Today

- Quiz 3

- Wrap up IV drug delivery and $y' = ay + b$
A drug delivered by IV accumulates at a constant rate $k_{IV}$. The body metabolizes the drug proportional to the amount of the drug.

$$d'(t) = k_{IV} - k_md(t), \quad d(0) = 0.$$ 

- What is the dose actually received by the patient?
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$$\lim_{t \to \infty} d(t) = \frac{k_{IV}}{k_m}$$

- When does the patient start getting that dose?

By $t = 1/k_m$ (characteristic time), the patient is getting “close” to the target dose.
Stable steady state case

Any problem of the form \( y' = c - dy \) (\( d > 0 \)) with IC \( y(0) = y_0 \) has solution

\[
y(t) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^{-dt}
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Stable steady state case

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If you are given an alleged solution, you can always check that it solves the IVP:

- **LHS:** $y'(t) = \text{(on the doc cam)}$
- **RHS:** $c - dy = \text{(on the doc cam)}$

$$y(0) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^0 = y_0$$
Stable steady state case

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With \( d>0 \), as \( t \to \infty \), \( y(t) \to \frac{c}{d} \), a stable steady state.
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- With \( d > 0 \), as \( t \to \infty \), \( y(t) \to \frac{c}{d} \), a stable steady state.

- With \( d > 0 \), the characteristic time is \( \frac{1}{d} \).
Stable steady state case

\[ y(t) = \frac{c}{d} + \left( y_0 - \frac{c}{d} \right) e^{-\frac{d}{t}} \]
Where is $y(t)$ going?

Stable steady state case

$y(t) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^{-\frac{d}{t}}$
Stable steady state case

\[ y(t) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^{-dt} \]

Where is \( y(t) \) going? To the steady state \( \frac{c}{d} \).
Stable steady state case

\[ y(t) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^{-\frac{d}{t}} \]

When will it get there?
$y(t) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^{-\frac{d}{t}}$

When will it get there?
Never but at $t=\frac{1}{d}$ it will be $\frac{1}{e}$ of the way.
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Look different but same same.

- Newton’s Law of Cooling: $T'(t) = k(E - T(t))$
  
  $c = kE, \ d = k.$

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$
  
  $c = k_{IV}, \ d = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$
  
  $c = g, \ d = \delta$

- General form, factored: $y'(t) = d \left( \frac{c}{d} - y \right).$
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steady state
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\[ \text{steady state} \]

\[ \text{1 / characteristic time} \]
What do you need to know?

Given a word description, write down an equation for the quantity \( q(t) \) described.

Ex. Blah is added at a constant rate and is removed proportional to how much is there...

Ex. Blah changes proportional to the difference between blah and fixed #.

Use the shift substitution to get \( z' = az \) and state that \( z(t) = C_0 e^{at} \) solves it.

Substitute back to find \( q(t) \).

Determine \( C_0 \) using the initial condition.

Answer questions about the resulting exponential \( q(t) \).