# Today

- Composition and chain rule, quotient rule
- Antiderivatives of power functions and polynomials
- Tangent lines
- Reminders:
  - Assignment3 Thursday 7am,
  - OSH 2 Friday 11:59 pm.
  - Sign up for midterm time/room.

# Composition of functions

If $f(x) = 2x+3$ and $g(x) = -4x+2$,

A. $h(x) = f(\ g(x)\ ) = -8x+7$

B. $h(x) = f(\ g(x)\ ) = -8x-10$

C. $h(x) = f(\ g(x)\ ) = -8x^2-8x+6$

D. $h(x) = f(\ g(x)\ ) = -8x+5$

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If $h(x) = f(g(x))$, then

A. $h'(x) = f'(x)\, g'(x)$

B. $h'(x) = f'(x)\, g(x) + f(x)\, g'(x)$

C. $h'(x) = f'(g'(x))$

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C. $h'(x) = f'(g'(x))$

D. $h'(x) = f'(g(x))\, g'(x)$ <---- Chain Rule

# Composition of functions

If $h(x)= (x^3-2x+1)^6$, then $h'(x)=?$

A. $6\ (x^3-2x+1)^5$

B. $(x^3-2x+1)^6\ (3x^2-2)$

C. $6\ (x^3-2x+1)^5\ (3x^2-2)$

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E. Are you kidding? It will take me weeks to multiply those out.

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$f(g) = g^6$

$g(x) = x^3-2x+1$

$h(x) = f(g(x))$

# Rules for differentiation - summary

- Addition rule:
  - $f(x) = g(x) + h(x)$ ----> $f'(x) = g'(x) + h'(x)$
- Product rule:
  - $f(x) = g(x)\, h(x)$ ----> $f'(x) = g'(x)\, h(x) + g(x)\, h'(x)$
- Chain rule:
  - $f(x) = g(\, h(x)\, )$ ----> $f'(x) = g'(\, h(x)\, )\, h'(x)$
- Quotient rule:
  - $f(x) = g(x) / h(x) = g(x)\, (h(x))^{-1}$ <---- apply product and chain rules or

Suppose $f(x) = g(x)/k(x)$ and that

$$g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$$

---

- What is f'(2)?

(A) -13

(B) -13/25

(C) -13/9

(D) 17/25

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# Antiderivatives - going backward

If $f'(x) = 6x^2 + 4x - 1$, then

(A) $f(x) = 12x + 4$

(B) $f(x) = 2x^3 + 2x^2 - x$

(C) $f(x) = 2x^3 + 2x^2 - x + 2$

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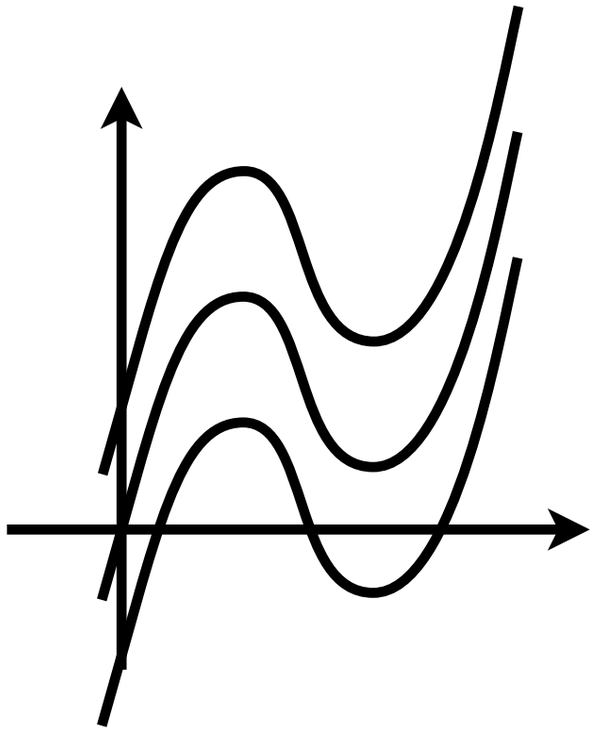
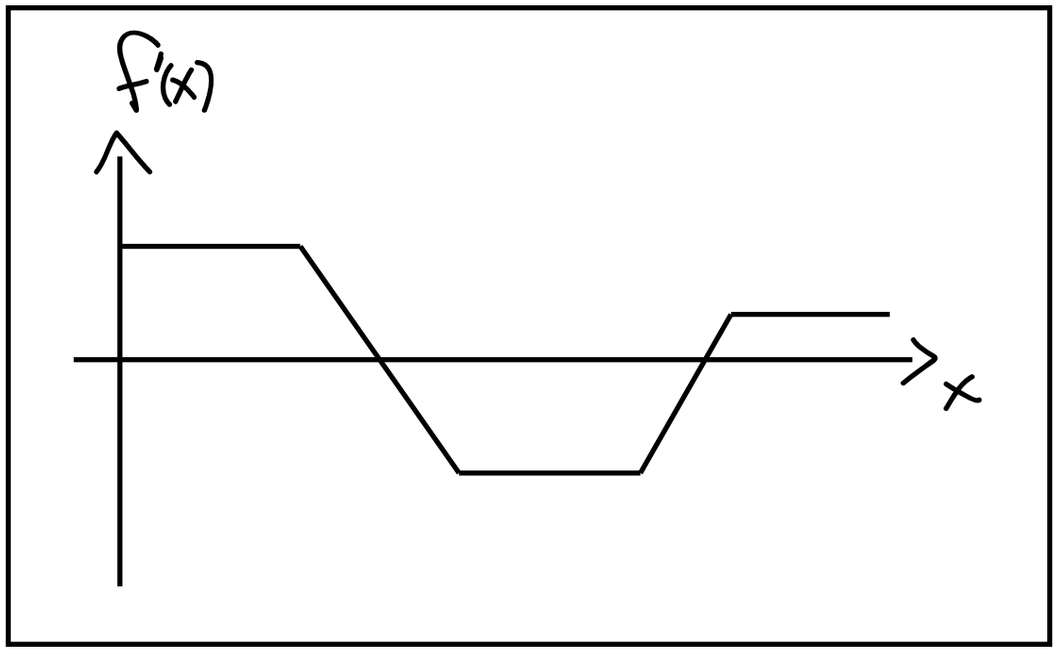
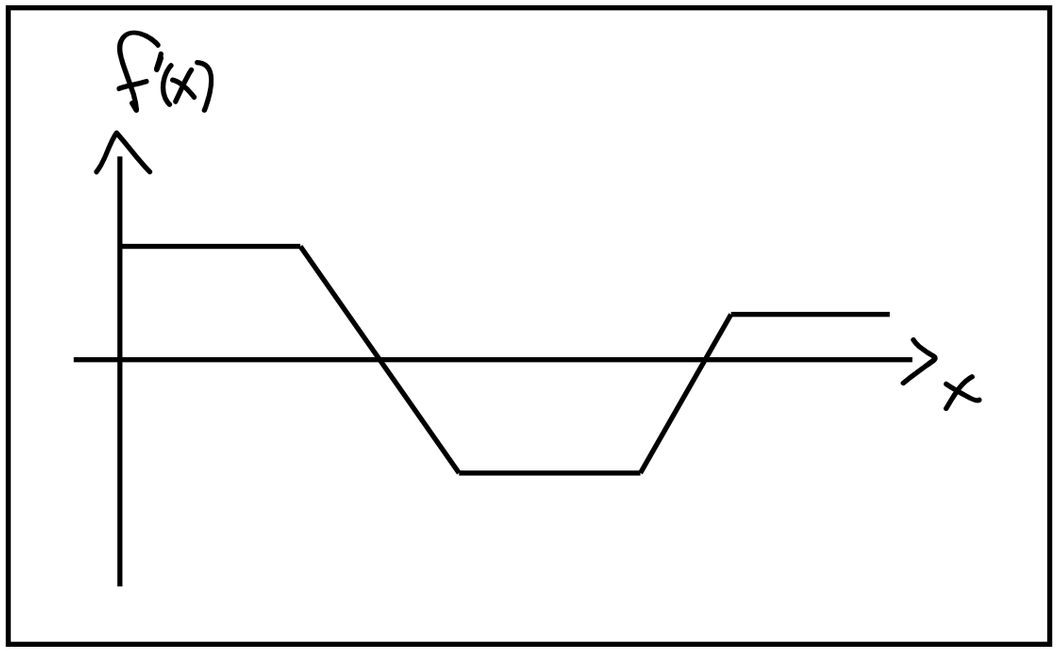


Slopes at each x don't change with vertical shift.




This is f'(x). Draw f(x).

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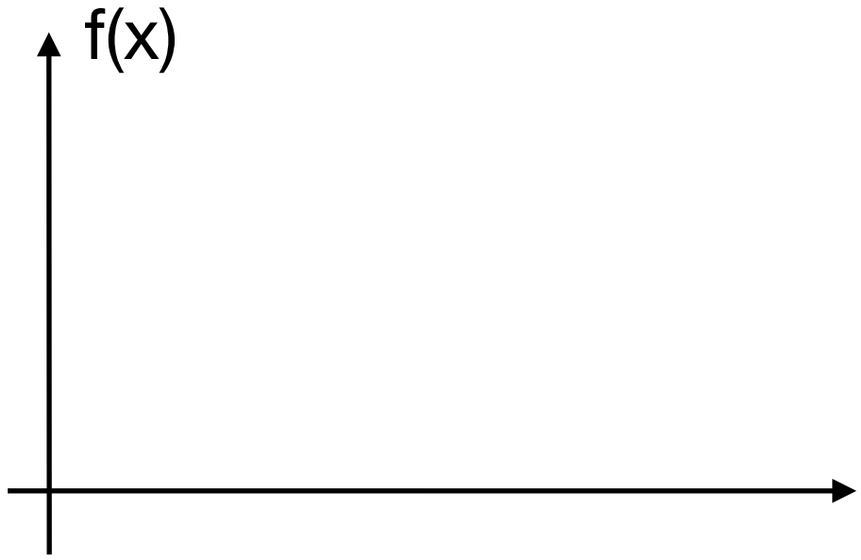

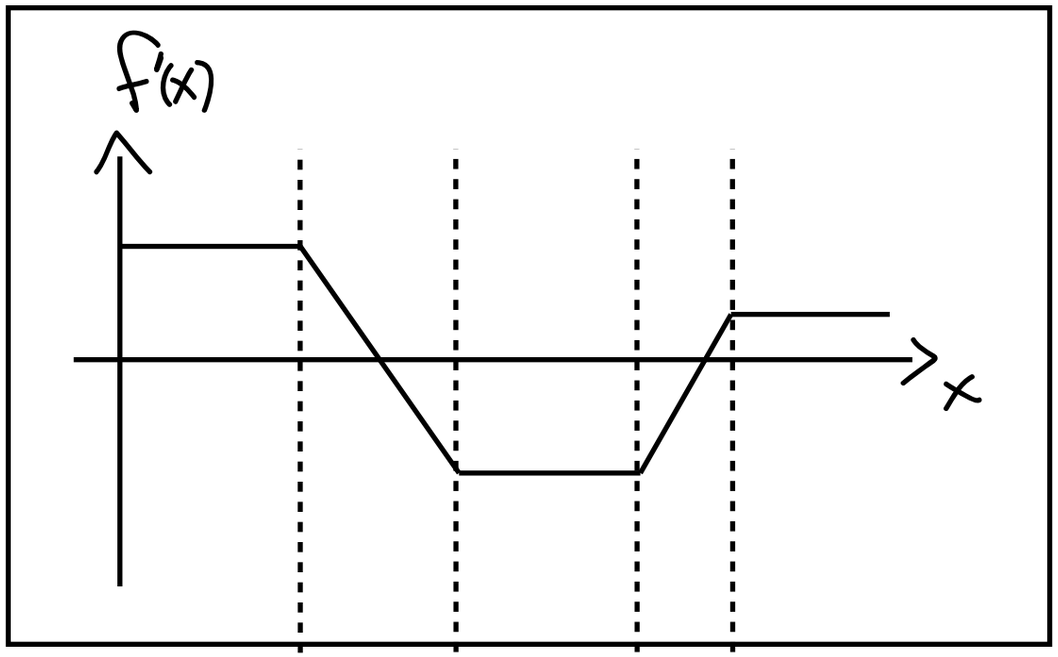


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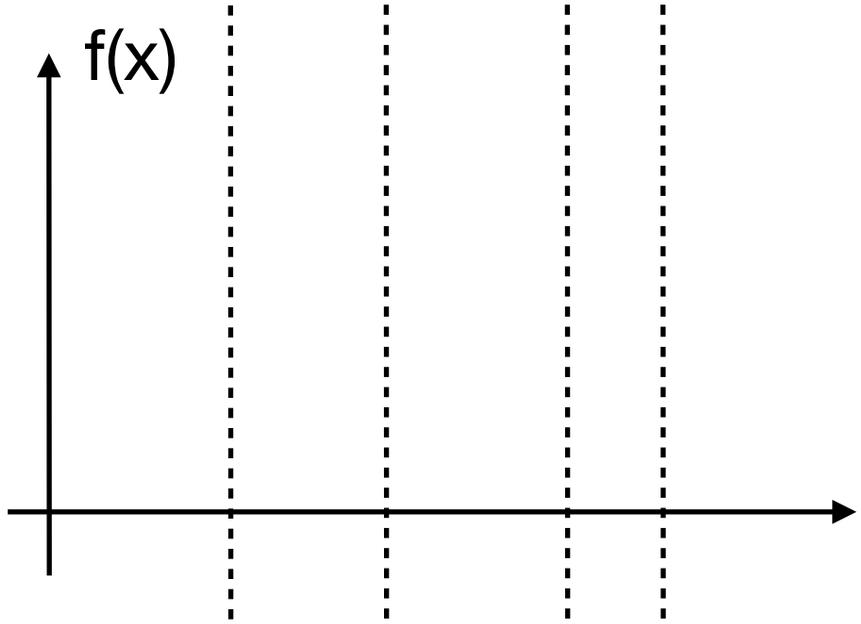

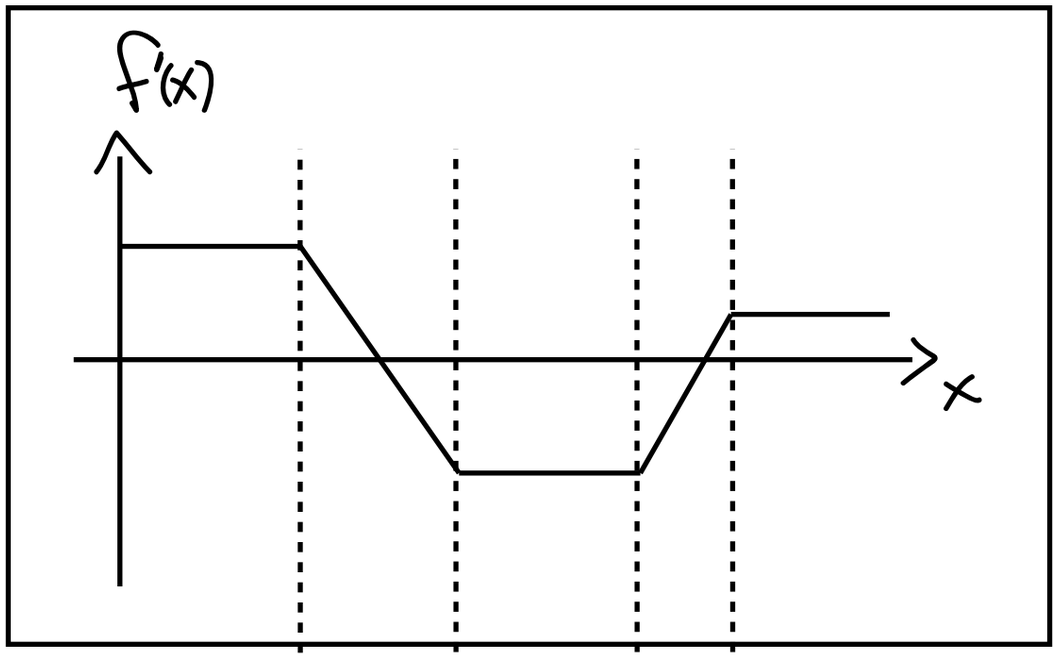


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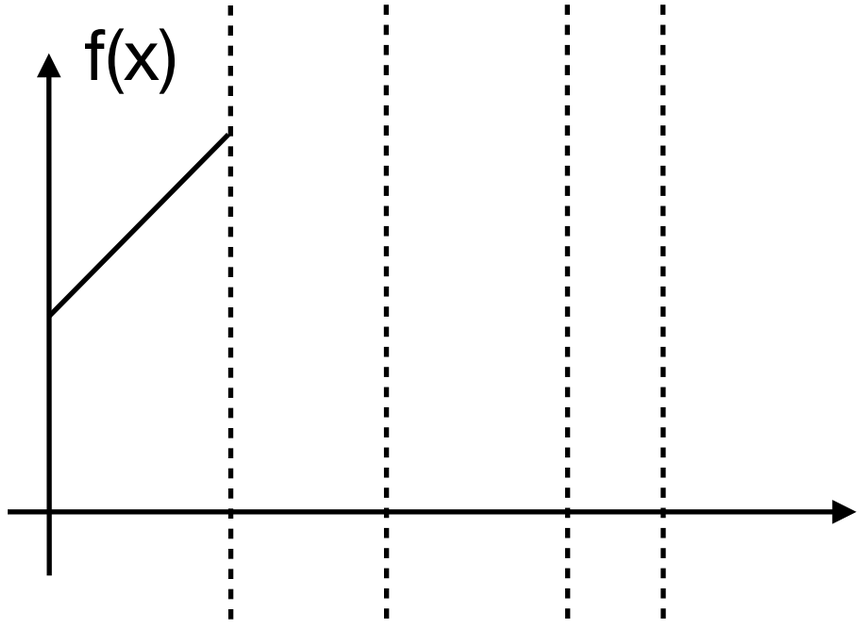

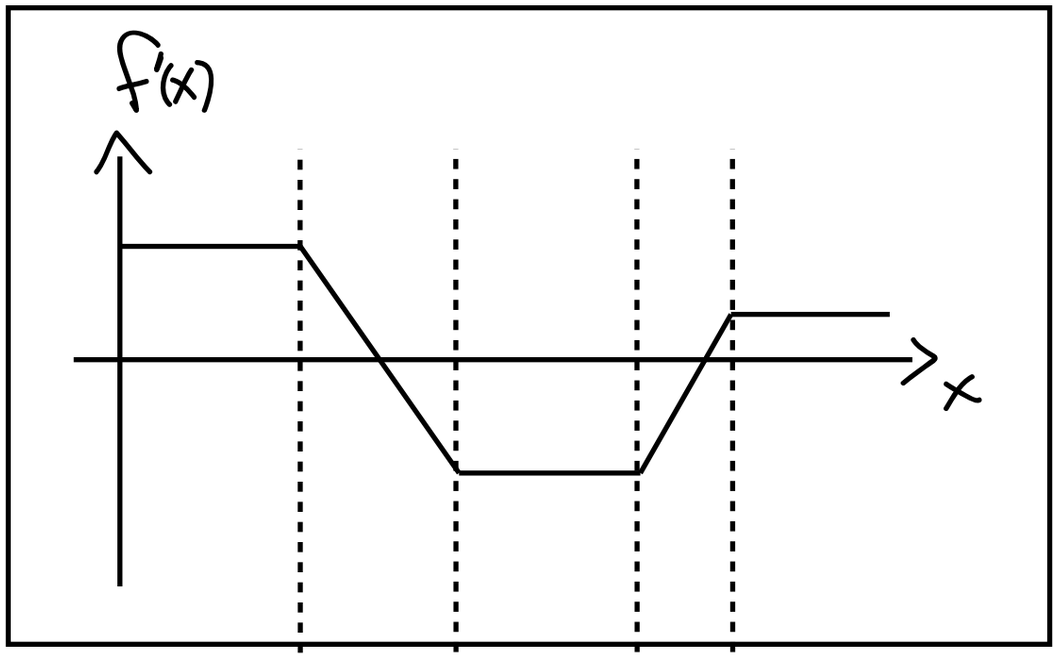


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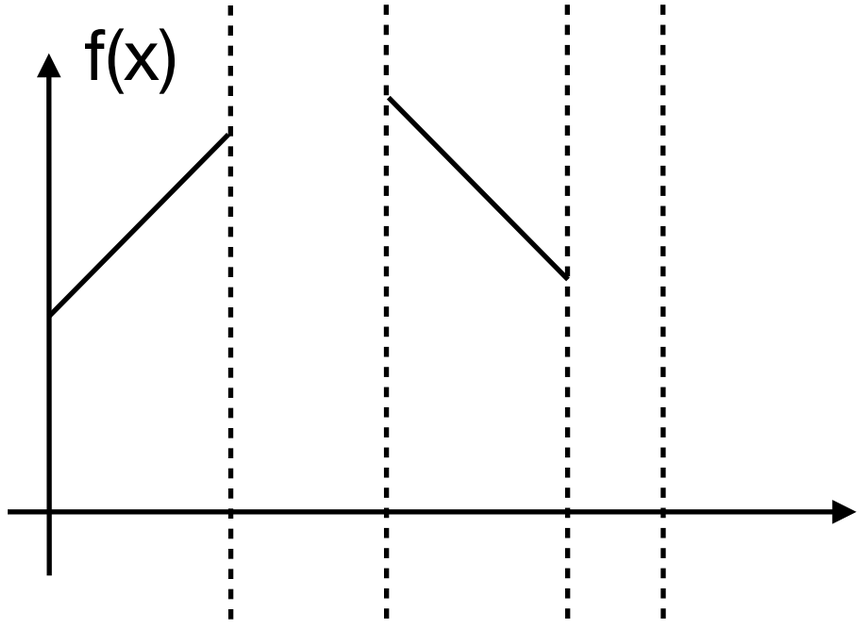

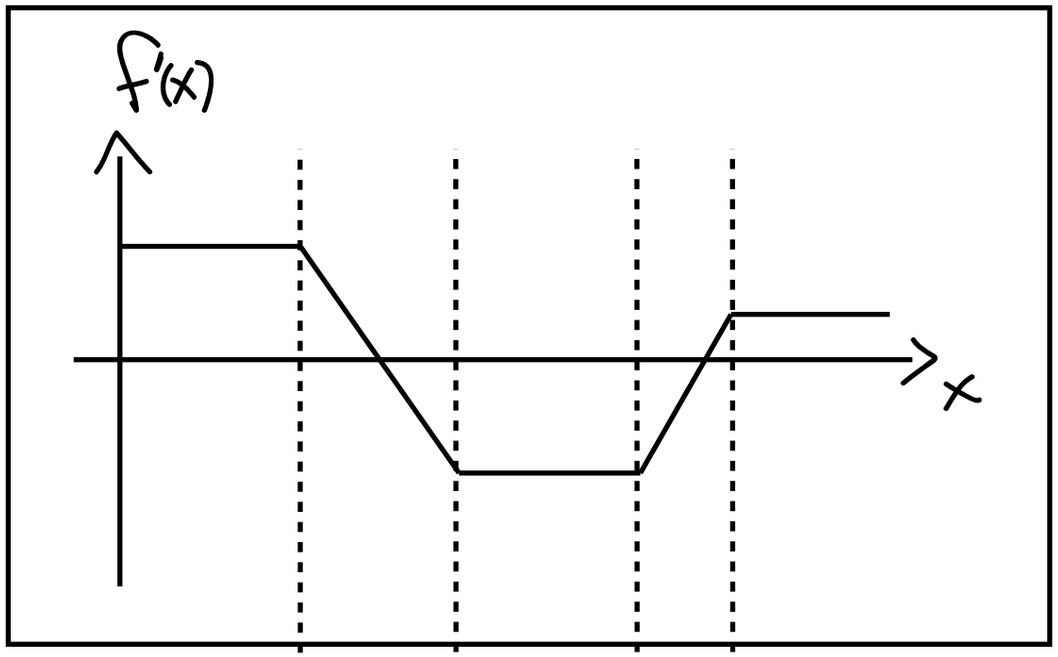


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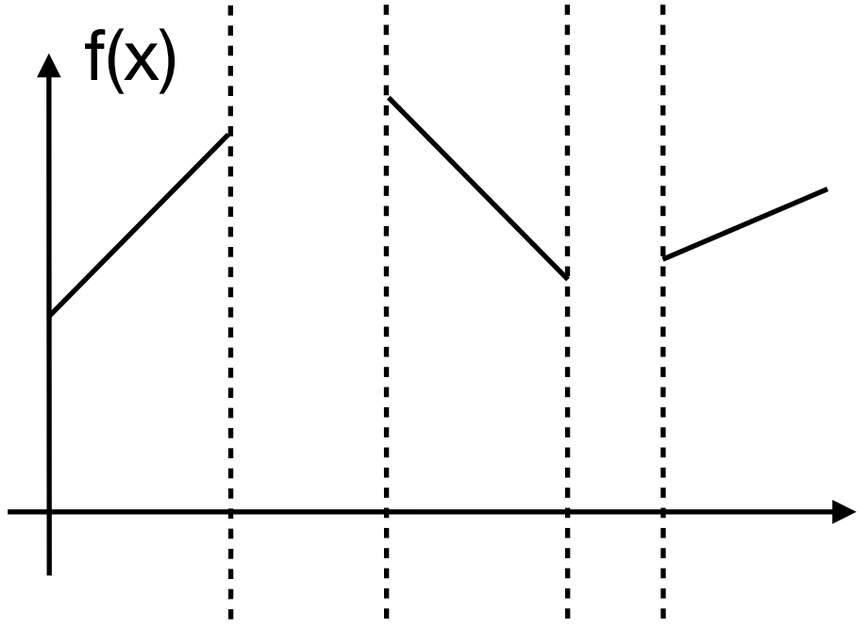

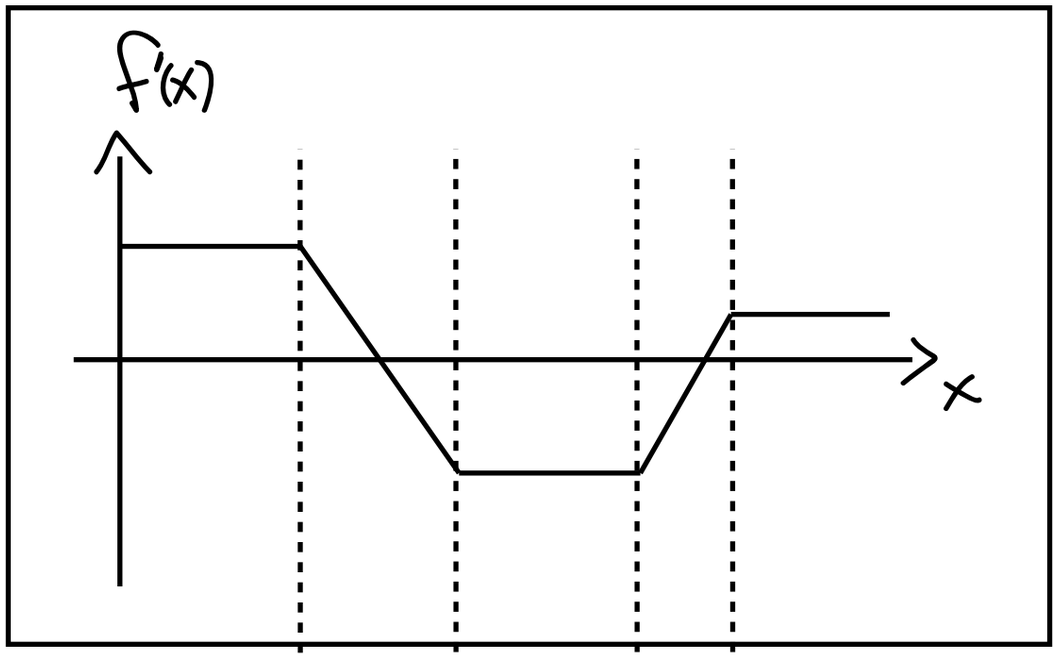


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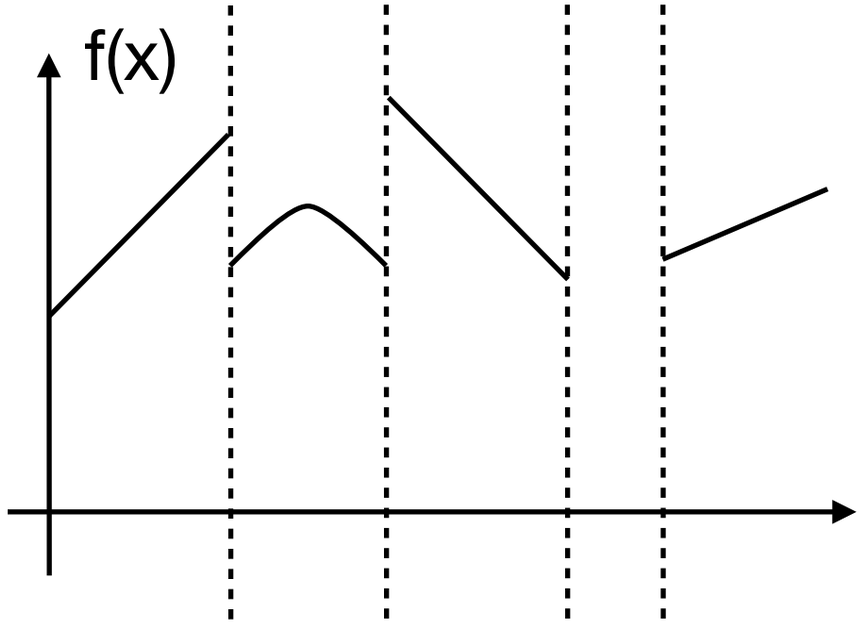

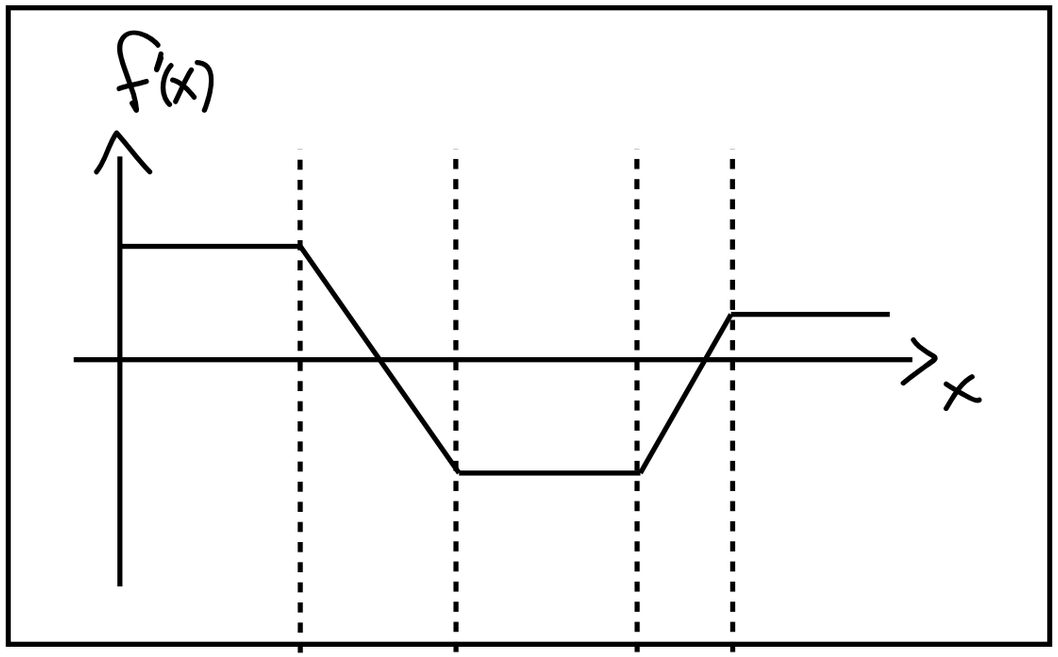


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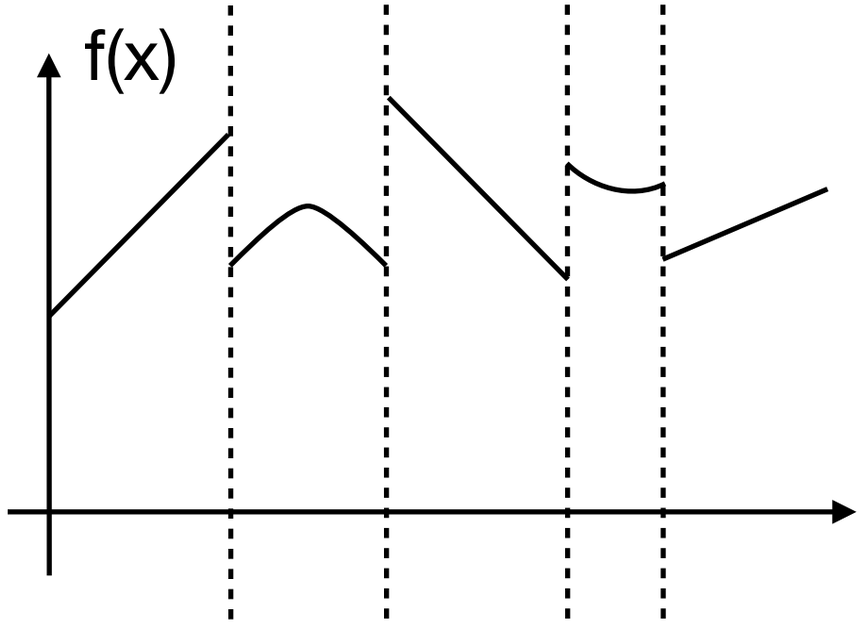

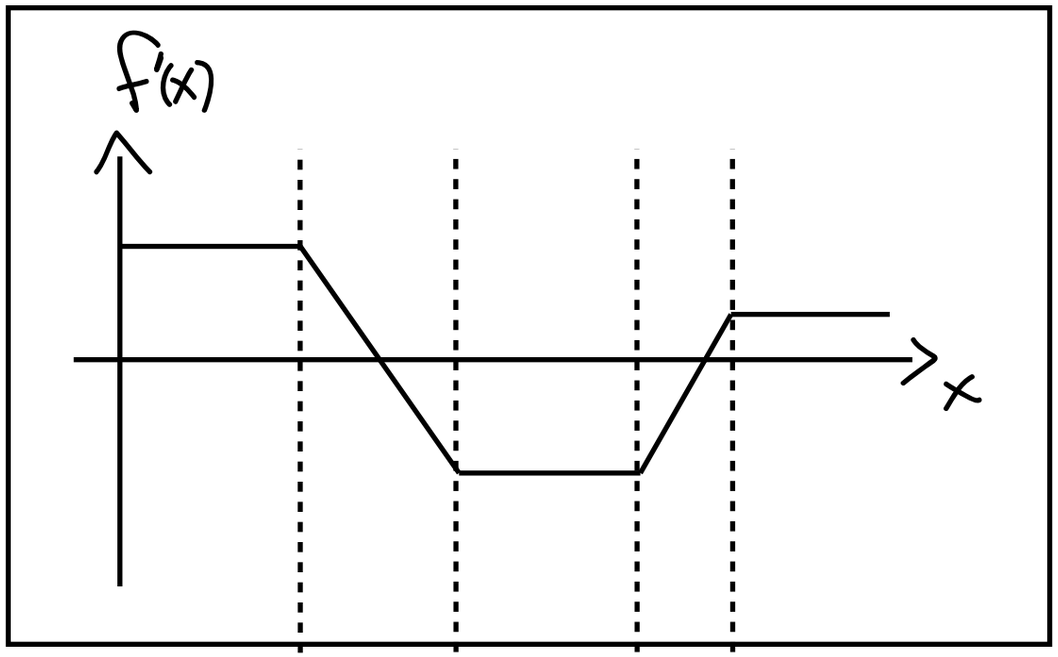


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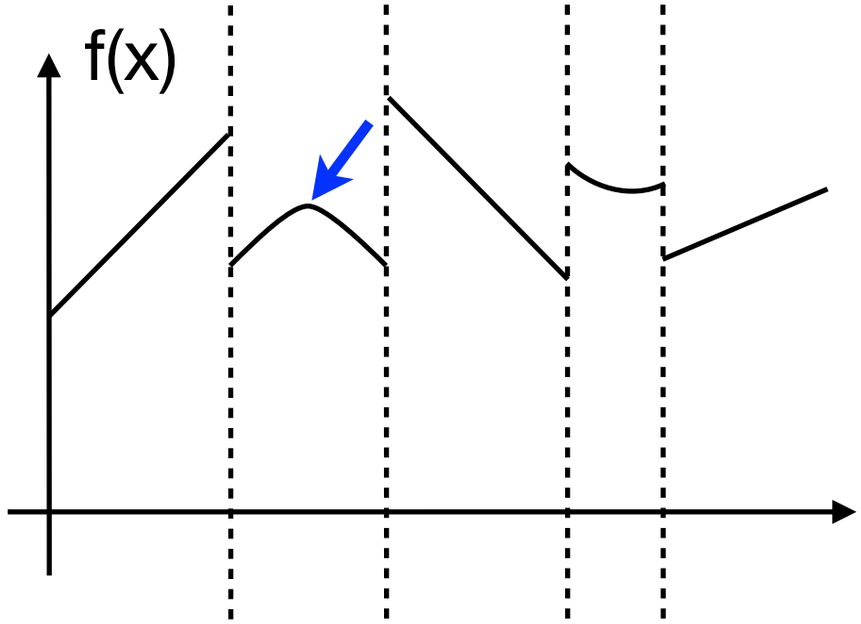

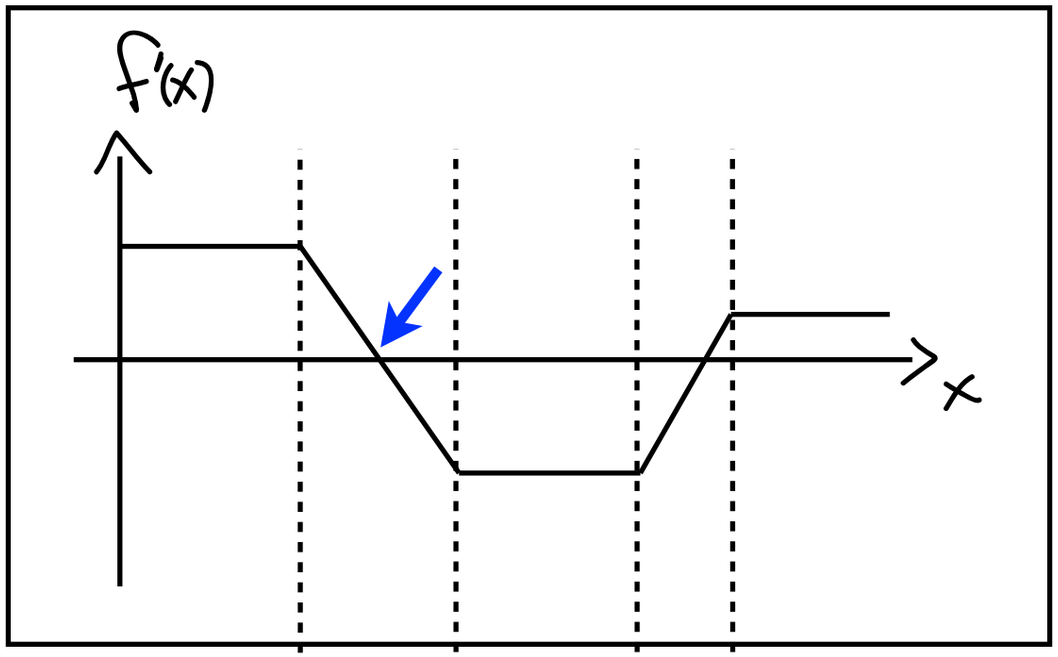


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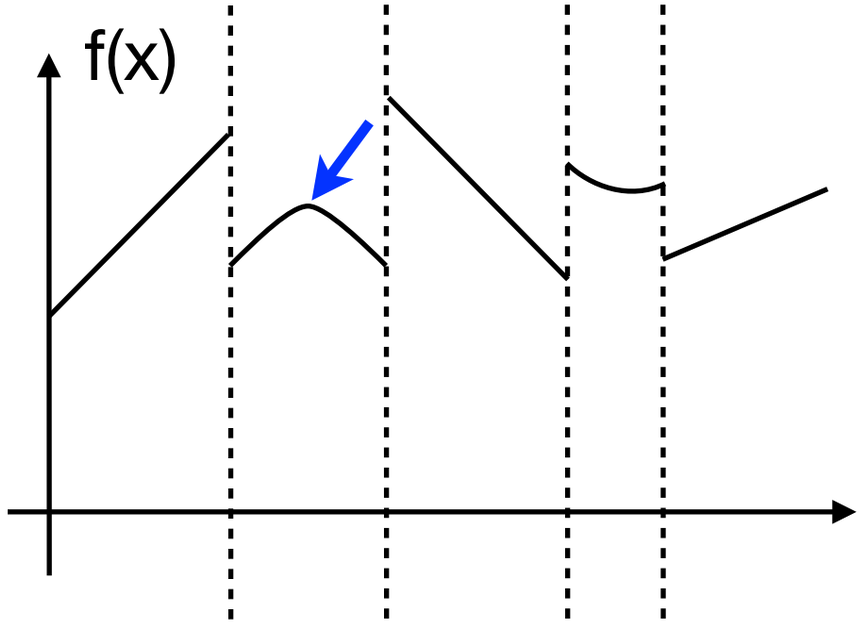

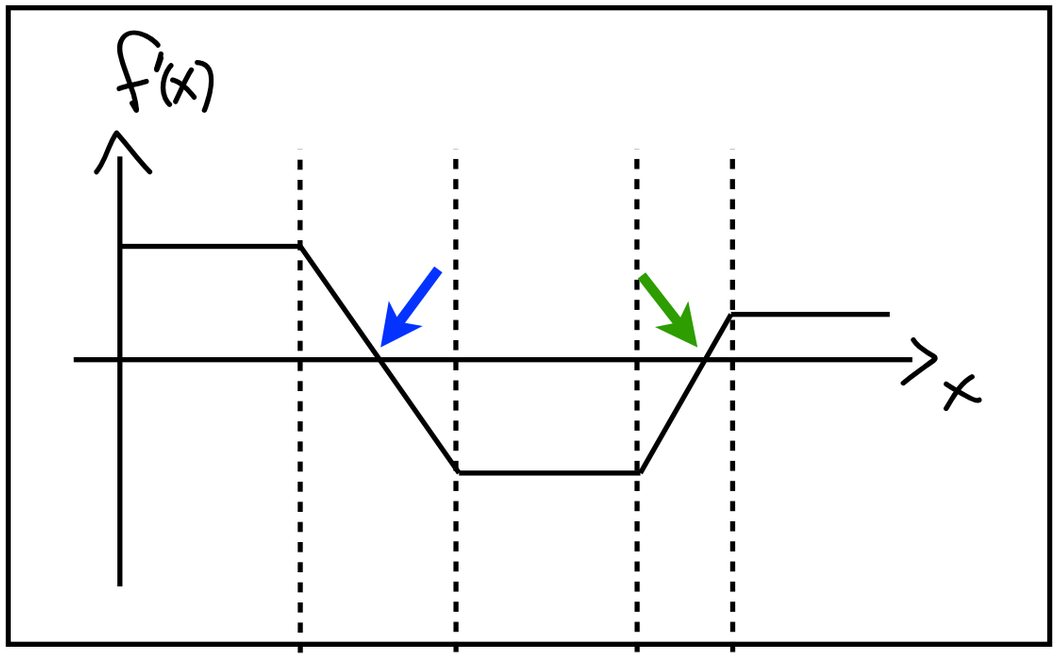


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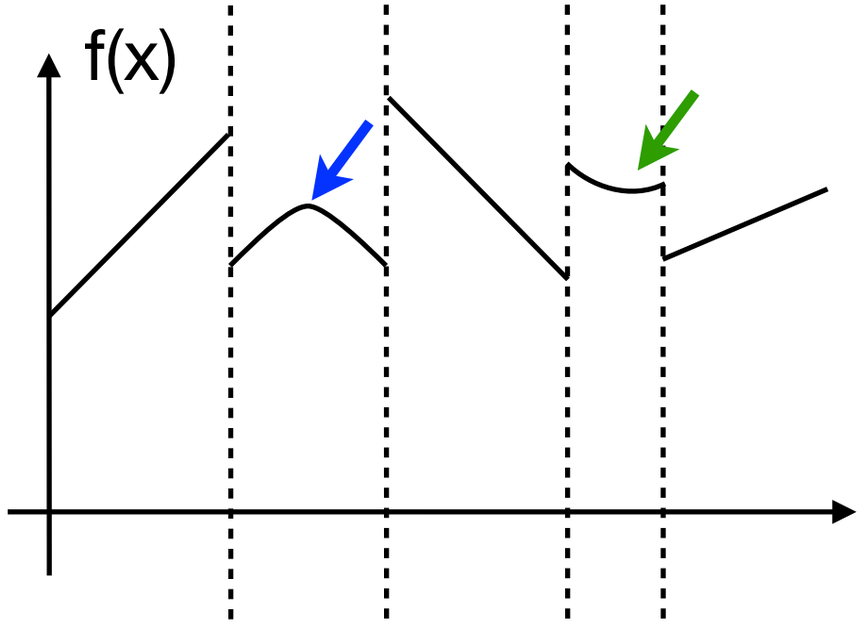

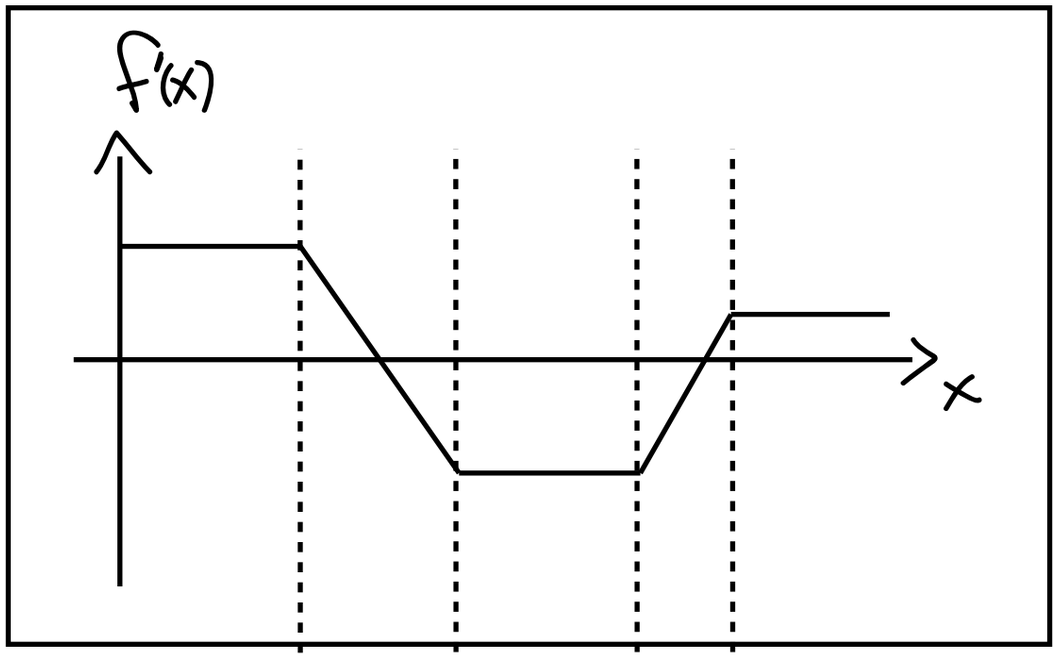


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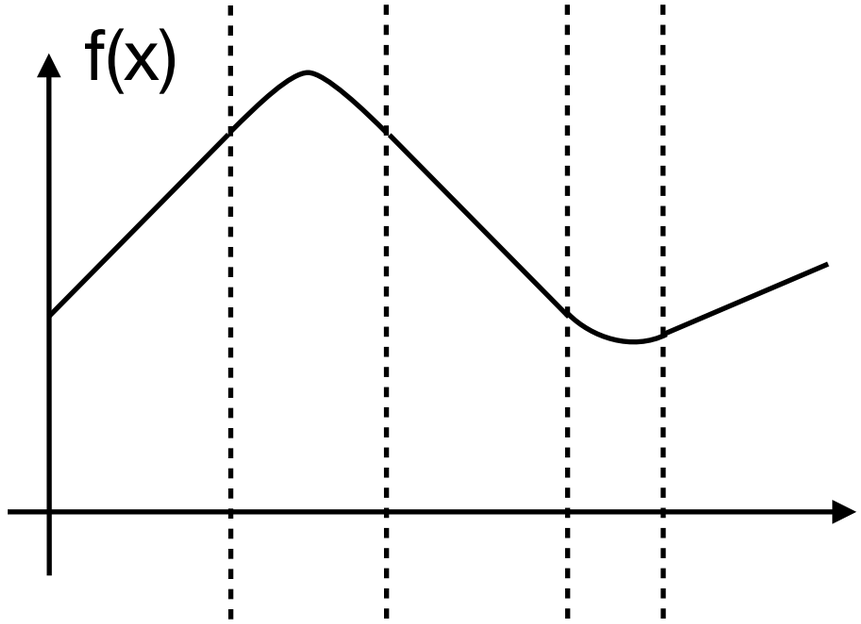

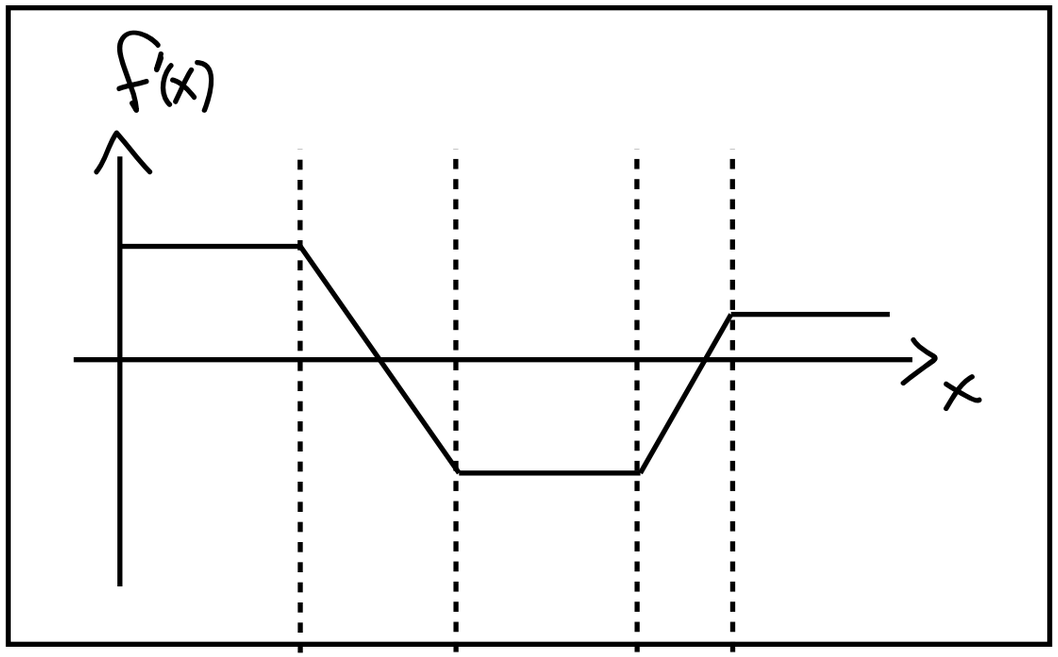


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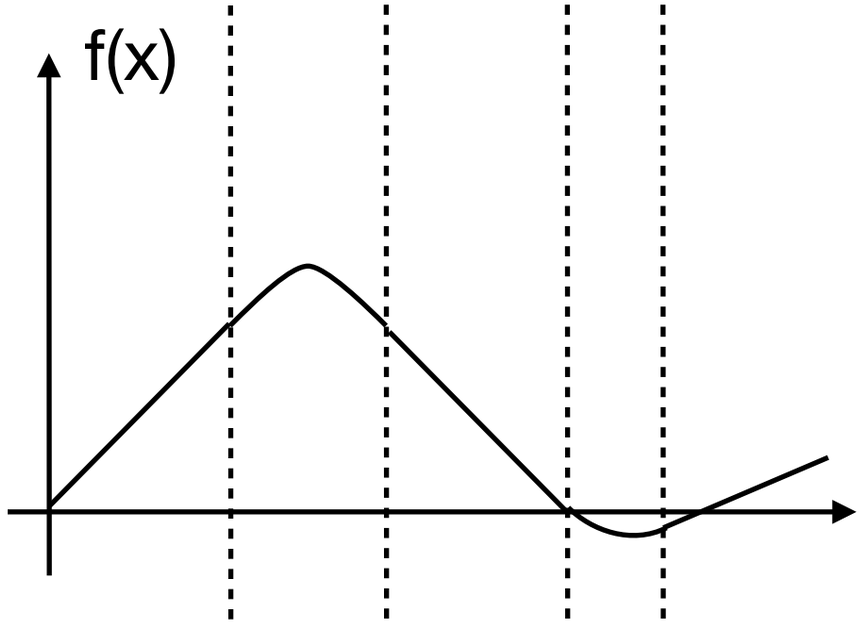


Only determined up to a vertical shift.

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  - $x(t) = a/2\ t^2 + v_0 t + D = a/2\ t^2 + v_0 t + x_0$
  - Classic "projectile motion" (ball falling)

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