Today...

- Points for 3 cases on OSH 1.
- Limits at infinity

Limits at infinity

- Polynomials
- Rational functions
  - power tricks (approx or divide by x^m)

Polynomials first...

- ex. $\lim_{x \to \infty} -x^3 + x^2 = ?$

- ex. $\lim_{x \to -\infty} x^6 + 10x^5 = ?$

- ex. $\lim_{x \to -\infty} 10x^5 + x^4 = ?$

Polynomials first...

- ex. $\lim_{x \to \infty} -x^3 + x^2 = ? \ -\infty$

  ➡ Highest power wins

- ex. $\lim_{x \to -\infty} x^6 + 10x^5 = ?$

- ex. $\lim_{x \to -\infty} 10x^5 + x^4 = ?$

Polynomials first...

- ex. $\lim_{x \to \infty} -x^3 + x^2 = ? \quad -\infty$

  ➡ Highest power wins

- ex. $\lim_{x \to -\infty} x^6 + 10x^5 = ? \quad \infty$

  ➡ Even powers kill minus signs.

- ex. $\lim_{x \to -\infty} 10x^5 + x^4 = ?$

Polynomials first...

- ex. $\lim_{x \to \infty} -x^3 + x^2 = ? \quad -\infty$

  ➡ Highest power wins

- ex. $\lim_{x \to -\infty} x^6 + 10x^5 = ? \quad \infty$

  ➡ Even powers kill minus signs.

- ex. $\lim_{x \to -\infty} 10x^5 + x^4 = ? \quad -\infty$

  ➡ Odd powers don't.

Polynomials first...

- ex. $\lim_{x \to \infty} -x^3 + x^2 = ? \quad -\infty$

  ➡ Highest power wins

- ex. $\lim_{x \to -\infty} x^6 + 10x^5 = ? \quad \infty$

  ➡ Even powers kill minus signs.

- ex. $\lim_{x \to -\infty} 10x^5 + x^4 = ? \quad -\infty$

  ➡ Odd powers don't.

On the board - technique for rational functions...

Two ways to find limits at infinity

$$\lim_{x \to -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$$

For |x| large, $4x^7 - 2x^4 + 4$

(A) ≈ 4

(B) $\approx 4x^7$

(C) $\longrightarrow \infty$

(D) $\longrightarrow -\infty$

For |x| large, $4x^7 - 2x^4 + 4$

(A) ≈ 4

(B) $\approx 4x^7$

(C) $\longrightarrow \infty$

(D) $\longrightarrow -\infty$

For |x| large, $10x^5 - 3x^3 + 4x$

(A) $\approx 4x$

(B) $\approx 10x^5$

(C) $\longrightarrow \infty$

(D) $\longrightarrow -\infty$

For |x| large, $10x^5 - 3x^3 + 4x$

(A) $\approx 4x$

(B) $\approx 10x^5$

(C) $\longrightarrow \infty$

(D) $\longrightarrow -\infty$

For x large, $\dfrac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$

(A) ≈ 4 / 10

(B) ≈ 2 / 5

(C) ≈ 2 / ($5x^2$)

(D) ≈ $2x^2$ / 5

For |x| large, $\dfrac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$

(A) ≈ 4 / 10

(B) ≈ 2 / 5

(C) ≈ 2 / (5x²)

(D) ≈ 2x² / 5

For |x| large, $\dfrac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$

(A) ≈ 4 / 10

(B) ≈ 2 / 5

(C) ≈ 2 / (5x²)

(D) ≈ 2x² / 5

$$\lim_{x \to -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x} =$$

For |x| large, $\dfrac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$

(A) ≈ 4 / 10

(B) ≈ 2 / 5

(C) ≈ 2 / (5x²)

(D) ≈ 2x² / 5

$$\lim_{x \to -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x} = \infty$$

Or divide top/bottom by highest power in denom.

$$\lim_{x \to -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x} =$$

Or divide top/bottom by highest power in denom.

$$\lim_{x \to -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x} =$$

$$\lim_{x \to -\infty} \frac{4x^2 - \frac{2}{x} + \frac{4}{x^5}}{10 - \frac{3}{x^2} + \frac{4}{x^4}}$$

Or divide top/bottom by highest power in denom.

$$\lim_{x\to-\infty}\frac{4x^7-2x^4+4}{10x^5-3x^3+4x}=$$

$$\lim_{x\to-\infty}\frac{4x^2-\cancel{\frac{2}{x}}+\frac{4}{x^5}}{10-\frac{3}{x^2}+\frac{4}{x^4}}$$

Or divide top/bottom by highest power in denom.

$$\lim_{x \to -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x} =$$

$$\lim_{x \to -\infty} \frac{4x^2 - \cancel{\frac{2}{x}} + \cancel{\frac{4}{x^5}}}{10 - \frac{3}{x^2} + \frac{4}{x^4}}$$

Or divide top/bottom by highest power in denom.

$$\lim_{x\to-\infty}\frac{4x^7-2x^4+4}{10x^5-3x^3+4x}=$$

$$\lim_{x\to-\infty}\frac{4x^2-\cancel{\frac{2}{x}}+\cancel{\frac{4}{x^5}}}{10-\cancel{\frac{3}{x^2}}+\frac{4}{x^4}}$$

Or divide top/bottom by highest power in denom.

$$\lim_{x\to-\infty}\frac{4x^7-2x^4+4}{10x^5-3x^3+4x}=$$

$$\lim_{x\to-\infty}\frac{4x^2-\cancel{\frac{2}{x}}+\cancel{\frac{4}{x^5}}}{10-\cancel{\frac{3}{x^2}}+\cancel{\frac{4}{x^4}}}$$

Or divide top/bottom by highest power in denom.

$$\lim_{x\to-\infty}\frac{4x^7-2x^4+4}{10x^5-3x^3+4x}=$$

$$\lim_{x\to-\infty}\frac{4x^2-\cancel{\frac{2}{x}}+\cancel{\frac{4}{x^5}}}{10-\cancel{\frac{3}{x^2}}+\cancel{\frac{4}{x^4}}}=\infty$$

What is $\lim_{x \to -\infty} \frac{3x^n + x^2 - 1}{x^3 + 4}$?

- (A) If n=2, the limit is $-\infty$.
- (B) If n=3, the limit is ∞ .
- (C) If n>3 and even, the limit is $-\infty$.
- (D) If n>3 and odd, the limit is $-\infty$.

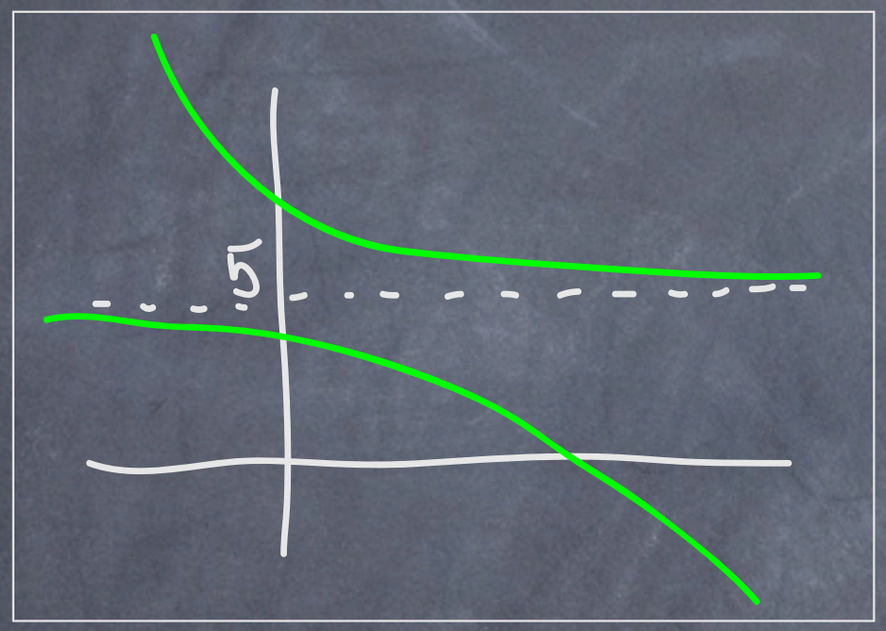
Suppose $\lim_{x \to \infty} f(x) = 3$.

What is $\lim_{x \to -\infty} f(x)$?

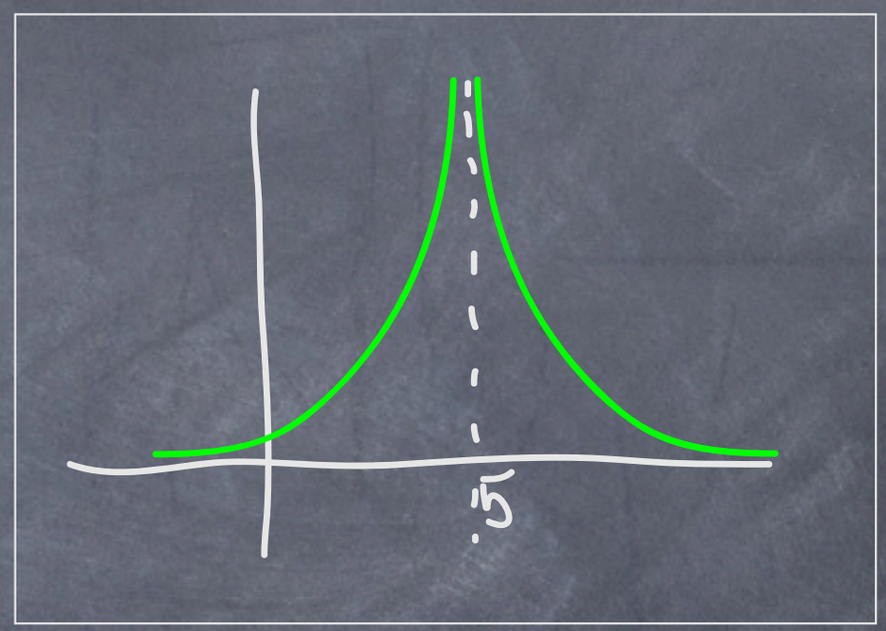
- (A) 3
- (B) -3
- (C) If f(x) is even, then the limit is 3.
- (D) If f(x) is odd, then the limit is 3.

Which of the following could be f(x) where $\lim_{x \to \infty} f(x) = 5$?

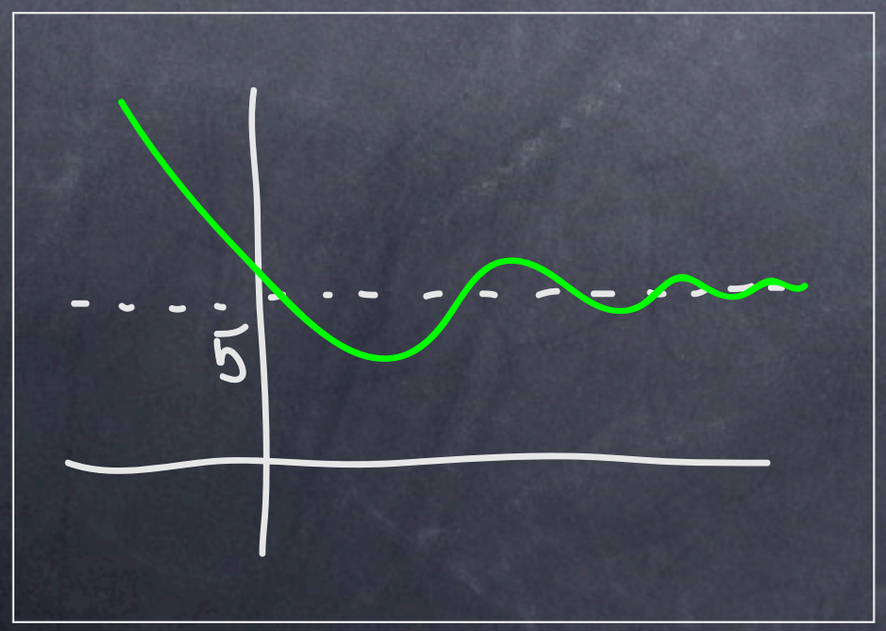
(A)



(C)



(B)



(D)