Exponents, Logs and Inverse Functions

Review: What is the equation of the tangent line to the curve \( y^2x - y = 1 \) at (0,1)?

1) The Natural Logarithm ln\( (x) \) is the inverse function of \( e^x \), that is ln\( (e^x) = x \). Let’s briefly recall the rules of logs and exponents:

\[
e^x e^y = \quad e^x / e^y = \quad (e^x)^y = \quad 2^x = \\
\ln(xy) = \quad \ln(y) - \ln(y) = \quad \ln x^y = \quad \log_a x =
\]

Use implicit differentiation to find the derivative of \( y = \ln(x) \).

2) Derivatives with Natural Log Compute the following derivatives:

\[
\frac{d}{dx} \ln(x^2 - 3x + 1) = \quad \frac{d}{dx} \ln \frac{x}{x - 1} = \quad \frac{d}{dx} x^2 \ln x =
\]

3) Inverse Functions For a function \( f(x) \), the “inverse function” is defined as the function \( g(x) \) such that \( f(g(x)) = 1 \). The inverse function can be visualized by reflecting the graph \( y = f(x) \) over the line \( y = x \).
Which of the following statements about inverse functions are true:

1. If \((a, b)\) is a point on \(y = f(x)\), then \((b, a)\) is a point on the graph of the inverse function
2. If \(f(a) = b\), then the slope of the inverse function at \(x = b\) is \(1/f'(x)\).
3. The inverse function of \(f(x)\) isn’t defined at a critical point of \(f(x)\)
4. The inverse function is always well defined everywhere.

**Example** What is the relationship between \(\frac{d}{dx}\sqrt{x}\) at \(x = 9\) and \(\frac{d}{dx}x^2\) at \(x = 3\)?

**Example** Assume \(f(x)\) is an increasing function with \(f(1) = 2, f(2) = 4, f'(1) = 3\) and \(f'(2) = 5\). If \(g(x)\) is the inverse function of \(f(x)\), what is \(g(2)\)?

(Bonus) Consider the function \(f(x) = x^3 - x\) and let \(g(x)\) be its inverse function. Is \(g(x)\) defined everywhere? When does \(g(x)\) have a maximum?

(Bonus) Prove that \(\frac{d}{dx}a^x = \ln(a) a^x\).

4) **Semi-Log Plots** When fitting nonlinear data, it can be useful to plot not the data itself but the log of the data. If we expect that data to be of the form \(y_i \approx a e^{k x_i}\) for some constants \(k\) and \(a\), how does plotting \(\ln(y)\) effect the data?

5) **Log-Log Plots** What is a Log-Log plot and when do we use one?

If we use numerical software tells us that the best fit line to a Log-Log plot is \(y = mx + b\), what function do we expect to fit the data?