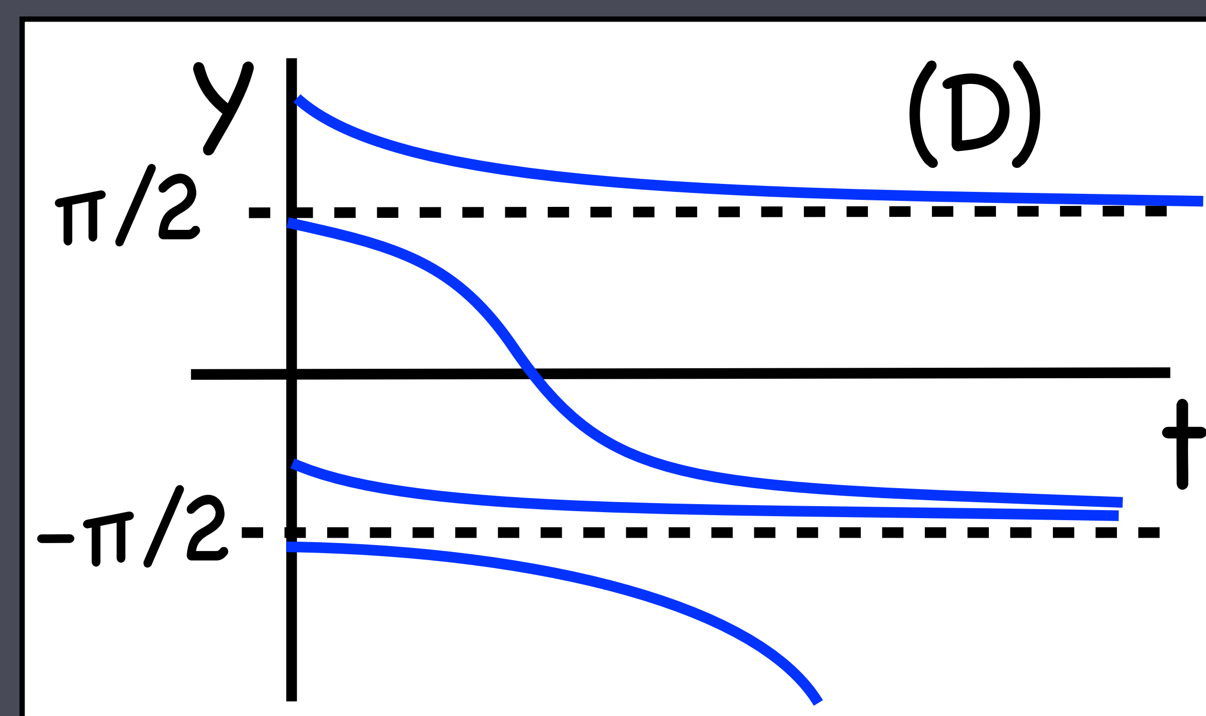

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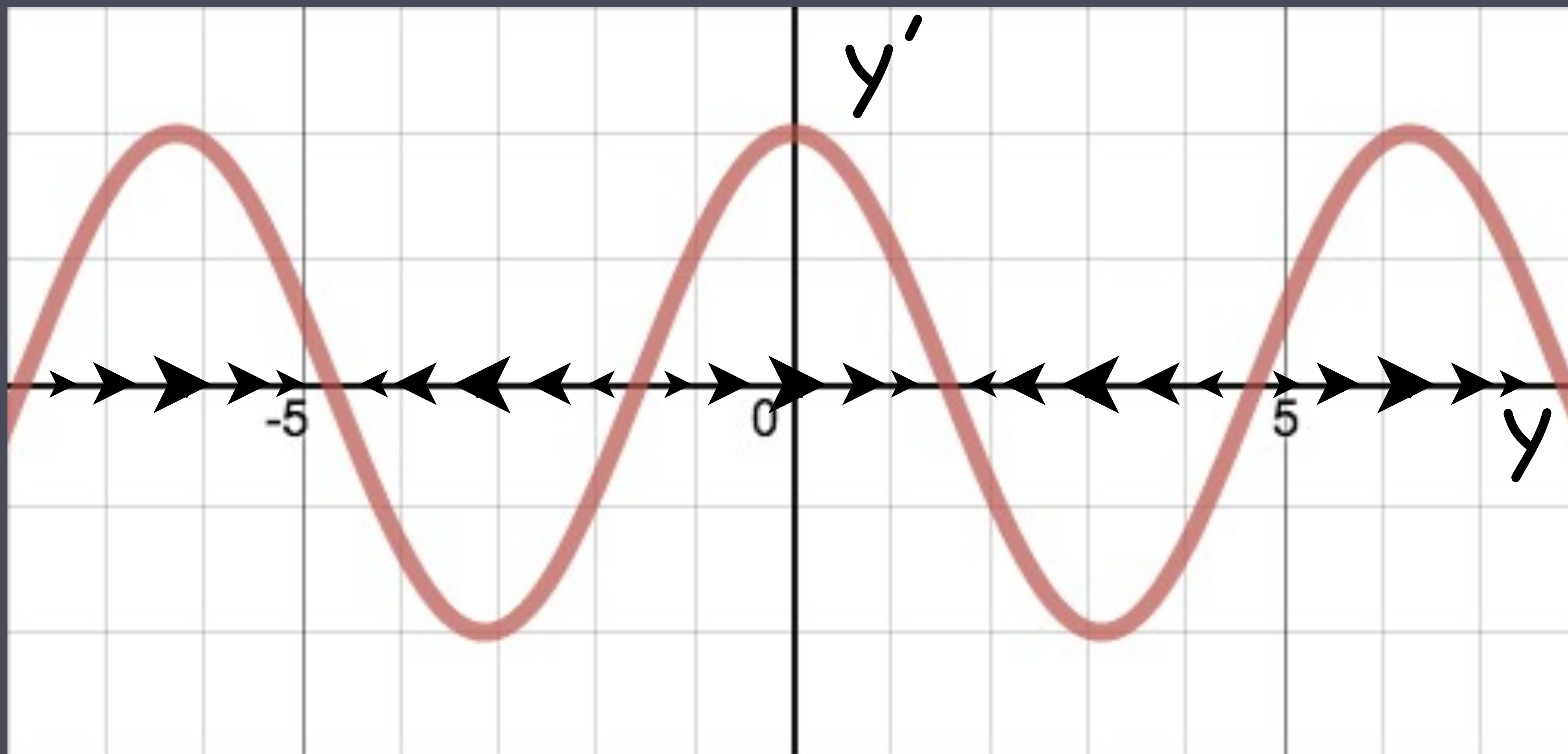

What does a solution look like as it approaches $\pi/2$?

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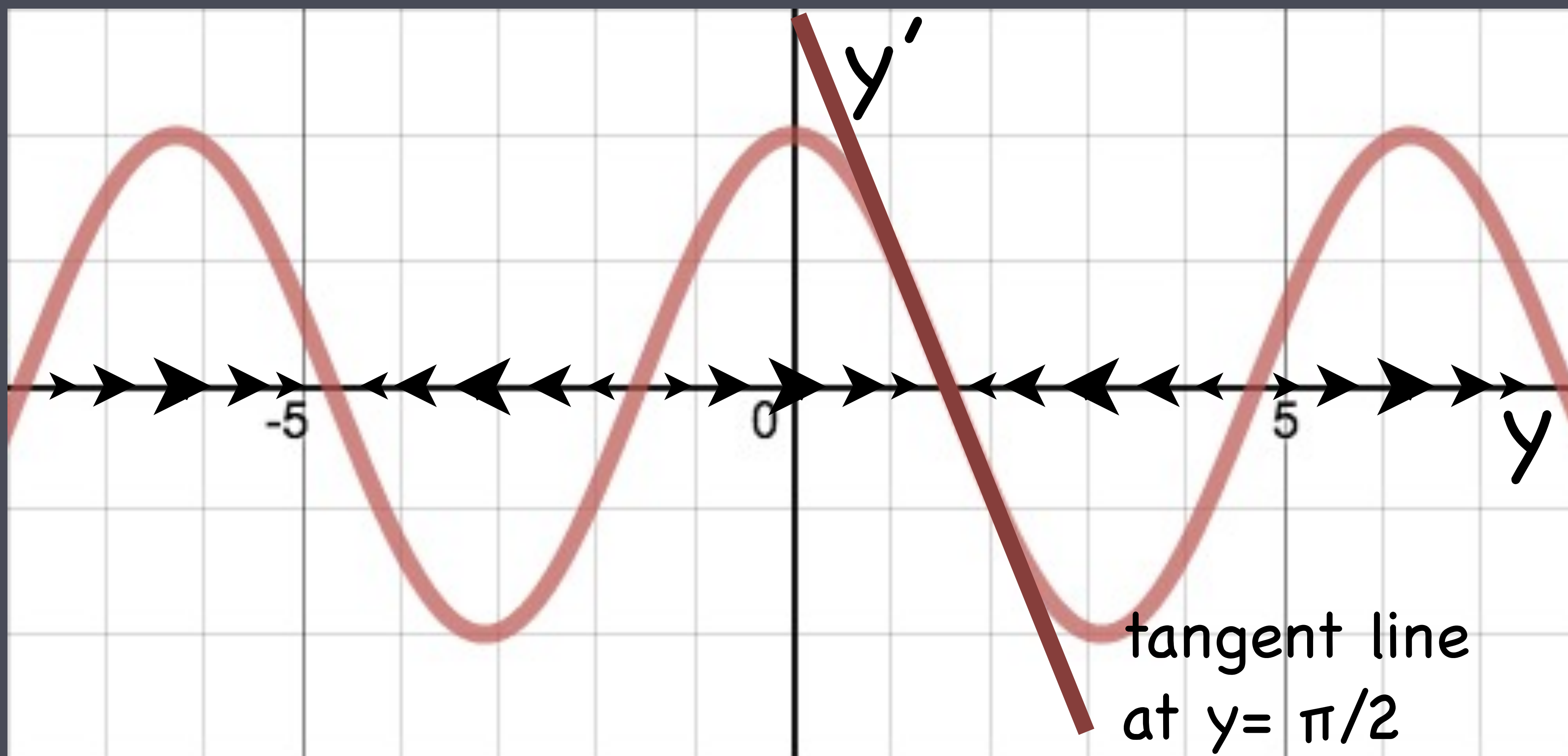

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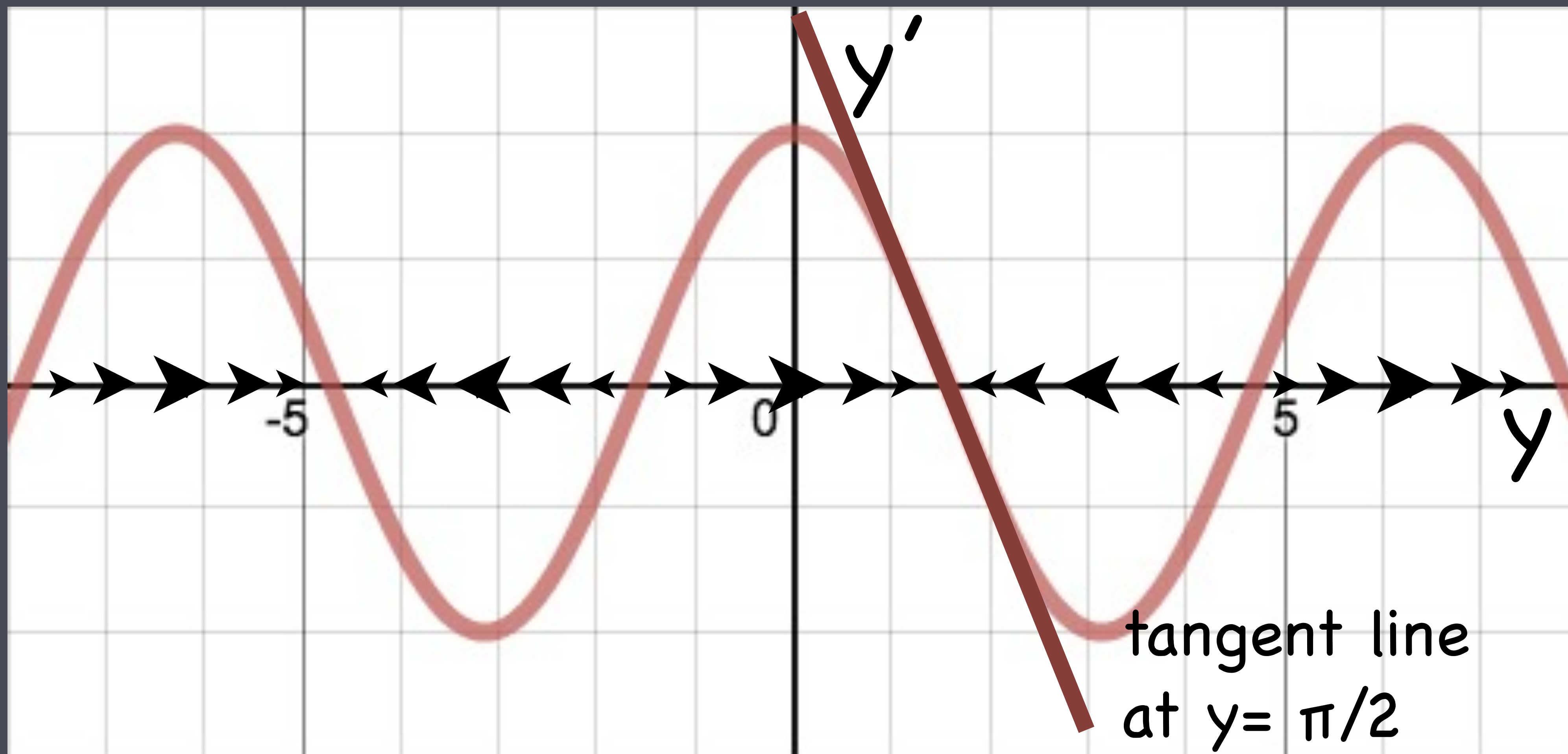

What does a solution look like as it approaches $\pi/2$?

The equation looks like $y' = -y + \pi/2$ so solutions start to look like $y(t) = \pi/2 + Ce^{-t}$ as they get close.

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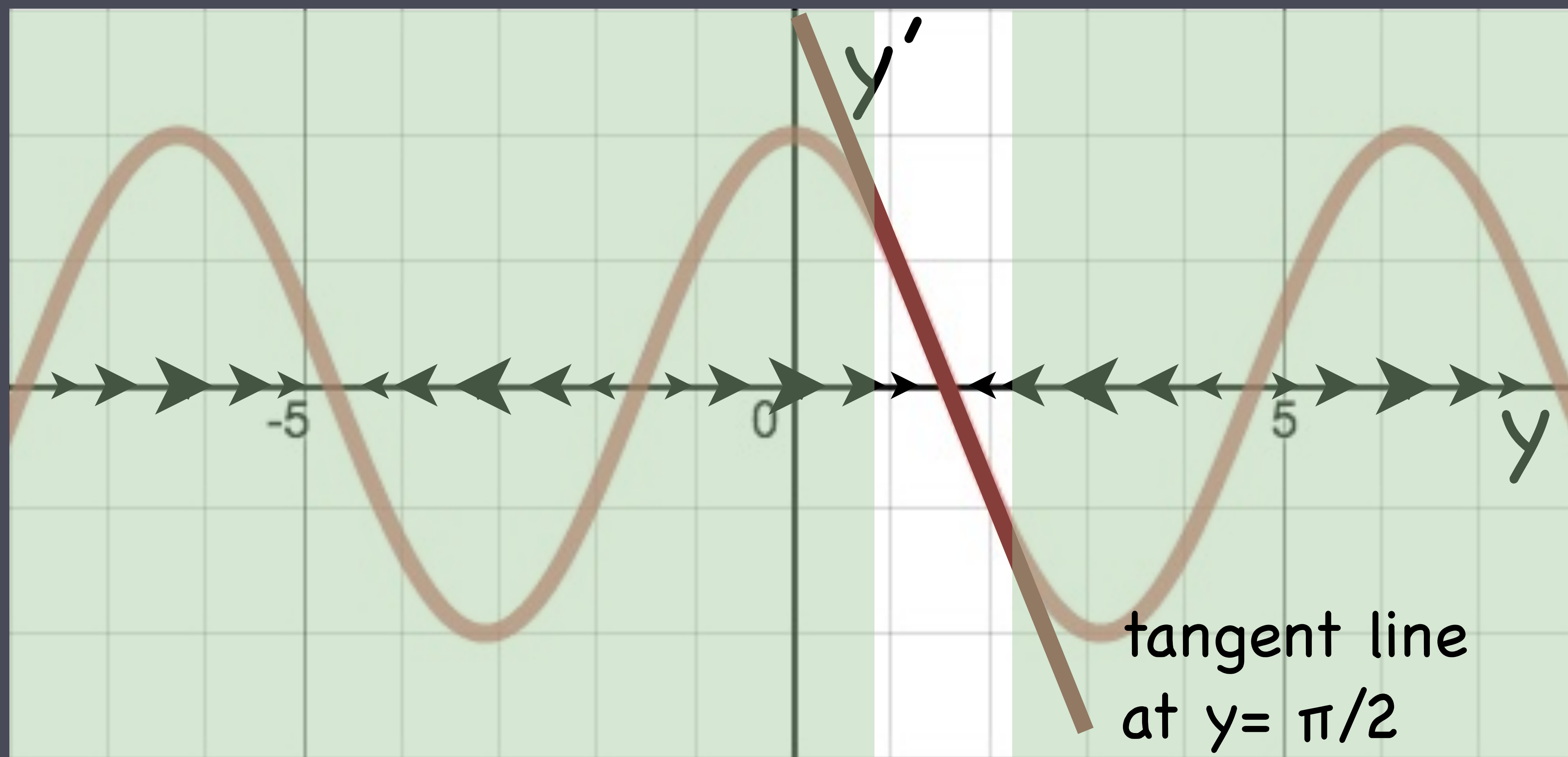

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What you should be able to do:

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states.
- Determine long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, h-asymptotes).