Today

- Euler's method (cont)
- Qualitative analysis of differential equations
  - Steady states
  - Slope fields
  - Stability of steady states
  - Velocity (y') versus position (y)

Euler's method (cont)

- Spreadsheet demo

DEs - a broad view

- We have focussed on linear DEs so far:

$$y' = a + by$$

- A linear DE is one in which the y' and the y appear linearly (e.g. not squared).

- Some nonlinear equations:

$$v' = g - v^2, \quad y' = -\sin(y), \quad (h')^2 = bh.$$

object falling through air	pendulum under water	water draining from a vessel

DEs - a broad view

- Where do nonlinear equations come from?
- Population growth:

$$N' = bN - dN = kN \qquad \text{(linear)}$$

where b is per-capita birth rate, d is per-capita death rate and $k=b-d$.

- Suppose the per-capita death rate is not constant but increases with population size (more death at high density) so $d = cN$.

$$N' = bN-(cN)N = bN-cN^2$$

DEs - a broad view

$$\frac{dN}{dt} = bN - cN^2$$

This is called the logistic equation, usually written as

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

where r=b and K=1/c. This is a nonlinear DE because of the N^2.

Qualitative analysis

- Finding a formula for a solution to a DE is ideal but what if you can't?
- Qualitative analysis - extract information about the general solution without solving.
    - Steady states
    - Slope fields
    - Stability of steady states
    - Plotting y′ versus y (state space/phase line)

$$x' = x(1 - x)$$

velocity   position

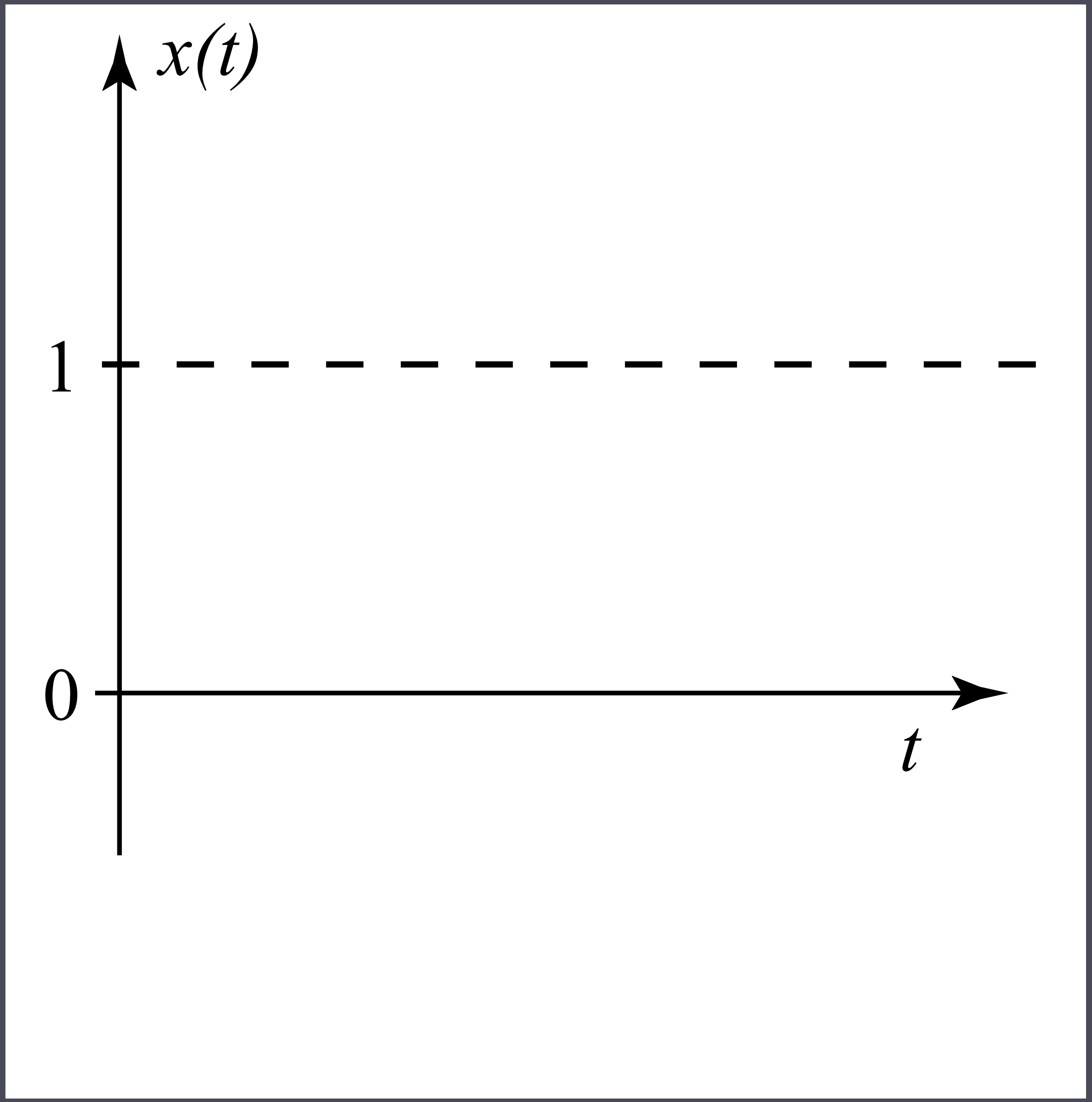
- Steady state. Where can you stand so that the DE tells you not to move?

  (A) x=-1

  (B) x=0

  (C) x=1/2

  (D) x=1

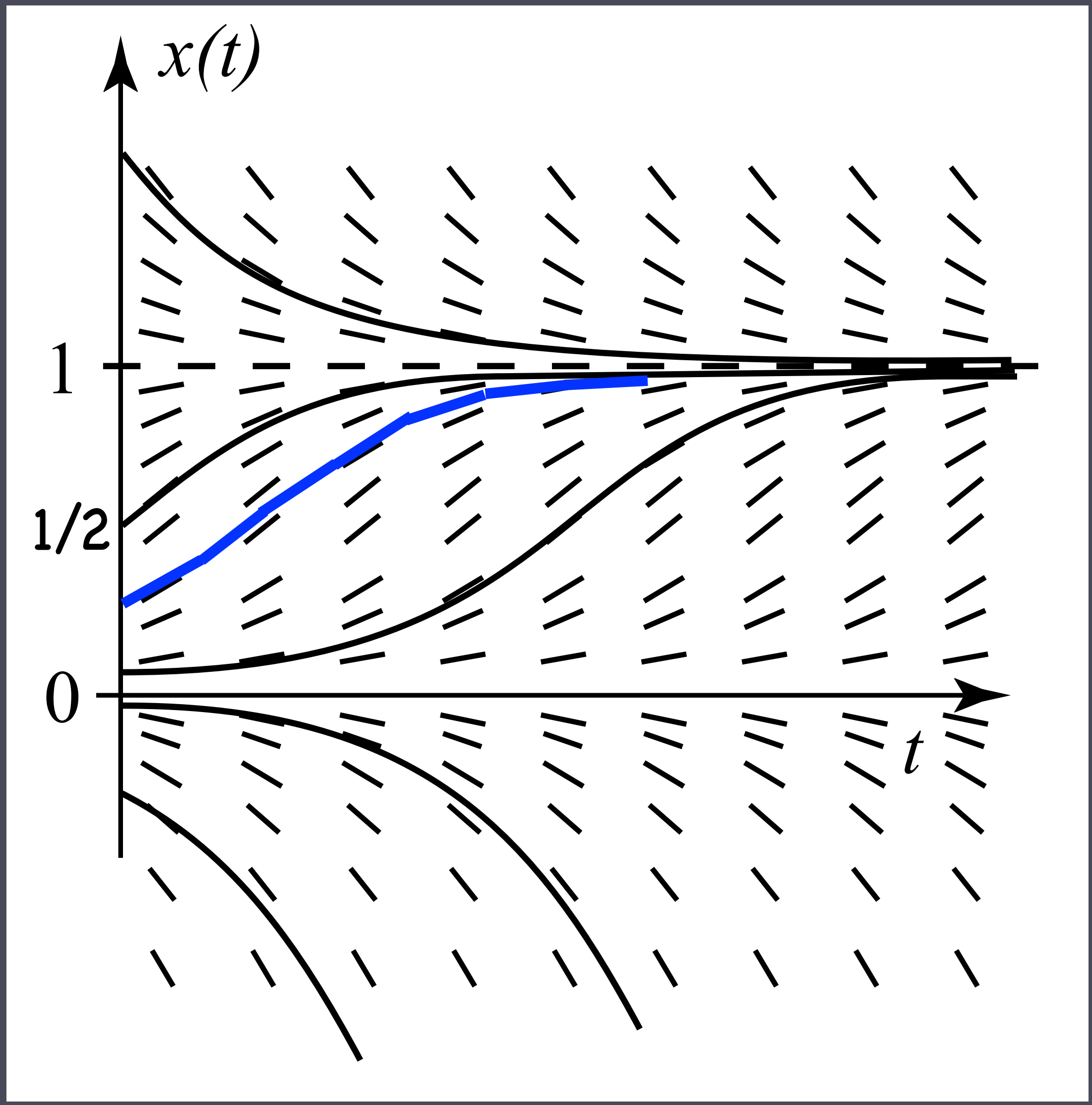


A steady state is a constant solution.

$$x' = x(1-x)$$

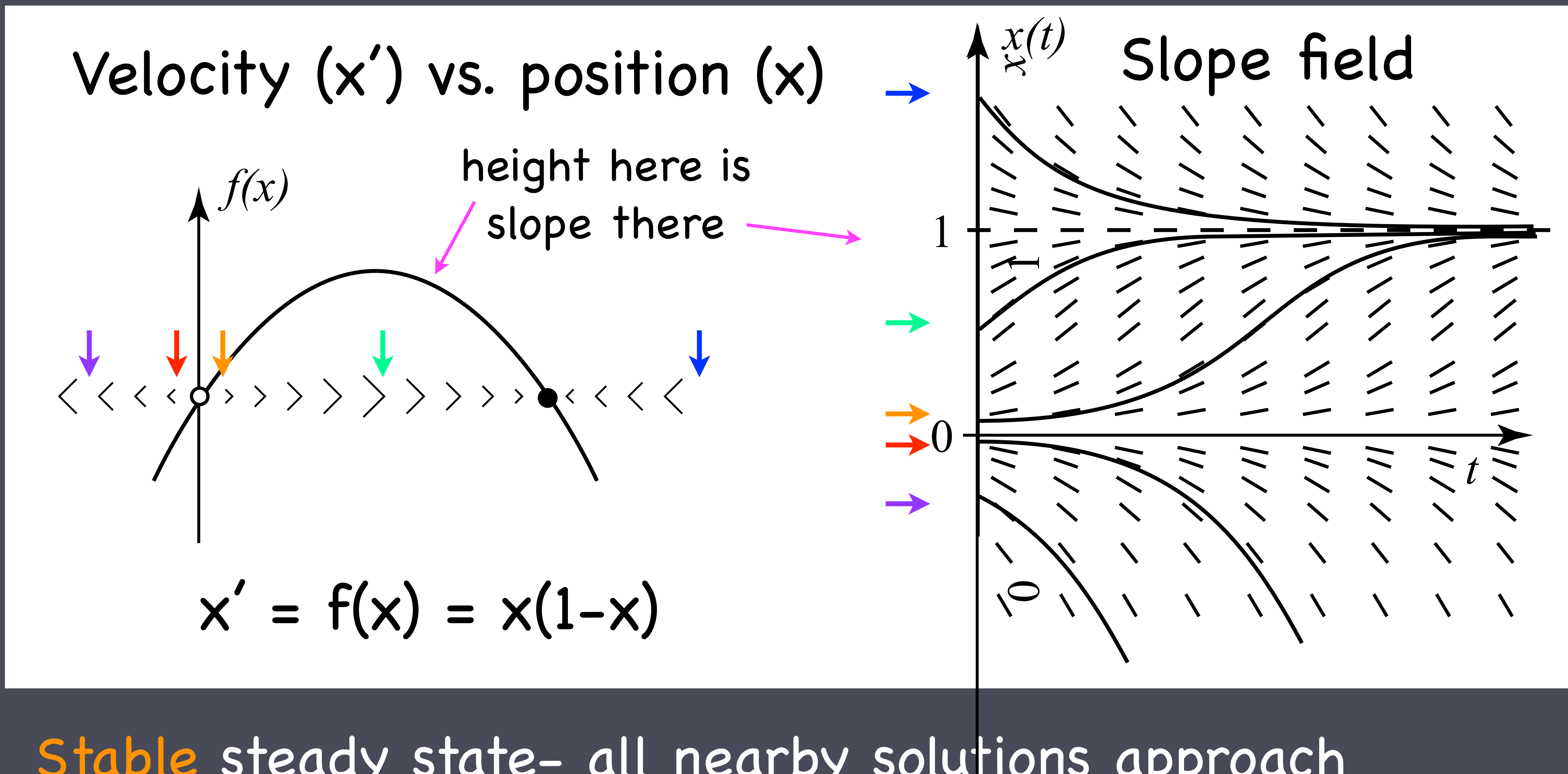
velocity — position

Slope field



- Slope field.
- At any t, don't know x yet so plot all possible x' values
- Now draw them for all t.
- When x(t)=1/2 what is x'?
  - (A) 0
  - (B) 1/4
  - (C) 1/2
  - (D) 1
- Solution curves must be tangent to slope field everywhere.

Velocity versus position

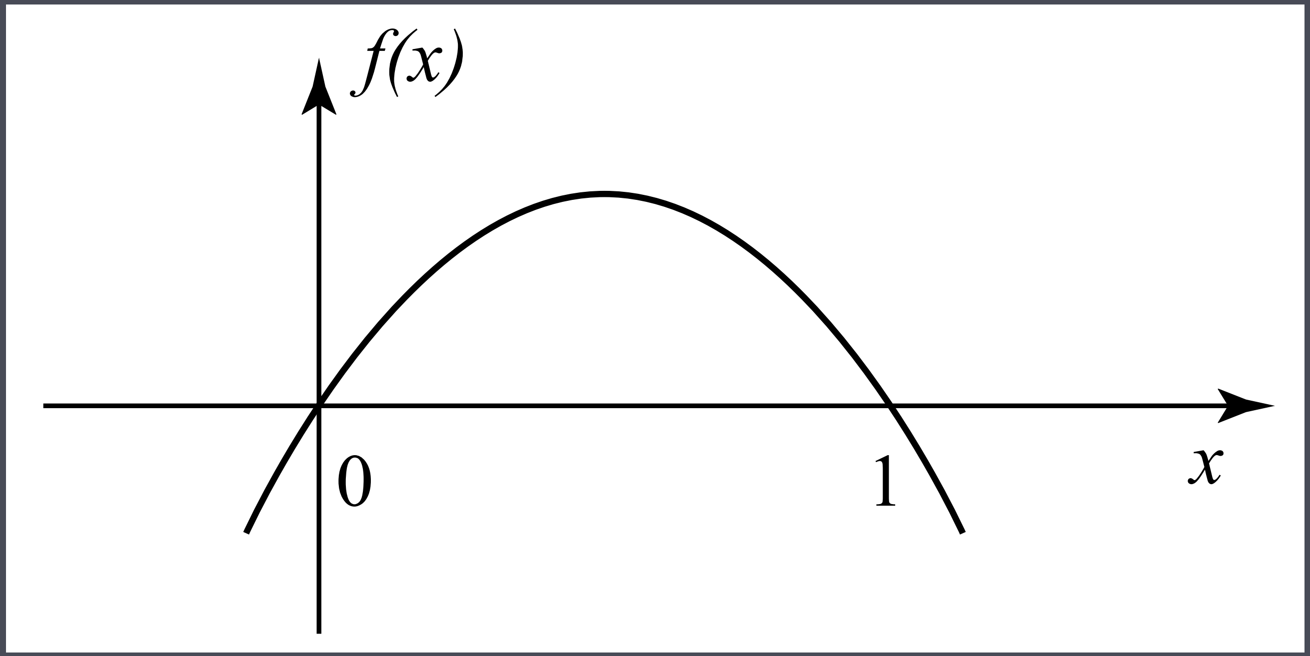


Stable steady state- all nearby solutions approach

Unstable steady state - not stable

Determine stability

$$x' = x(1-x)$$



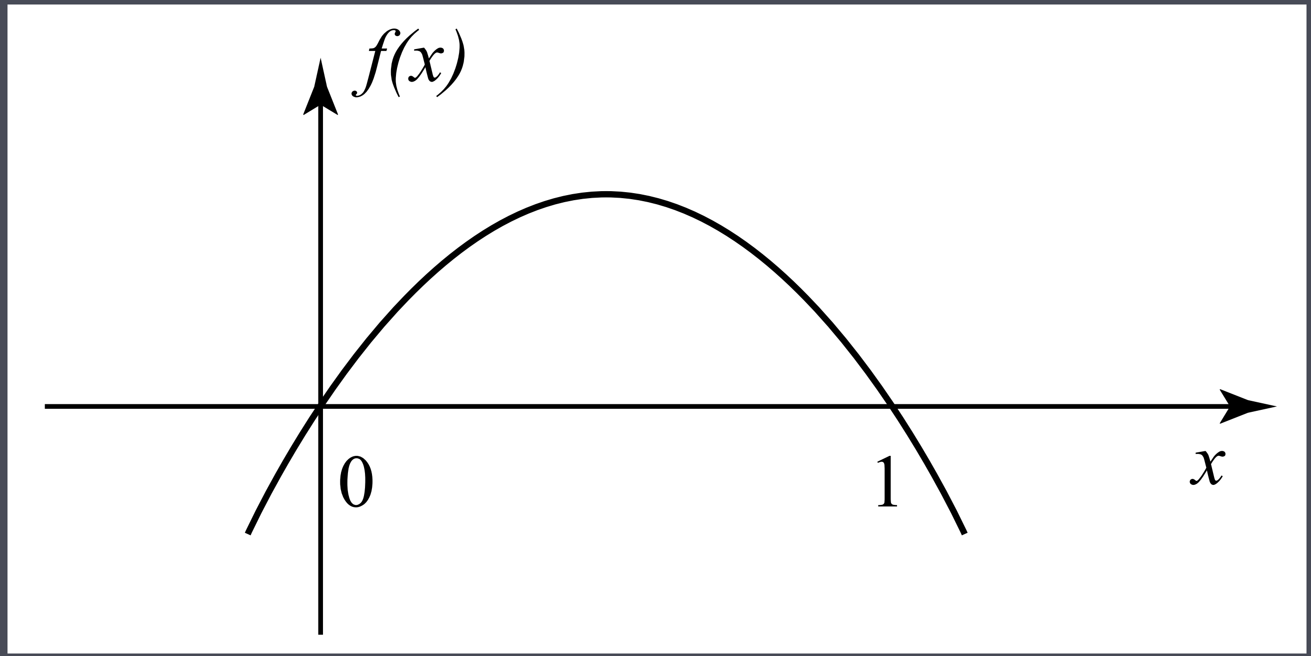
- If you start at x(0) = -0.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



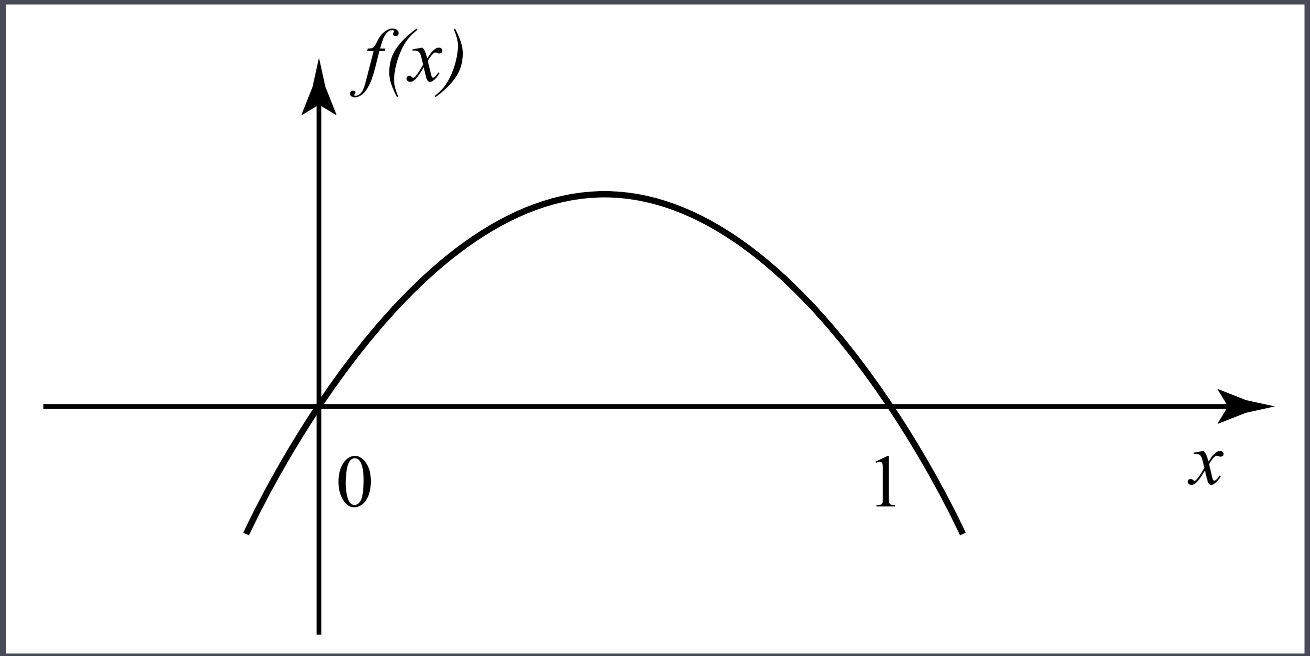
- If you start at x(0) = 0.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



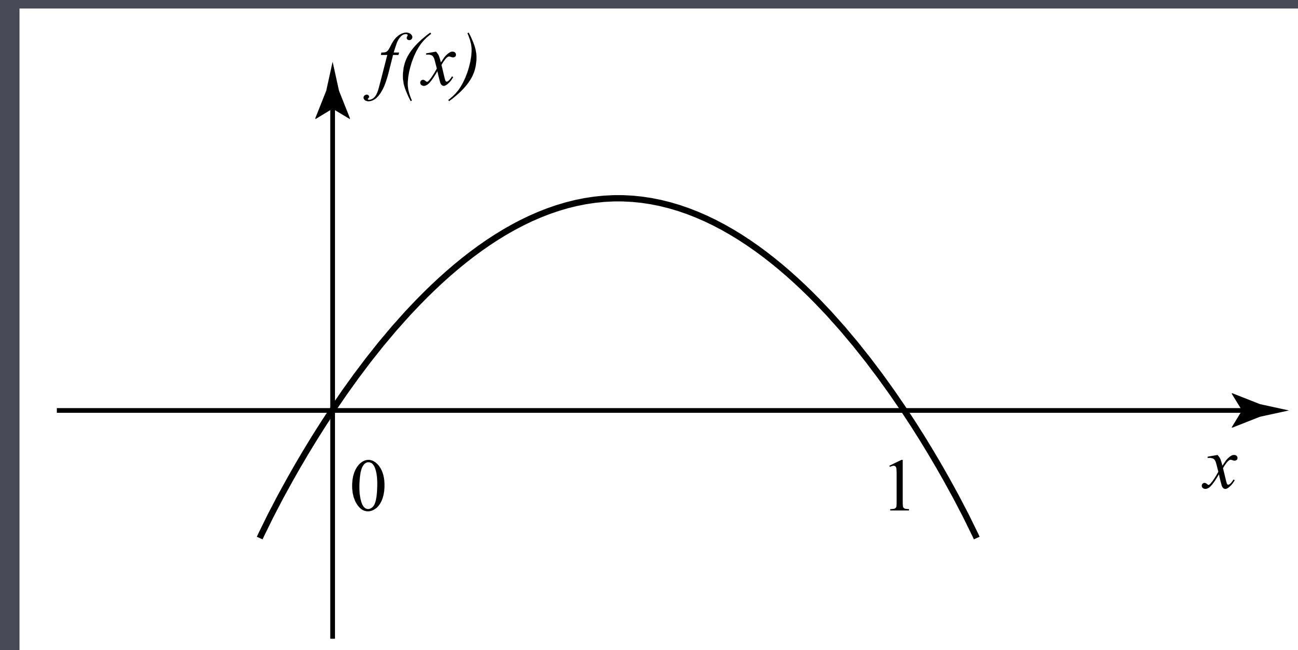
- If you start at x(0) = 0.99, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



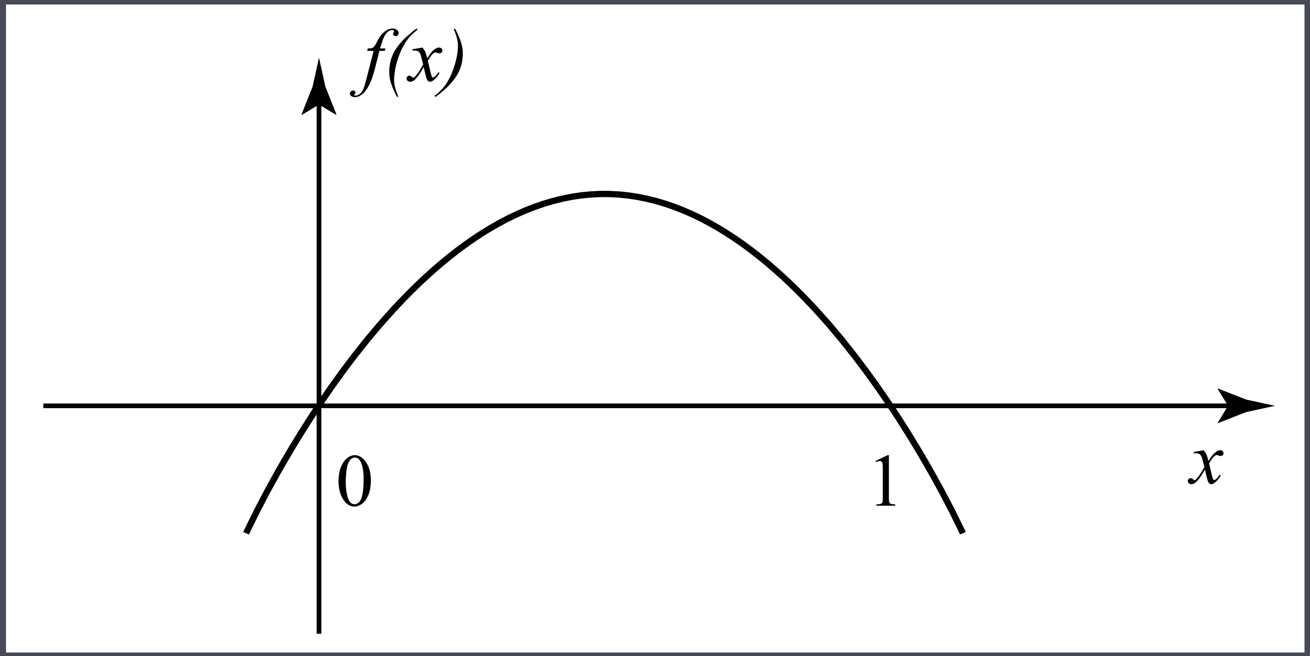
- If you start at x(0) = 1.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1 - x)$$



(A) Both x(t)=0 and x(t)=1 are stable steady states.

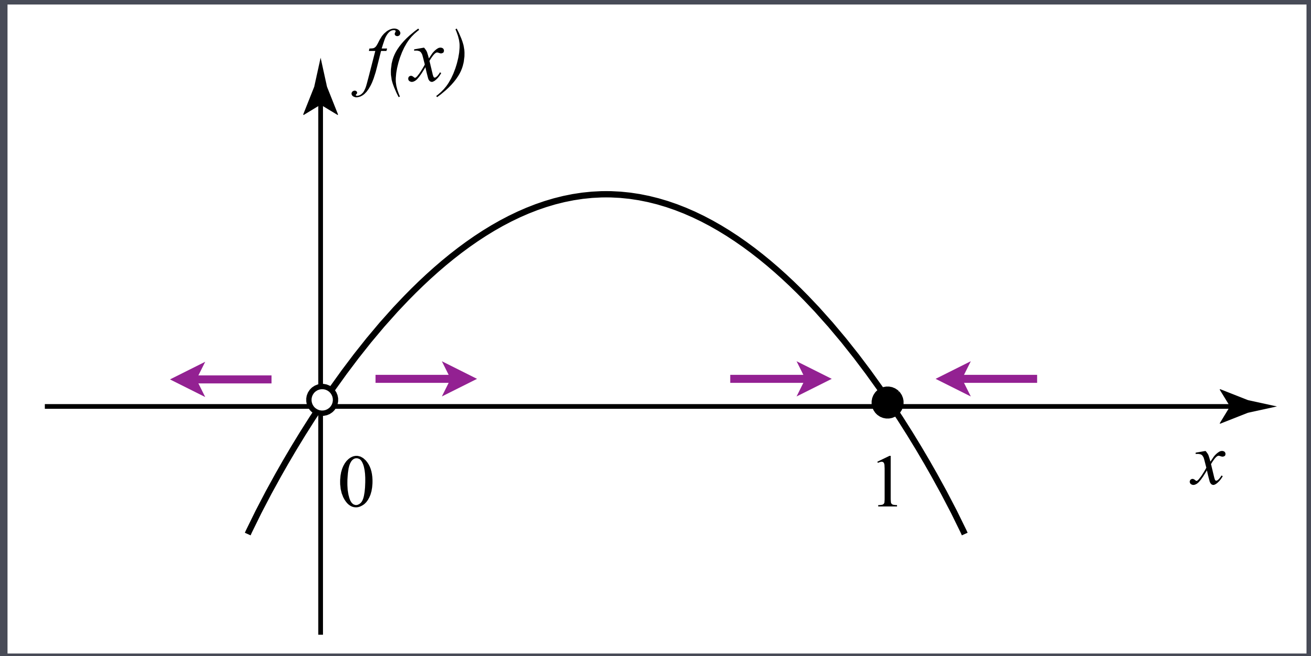
(B) x(t)=0 is stable and x(t)=1 is unstable.

(C) x(t)=0 is unstable and x(t)=1 is stable.

(D) Both x(t)=0 and x(t)=1 are unstable steady states.

Determine stability

$$x' = x(1-x)$$



(A) Both x(t)=0 and x(t)=1 are stable steady states.

(B) x(t)=0 is stable and x(t)=1 is unstable.

(C) x(t)=0 is unstable and x(t)=1 is stable.

(D) Both x(t)=0 and x(t)=1 are unstable steady states.

Stable - solid dot. Unstable - empty dot.