Today

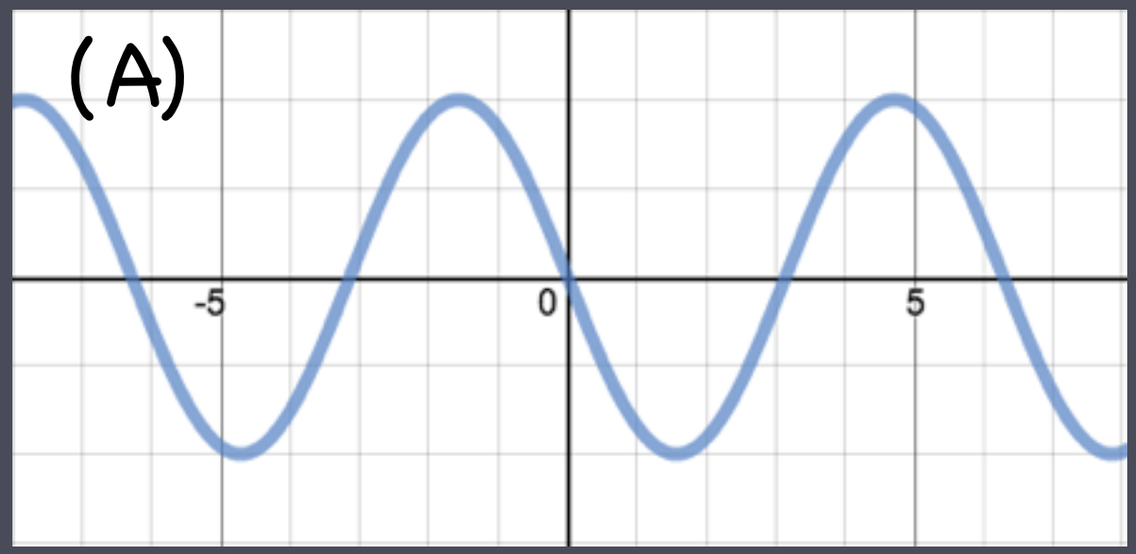
- Quiz 3 - what's on it?
- $y' = \sin(y)$
- Newton's Law of Cooling.
- Solving a linear ODE ($y'=ay+b$) using a shift.
- Translating from word problem to IVP.

Quiz 3

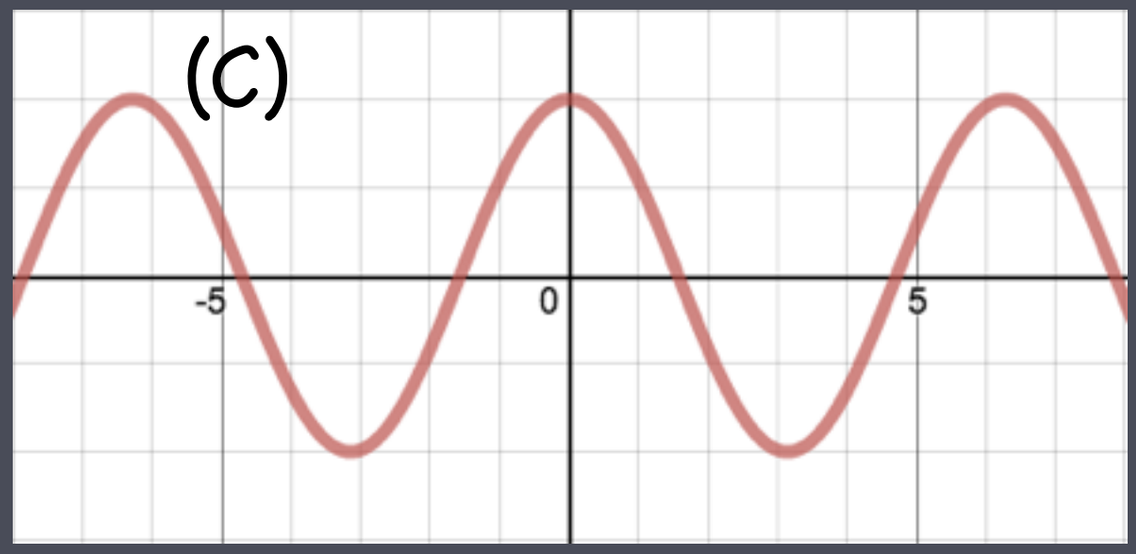
- Exponential functions and their derivatives
- Inverse functions, the log function and its derivative
- Applications using exponential functions
- Qualitative analysis of ODEs (slope fields, phase lines, steady states, stability)
- This means part of Week 8, all of Weeks 9 & 10.
- Not appearing but still important for the final exam: least squares, related rates, implicit differentiation

f(y) = sin(y)

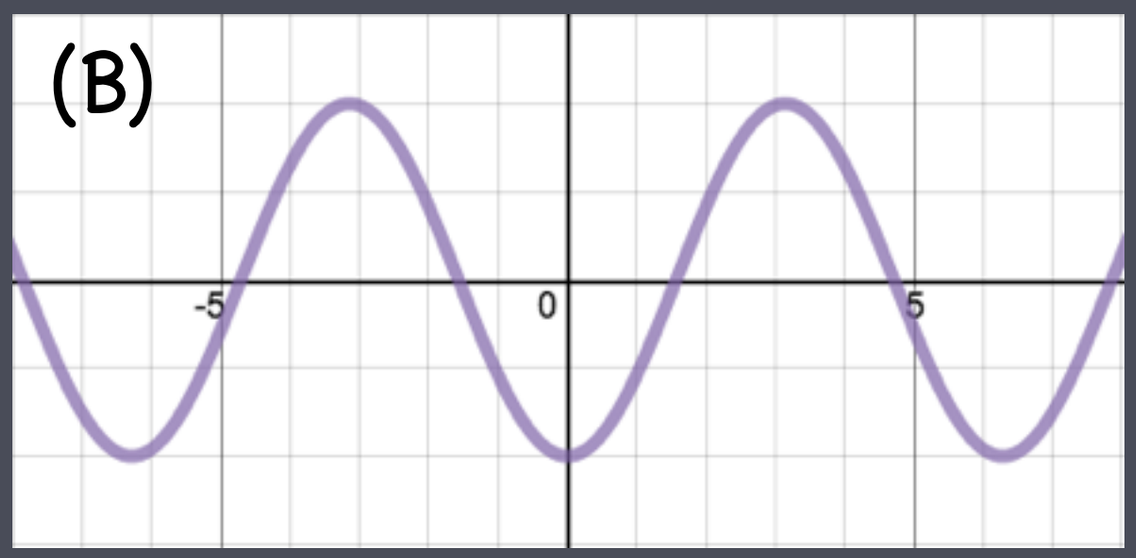
(A)



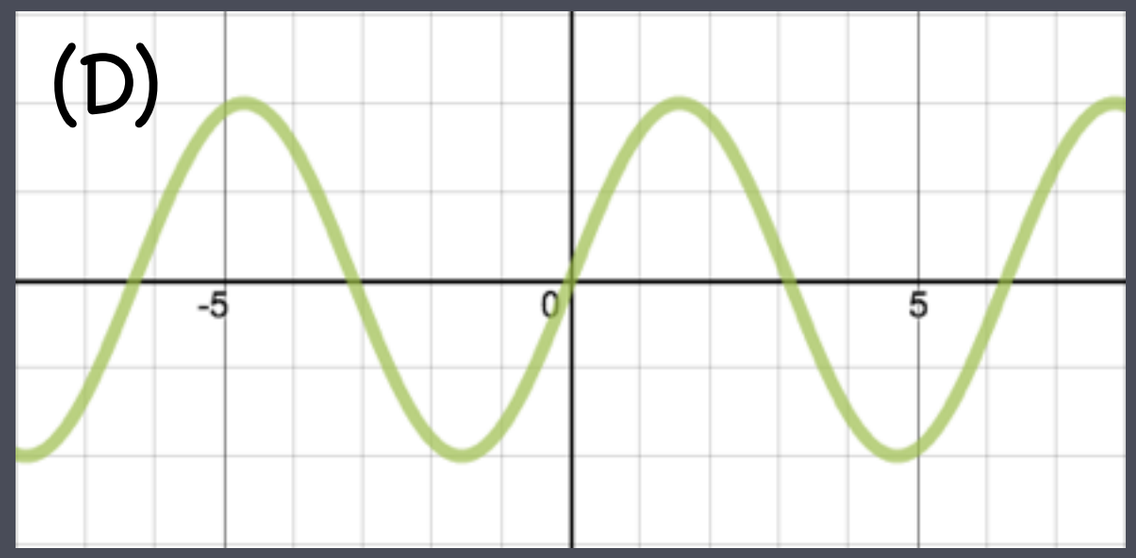
(C)



(B)



(D)



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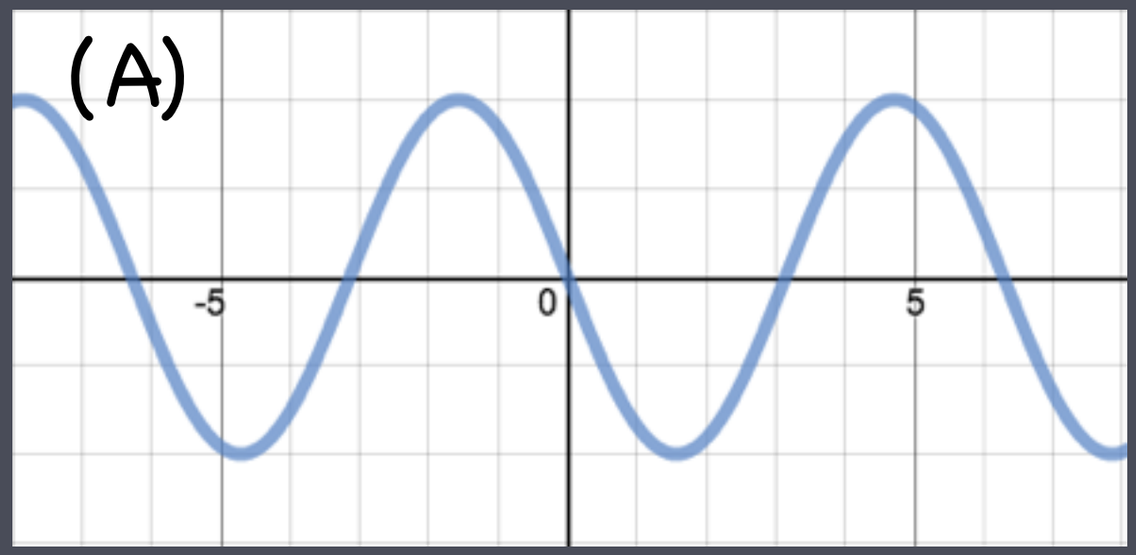
(A)



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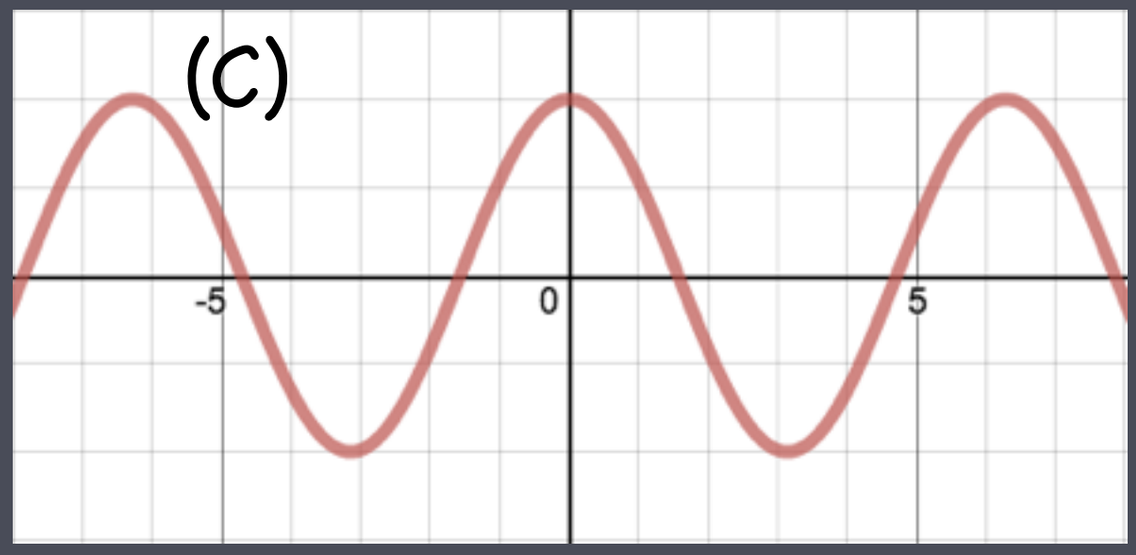
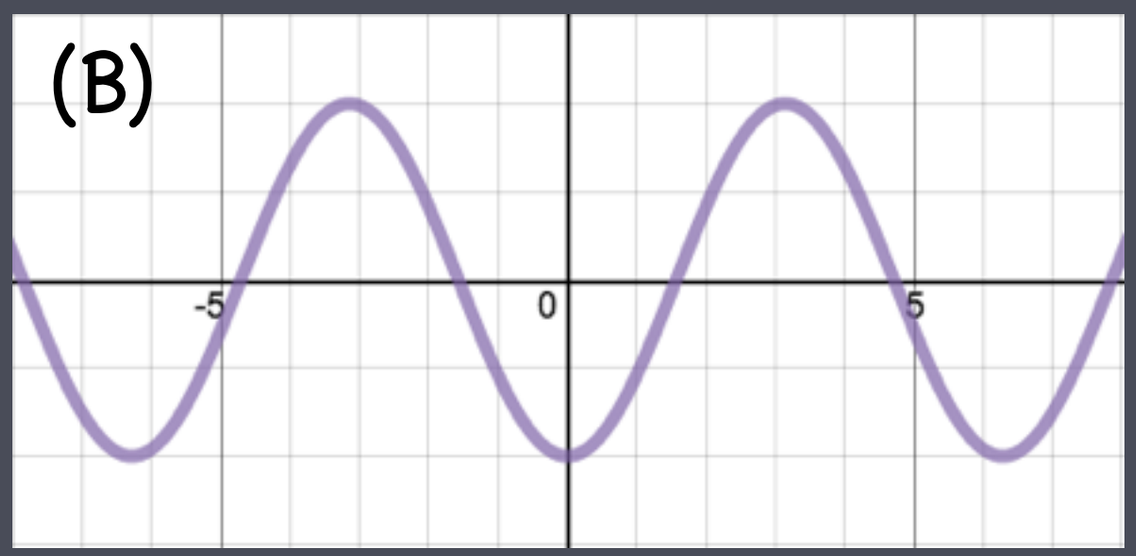


(B)



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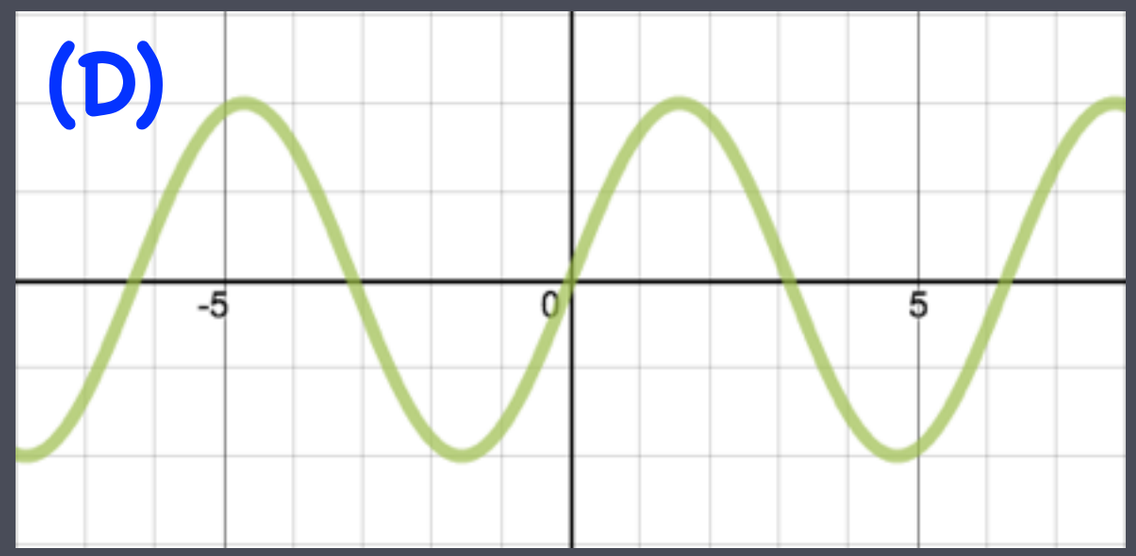
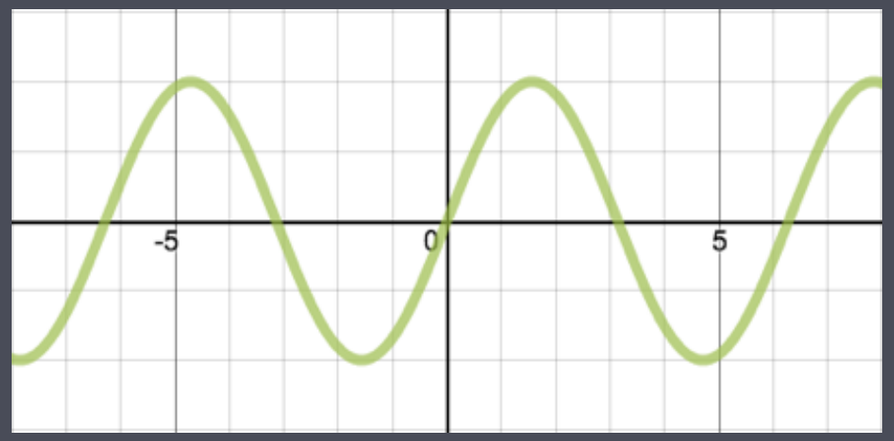


$y' = \sin(y)$



A solution satisfying the initial condition $y(0)=y_0$ will approach y^* as $t \dashrightarrow \infty$. Which y_0 and y^* pair is correct?

(A) $y_0 = -3\pi/2$, $y^* = -2\pi$. X ---> $y^* = -\pi$

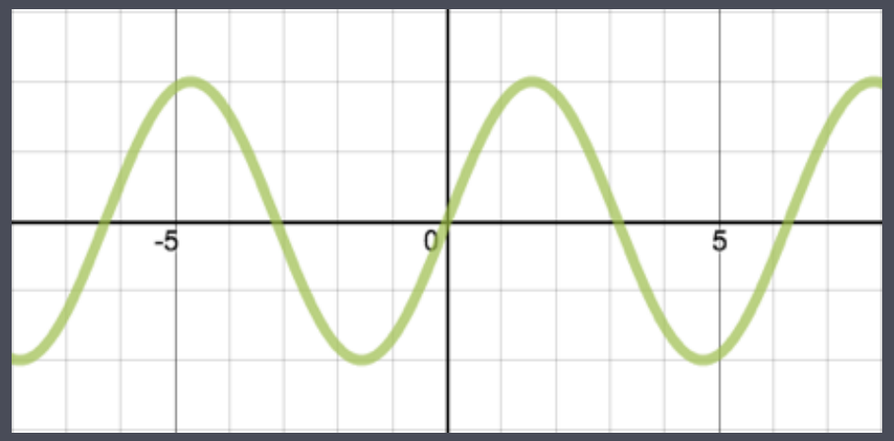
(B) $y_0 = -\pi/2$, $y^* = -\pi/2$. X ---> $y^* = -\pi$

(C) $y_0 = \pi/4$, $y^* = \pi/2$. X ---> $y^* = \pi$

(D) $y_0 = 3\pi/4$, $y^* = \pi$.

(E) $y_0 = \pi$, $y^* = 0$. X ---> $y^* = \pi$

$y' = \sin(y)$



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(D) $y_0 = 3\pi/4$, $y^* = \pi$.

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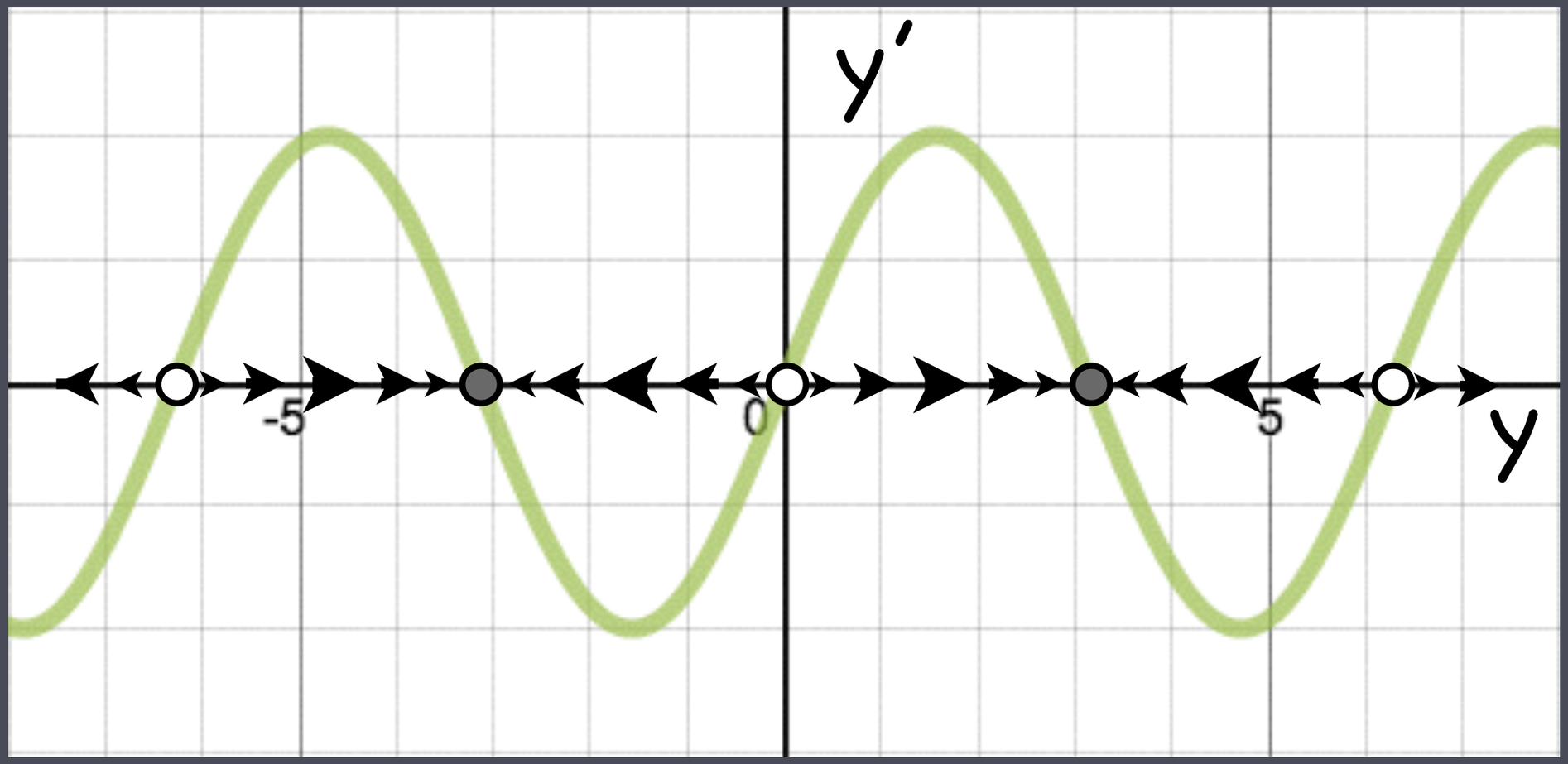
$y' = \sin(y)$

Fill in the arrows and steady states on the phase line. Sketch a few solutions.

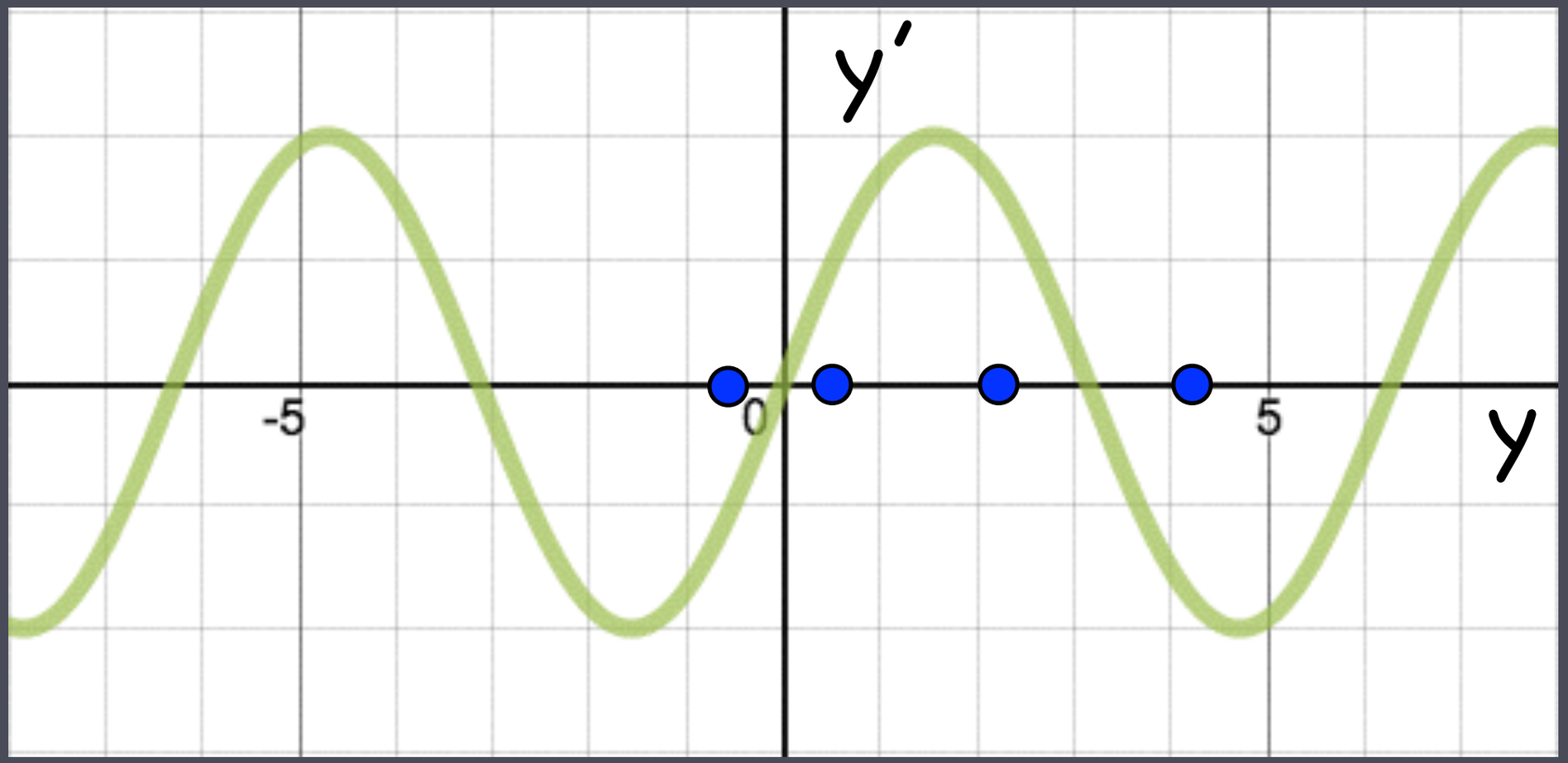


Filled circle ● - stable steady state

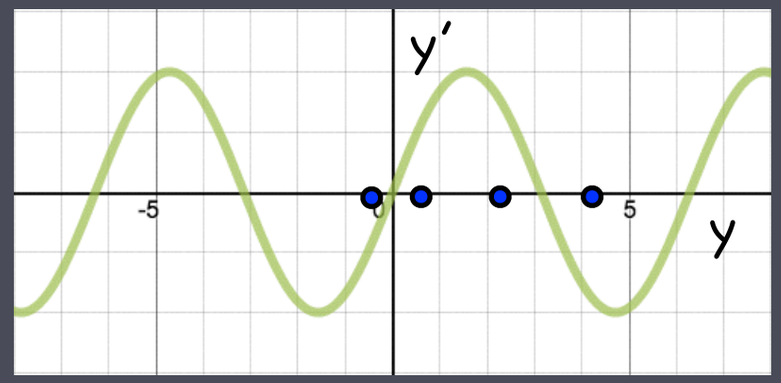
Empty circle ○ - unstable steady state

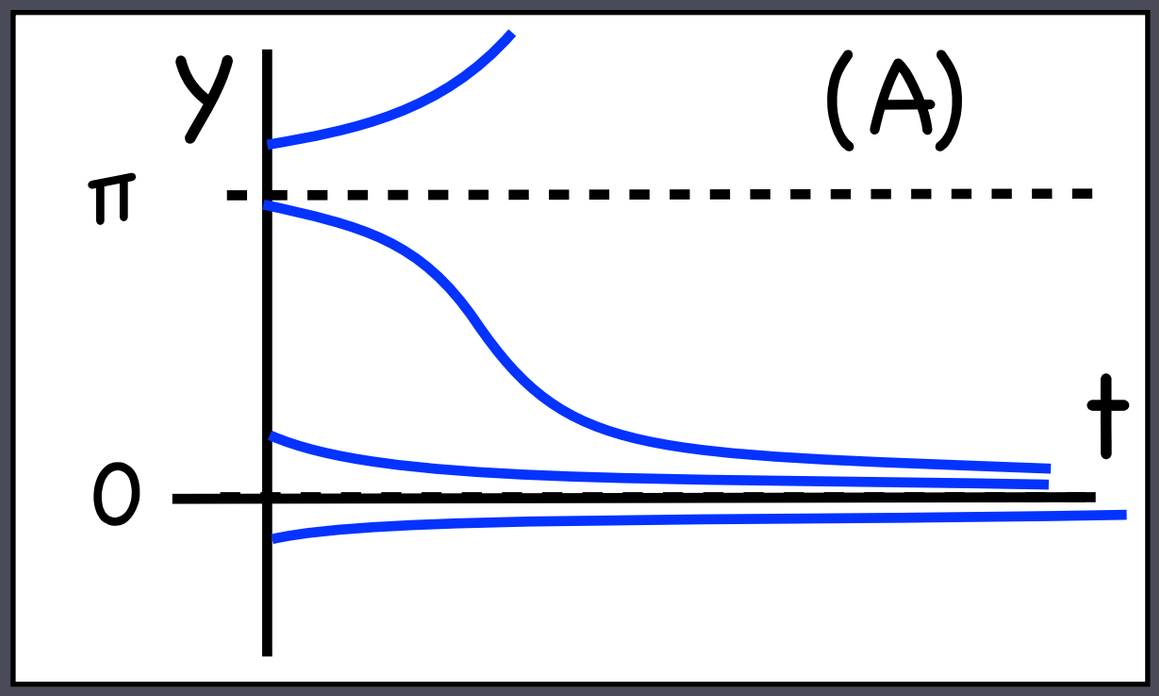
$$y' = \sin(y)$$

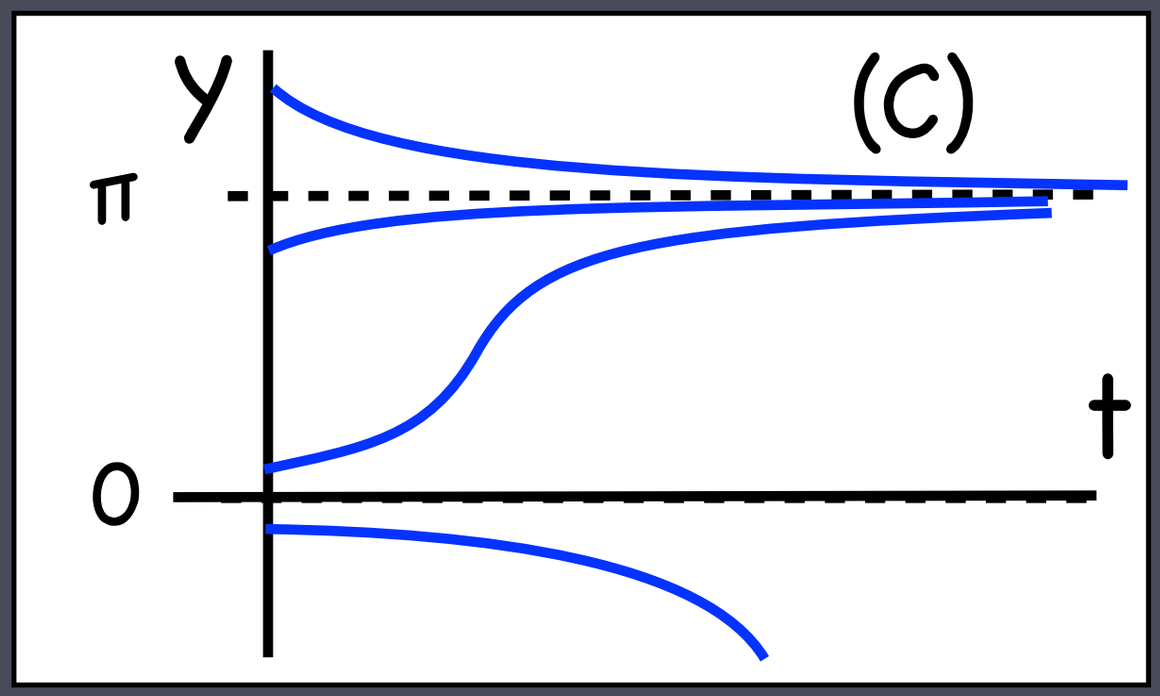
Sketch a few solutions y(t).

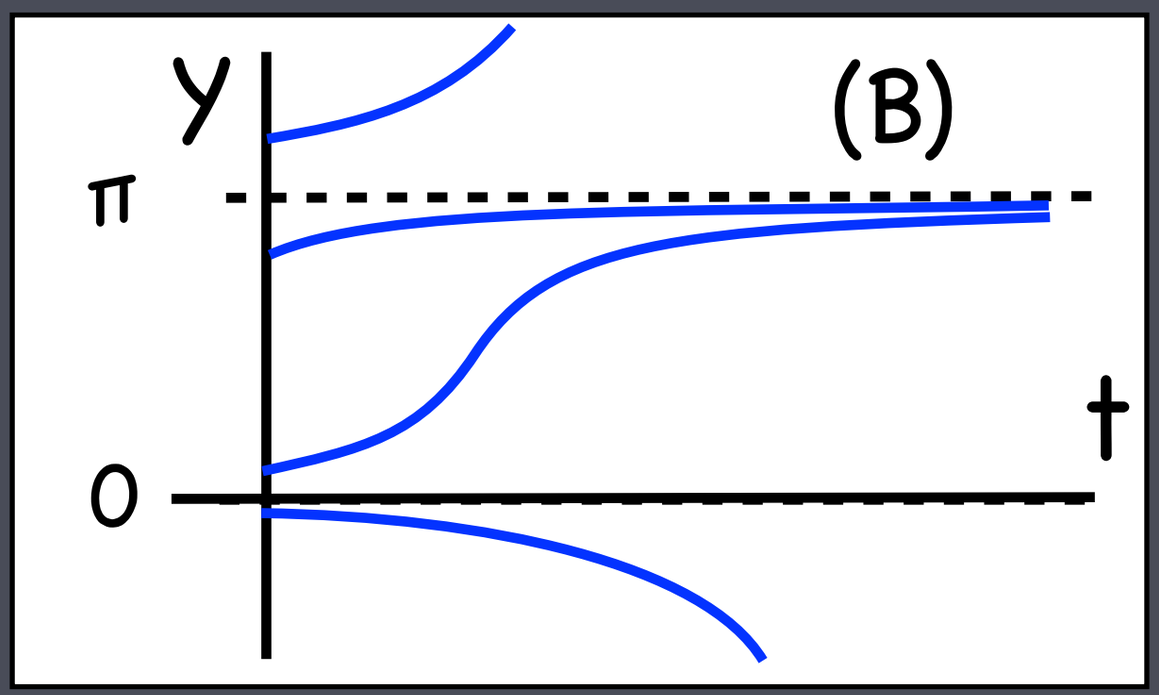


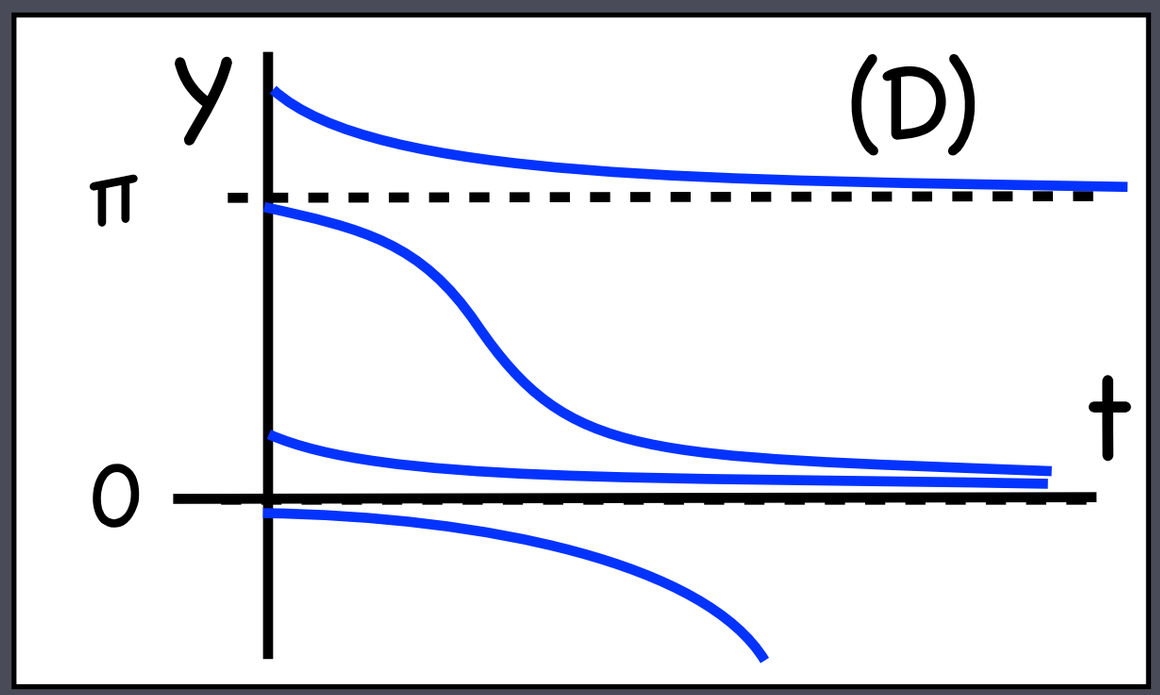
$y' = \sin(y)$



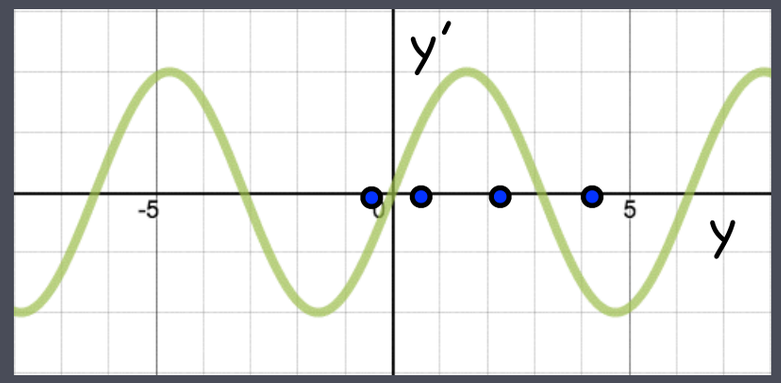


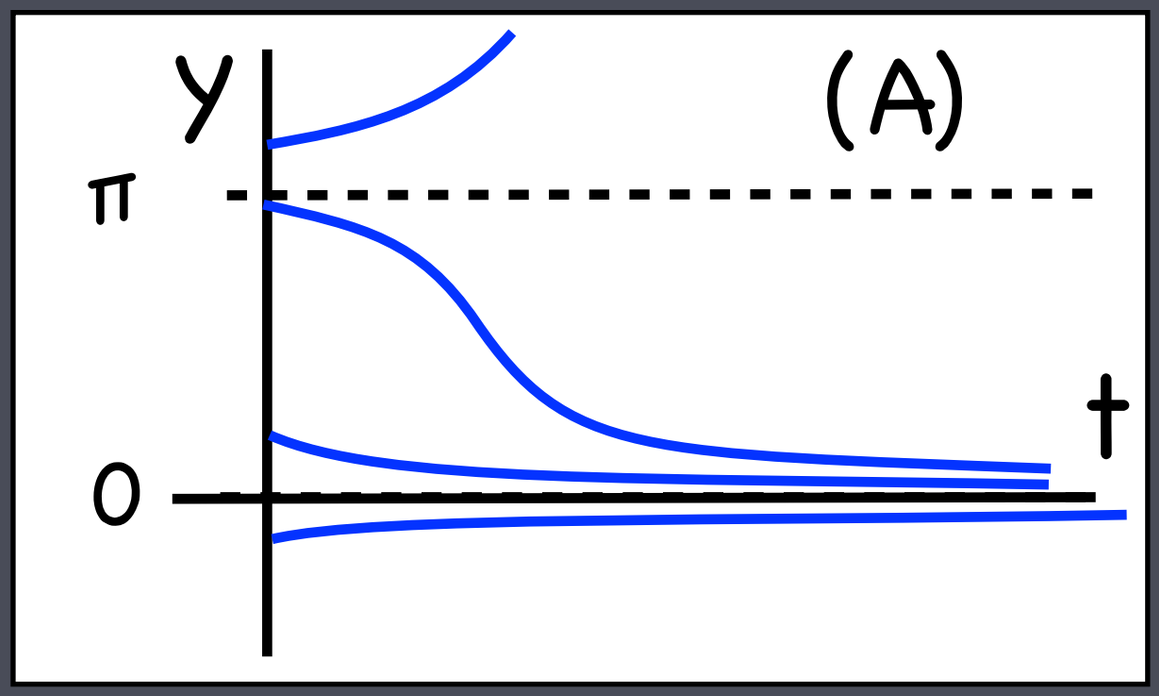


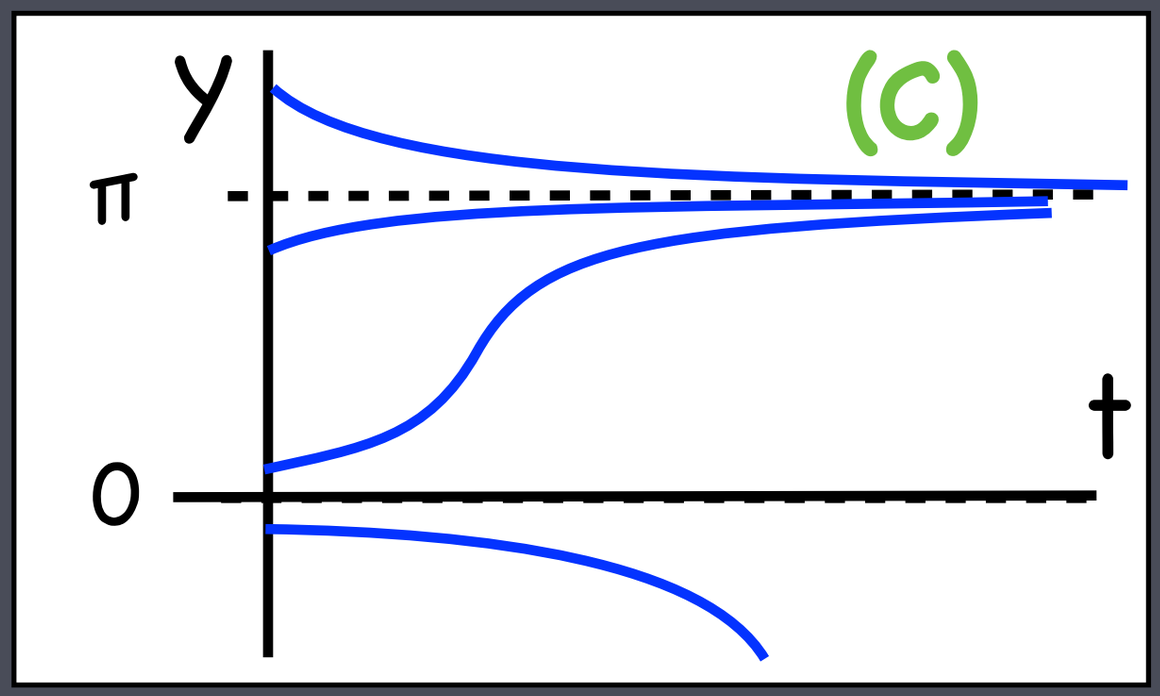


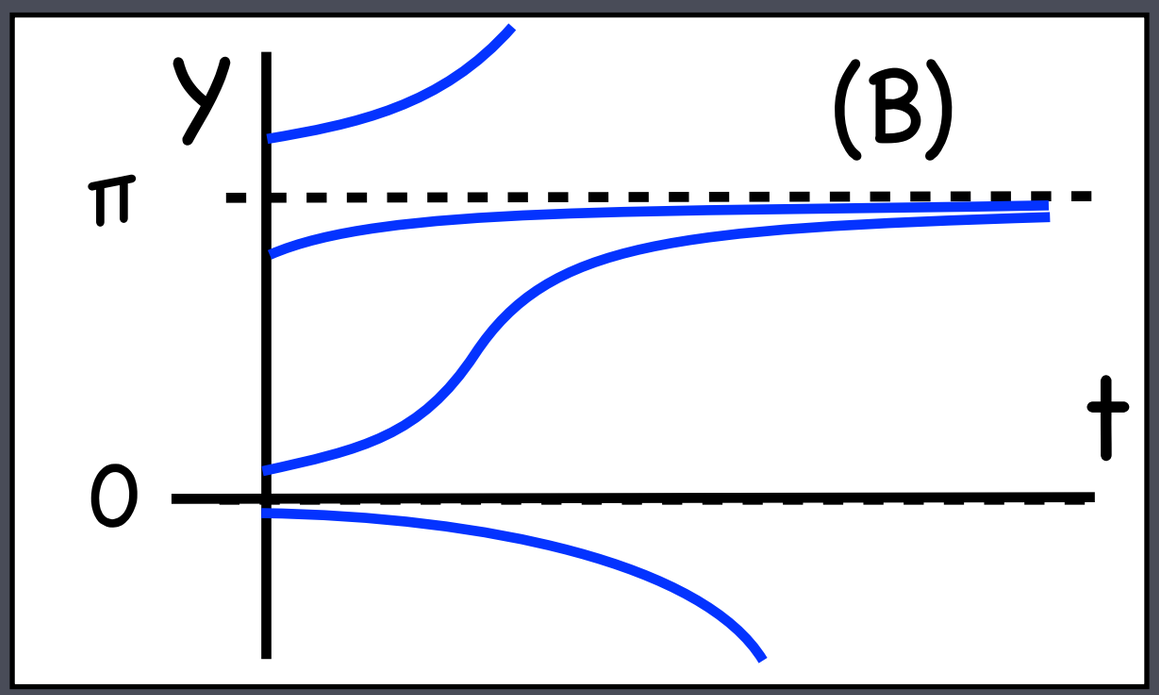


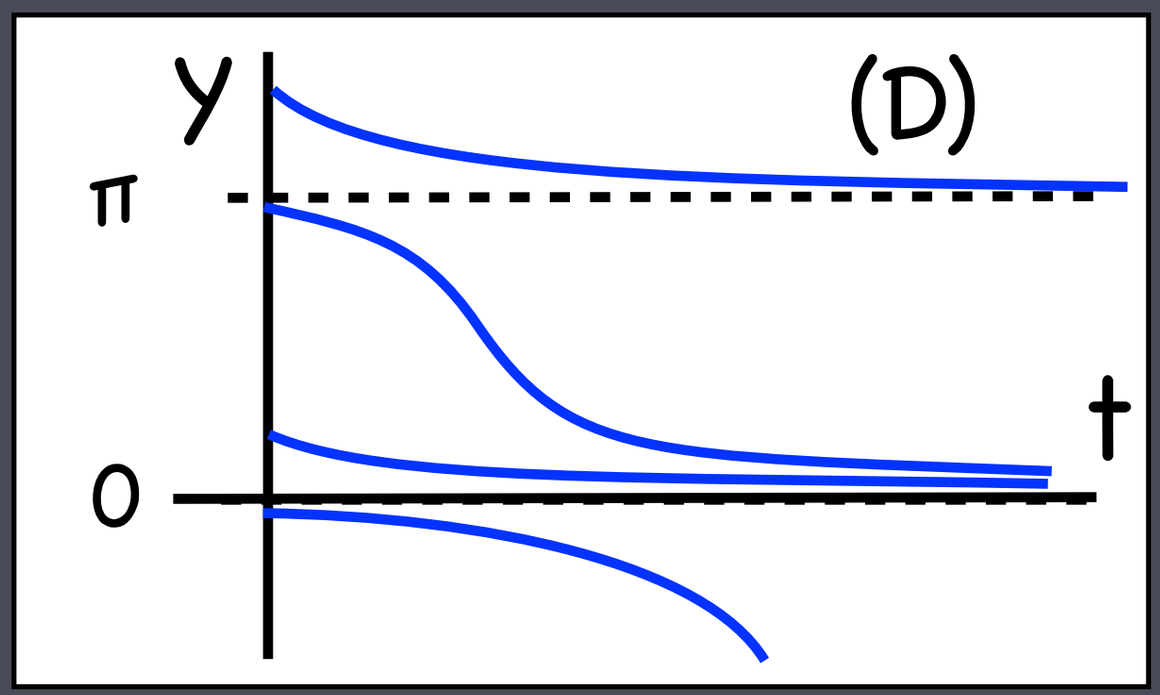
$$y' = \sin(y)$$











What you should be able to do:

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states.
- Determine long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, h-asymptotes).

The rate of change of an object's temperature is proportional to the difference between the object's temperature and the surrounding environment.

(A) $T'(t) = k\,(\,T(t) - E\,)$

(B) $T'(t) = k\,(\,E - T(t)\,)$

(C) $T'(t) = E - kT(t)$

(D) $T'(t) = kT(t) - E$

Assume $k>0$ and E is the temperature of the environment.

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Make sure eq matches physical intuition.

If the coefficient on T(t) is +ive, the solution ---> ∞.

Units have to match!

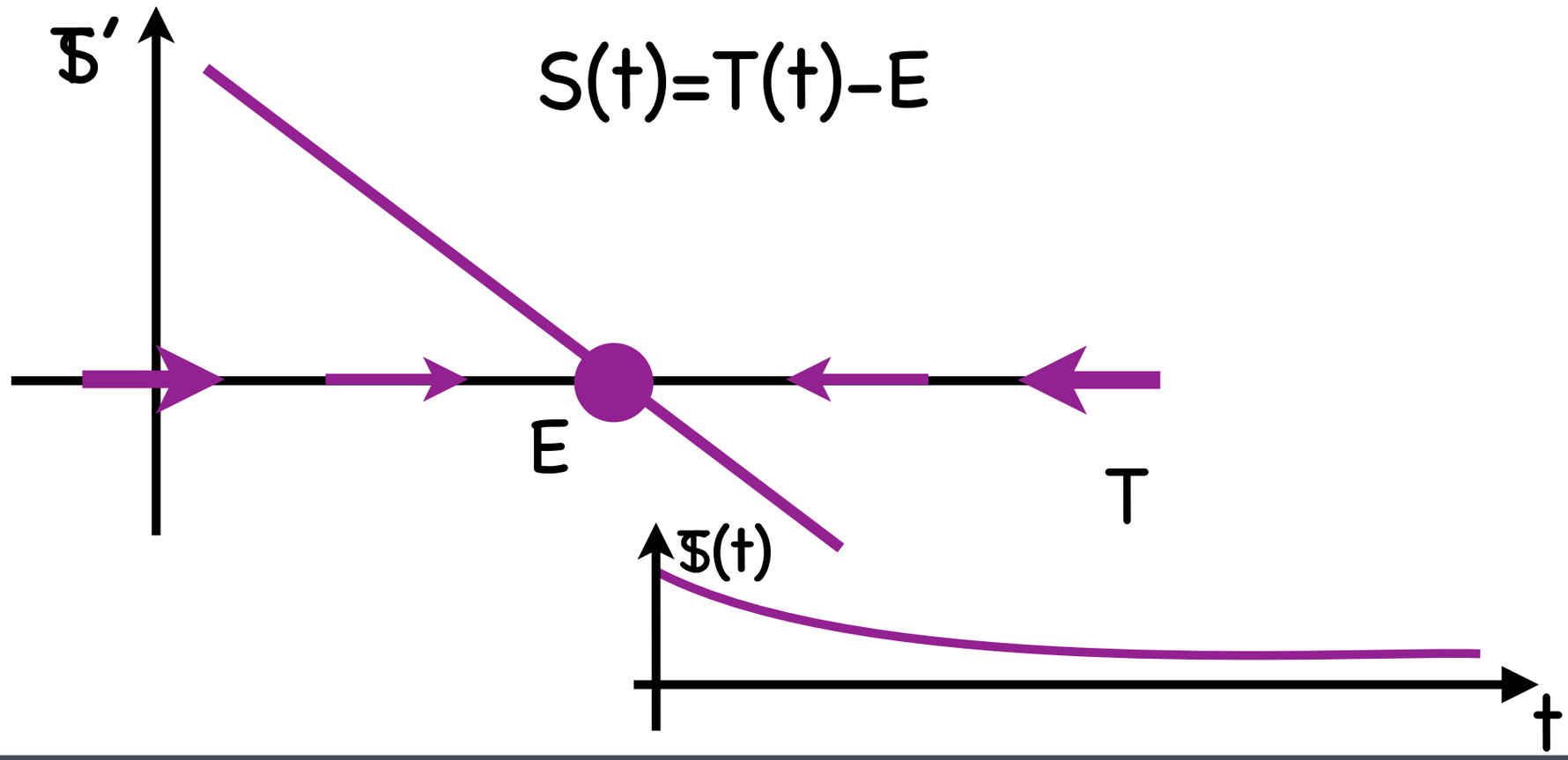
Assume k>0 and E is the temperature of the environment.

Solving T′ = k(E–T)

- Suppose T is measured in Kelvin and E = 273K (surrounding) is the freezing point of H_2O.
- Change to centigrade.
  - Define S(t) = T(t) – E.
    - S′(t) = T′(t)
    - S′ = T′ = k(E–T) = k (–S) = – kS
- Solution: $S(t) = S_0e^{-kt}$. Change back to T(t):
- $T(t) = S(t) + E = S_0e^{-kt} + E = (T_0–E)e^{-kt} + E$
- With $T(0)=T_0$, $T(0) = S_0+E = T_0$ so $S_0=T_0–E$.

Phase line for NLC:

$$\frac{dT}{dt} = k(E - T)$$



Notice that the arrows are always the same for any E, just shifted left or right.