CALCULUS I

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Outline

Here is a listing and brief description of the material in this set of notes.

Review

Review : Functions – Here is a quick review of functions, function notation and a couple of fairly important ideas about functions.
Review : Inverse Functions – A quick review of inverse functions and the notation for inverse functions.
Review : Trig Functions – A review of trig functions, evaluation of trig functions and the unit circle. This section usually gets a quick review in my class.
Review : Solving Trig Equations – A reminder on how to solve trig equations. This section is always covered in my class.
Review : Solving Trig Equations with Calculators, Part I – The previous section worked problems whose answers were always the "standard" angles. In this section we work some problems whose answers are not "standard" and so a calculator is needed. This section is always covered in my class as most trig equations in the remainder will need a calculator.
Review : Solving Trig Equations with Calculators, Part II – Even more trig equations requiring a calculator to solve.
Review : Exponential Functions – A review of exponential functions. This section usually gets a quick review in my class.
Review : Logarithm Functions – A review of logarithm functions and logarithm properties. This section usually gets a quick review in my class.
Review : Exponential and Logarithm Equations – How to solve exponential and logarithm equations. This section is always covered in my class.
Review : Common Graphs – This section isn't much. It's mostly a collection of graphs of many of the common functions that are liable to be seen in a Calculus class.

Limits

Tangent Lines and Rates of Change – In this section we will take a look at two problems that we will see time and again in this course. These problems will be used to introduce the topic of limits.
The Limit – Here we will take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us.
One-Sided Limits – A brief introduction to one-sided limits.
Limit Properties – Properties of limits that we'll need to use in computing limits. We will also compute some basic limits in this section

Computing Limits – Many of the limits we'll be asked to compute will not be "simple" limits. In other words, we won't be able to just apply the properties and be done. In this section we will look at several types of limits that require some work before we can use the limit properties to compute them.

Infinite Limits – Here we will take a look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes.
Limits At Infinity, Part I – In this section we'll look at limits at infinity. In other words, limits in which the variable gets very large in either the positive or negative sense. We'll also take a brief look at horizontal asymptotes in this section. We'll be concentrating on polynomials and rational expression involving polynomials in this section.
Limits At Infinity, Part II – We'll continue to look at limits at infinity in this section, but this time we'll be looking at exponential, logarithms and inverse tangents.
Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Mean Value Theorem in this section.
The Definition of the Limit – We will give the exact definition of several of the limits covered in this section. We'll also give the exact definition of continuity.

Derivatives

The Definition of the Derivative – In this section we will be looking at the definition of the derivative.
Interpretation of the Derivative – Here we will take a quick look at some interpretations of the derivative.
Differentiation Formulas – Here we will start introducing some of the differentiation formulas used in a calculus course.
Product and Quotient Rule – In this section we will took at differentiating products and quotients of functions.
Derivatives of Trig Functions – We'll give the derivatives of the trig functions in this section.
Derivatives of Exponential and Logarithm Functions – In this section we will get the derivatives of the exponential and logarithm functions.
Derivatives of Inverse Trig Functions – Here we will look at the derivatives of inverse trig functions.
Derivatives of Hyperbolic Functions – Here we will look at the derivatives of hyperbolic functions.
Chain Rule – The Chain Rule is one of the more important differentiation rules and will allow us to differentiate a wider variety of functions. In this section we will take a look at it.
Implicit Differentiation – In this section we will be looking at implicit differentiation. Without this we won't be able to work some of the applications of derivatives.

Related Rates – In this section we will look at the lone application to derivatives in this chapter. This topic is here rather than the next chapter because it will help to cement in our minds one of the more important concepts about derivatives and because it requires implicit differentiation.

Higher Order Derivatives – Here we will introduce the idea of higher order derivatives.

Logarithmic Differentiation – The topic of logarithmic differentiation is not always presented in a standard calculus course. It is presented here for those who are interested in seeing how it is done and the types of functions on which it can be used.

Applications of Derivatives

Rates of Change – The point of this section is to remind us of the application/interpretation of derivatives that we were dealing with in the previous chapter. Namely, rates of change.

Critical Points – In this section we will define critical points. Critical points will show up in many of the sections in this chapter so it will be important to understand them.

Minimum and Maximum Values – In this section we will take a look at some of the basic definitions and facts involving minimum and maximum values of functions.

Finding Absolute Extrema – Here is the first application of derivatives that we'll look at in this chapter. We will be determining the largest and smallest value of a function on an interval.

The Shape of a Graph, Part I – We will start looking at the information that the first derivatives can tell us about the graph of a function. We will be looking at increasing/decreasing functions as well as the First Derivative Test.

The Shape of a Graph, Part II – In this section we will look at the information about the graph of a function that the second derivatives can tell us. We will look at inflection points, concavity, and the Second Derivative Test.

The Mean Value Theorem – Here we will take a look at the Mean Value Theorem.

Optimization Problems – This is the second major application of derivatives in this chapter. In this section we will look at optimizing a function, possibly subject to some constraint.

More Optimization Problems – Here are even more optimization problems.

L'Hospital's Rule and Indeterminate Forms – This isn't the first time that we've looked at indeterminate forms. In this section we will take a look at L'Hospital's Rule. This rule will allow us to compute some limits that we couldn't do until this section.

Linear Approximations – Here we will use derivatives to compute a linear approximation to a function. As we will see however, we've actually already done this.

Differentials – We will look at differentials in this section as well as an application for them.
Newton's Method – With this application of derivatives we'll see how to approximate solutions to an equation.
Business Applications – Here we will take a quick look at some applications of derivatives to the business field.