Today

• Composition and chain rule, quotient rule
• Antiderivatives of power functions and polynomials
• Tangent lines

• Reminders:
  • Assignment3 Thursday 7am,
  • OSH 2 Friday 11:59 pm.
  • Sign up for midterm time/room.
Composition of functions

If \( f(x) = 2x+3 \) and \( g(x) = -4x+2 \),

A. \( h(x) = f( g(x) ) = -8x+7 \)

B. \( h(x) = f( g(x) ) = -8x-10 \)

C. \( h(x) = f( g(x) ) = -8x^2-8x+6 \)

D. \( h(x) = f( g(x) ) = -8x+5 \)
Composition of functions

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C. \( h(x) = f( g(x) ) = -8x^2-8x+6 \)

D. \( h(x) = f( g(x) ) = -8x+5 \)
Composition of functions

If \( h(x) = f(g(x)) \), then

A. \( h'(x) = f'(x) g'(x) \)

B. \( h'(x) = f'(x) g(x) + f(x) g'(x) \)

C. \( h'(x) = f'(g'(x)) \)

D. \( h'(x) = f'(g(x)) g'(x) \)
Composition of functions

If $h(x) = f(g(x))$, then

A. $h'(x) = f'(x) g'(x)$

B. $h'(x) = f'(x) g(x) + f(x) g'(x)$

C. $h'(x) = f'(g'(x))$

D. $h'(x) = f'(g(x)) g'(x)$  <---- Chain Rule
Composition of functions

If \( h(x) = (x^3-2x+1)^6 \), then \( h'(x) = ? \)

A. \( 6 (x^3-2x+1)^5 \)

B. \( (x^3-2x+1)^6 (3x^2-2) \)

C. \( 6 (x^3-2x+1)^5 (3x^2-2) \)

D. \( 6 (x^3-2x+1)^5 (x^3-2x+1) \)

E. Are you kidding? It will take me weeks to multiply those out.
Composition of functions

If \( h(x) = (x^3 - 2x + 1)^6 \), then \( h'(x) = \)?

A. \( 6 (x^3 - 2x + 1)^5 \)

B. \( (x^3 - 2x + 1)^6 (3x^2 - 2) \)

C. \( 6 (x^3 - 2x + 1)^5 (3x^2 - 2) \)

D. \( 6 (x^3 - 2x + 1)^5 (x^3 - 2x + 1) \)

E. Are you kidding? It will take me weeks to multiply those out.
Rules for differentiation - summary

- **Addition rule:**
  - \( f(x) = g(x) + h(x) \quad \rightarrow \quad f'(x) = g'(x) + h'(x) \)

- **Product rule:**
  - \( f(x) = g(x) \cdot h(x) \quad \rightarrow \quad f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x) \)

- **Chain rule:**
  - \( f(x) = g( h(x) ) \quad \rightarrow \quad f'(x) = g'( h(x) ) \cdot h'(x) \)

- **Quotient rule:**
  - \( f(x) = \frac{g(x)}{h(x)} = g(x) \cdot (h(x))^{-1} \quad \text{<---- apply product and chain rules or} \)
Suppose \( f(x) = \frac{g(x)}{k(x)} \) and that 
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

• What is \( f'(2) \)?

(A) -13
(B) -13/25
(C) -13/9
(D) 17/25
Suppose $f(x) = g(x)/k(x)$ and that $g(2) = 3$, $k(2) = 1$, $g'(2) = 2$, $k'(2) = 5$.

• What is $f'(2)$?

(A) $-13$

(B) $-13/25$

(C) $-13/9$

(D) $17/25$
Antiderivatives – going backward

If \( f'(x) = 6x^2 + 4x - 1 \), then

(A) \( f(x) = 12x + 4 \)

(B) \( f(x) = 2x^3 + 2x^2 - x \)

(C) \( f(x) = 2x^3 + 2x^2 - x + 2 \)

(D) \( f(x) = 2x^3 + 2x^2 - x + C \)
Antiderivatives – going backward

If $f'(x) = 6x^2 + 4x - 1$, then

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Antiderivatives – going backward

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(D) \( f(x) = 2x^3 + 2x^2 - x + C \)

Slopes at each \( x \) don’t change with vertical shift.
This is $f'(x)$. Draw $f(x)$. 
This is $f'(x)$. Draw $f(x)$. 

\[ \begin{align*} 
\text{f(x)} & \quad \text{f(x)} \\
\end{align*} \]
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This is $f'(x)$. Draw $f(x)$. Only determined up to a vertical shift.
Position–Velocity–Acceleration

If $x(t)$ is position as a function of time,
Position–Velocity–Acceleration

If \( x(t) \) is position as a function of time,

velocity \( v(t) = x'(t) \),
Position–Velocity–Acceleration

- If $x(t)$ is position as a function of time,
- velocity $v(t) = x'(t)$,
- acceleration $a(t) = v'(t) = x''(t)$. 
Position-Velocity-Acceleration

If \( x(t) \) is position as a function of time,

- velocity \( v(t) = x'(t) \),
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Constant acceleration \( a \):
Position–Velocity–Acceleration

- If $x(t)$ is position as a function of time,
  - velocity $v(t) = x'(t)$,
  - acceleration $a(t) = v'(t) = x''(t)$.

Constant acceleration $a$:

- $v(t)$
Position–Velocity–Acceleration

If \( x(t) \) is position as a function of time,

- velocity \( v(t) = x'(t) \),
- acceleration \( a(t) = v'(t) = x''(t) \).

Constant acceleration \( a \):

- \( v(t) = at + C \)
Position–Velocity–Acceleration

- If $x(t)$ is position as a function of time, 
  - velocity $v(t) = x'(t)$, 
  - acceleration $a(t) = v'(t) = x''(t)$.
- Constant acceleration $a$:
  - $v(t) = at + C = at + v_0$
Position-Velocity-Acceleration

- If \( x(t) \) is position as a function of time,
  - velocity \( v(t) = x'(t) \),
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- Constant acceleration \( a \):
  - \( v(t) = at + C = at + v_0 \) so that \( v(0) = v_0 \).
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- If \( x(t) \) is position as a function of time, velocity \( v(t) = x'(t) \),
- acceleration \( a(t) = v'(t) = x''(t) \).
- Constant acceleration \( a \):
  - \( v(t) = at + C = at + v_0 \) so that \( v(0) = v_0 \).
  - \( x(t) \)
Position–Velocity–Acceleration

- If $x(t)$ is position as a function of time,
  - velocity $v(t) = x'(t)$,
  - acceleration $a(t) = v'(t) = x''(t)$.

- Constant acceleration $a$:
  - $v(t) = at + C = at + v_0$ so that $v(0) = v_0$.
  - $x(t) = a/2 \ t^2 + v_0t + D$
Position–Velocity–Acceleration

If \( x(t) \) is position as a function of time,

- velocity \( v(t) = x'(t) \),
- acceleration \( a(t) = v'(t) = x''(t) \).

Constant acceleration \( a \):

- \( v(t) = at + C = at + v_0 \) so that \( v(0) = v_0 \).
- \( x(t) = a/2 \ t^2 + v_0 t + D = a/2 \ t^2 + v_0 t + x_0 \)
Position–Velocity–Acceleration

- If \( x(t) \) is position as a function of time,
  - velocity \( v(t) = x'(t) \),
  - acceleration \( a(t) = v'(t) = x''(t) \).

- Constant acceleration \( a \):
  - \( v(t) = at + C = at + v_0 \) so that \( v(0) = v_0 \).
  - \( x(t) = \frac{a}{2} t^2 + v_0 t + D = \frac{a}{2} t^2 + v_0 t + x_0 \)
  - Classic “projectile motion” (ball falling)
Tangent lines - simple ex
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• Let $f(x) = x^3 + 2x^2 - x + 2$. 
Tangent lines – simple ex

• Let \( f(x) = x^3 + 2x^2 - x + 2 \).

• Find tangent line at \( x=3 \).
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- Find tangent line at \( x=3 \).
- Need equation of line
  - slope is \( m=f'(3) \), point on line is \( (3,f(3)) \)
Tangent lines – simple ex

• Let \( f(x) = x^3 + 2x^2 - x + 2 \).

• Find tangent line at \( x=3 \).

• Need equation of line
  
  • slope is \( m=f'(3) \), point on line is \( (3,f(3)) \)
  
  • Either \( y = mx + b \) or \( y = m(x-a) + f(a) \)...
Tangent lines - simple ex

• Let \( f(x) = x^3 + 2x^2 - x + 2 \).

• Find tangent line at \( x=3 \).

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  • slope is \( m=f'(3) \), point on line is \( (3,f(3)) \)

  • Either \( y = mx + b \) or \( y = m(x-a) + f(a) \)...

(A) \( y = 3x + 44 \)  
(B) \( y = 38x + 44 \)  
(C) \( y = 38(x-3) + 44 \)  
(D) \( y = 44 \)
Tangent lines – simple ex

• Let \( f(x) = x^3 + 2x^2 - x + 2 \).

• Find tangent line at \( x=3 \).

• Need equation of line
  • slope is \( m=f'(3) \), point on line is \((3,f(3))\)
  • Either \( y = mx + b \) or \( y = m(x-a) + f(a) \)...

\begin{align*}
(A) \quad y &= 3x + 44 \\
(B) \quad y &= 38x + 44 \\
(C) \quad y &= 38(x-3) + 44 \\
(D) \quad y &= 44
\end{align*}