

# Today

- Tangent lines
- Linear approximation
- Reminders:
  - OSH 3 on Monday
  - Midterm 1 on Tuesday @ 6pm
  - Assignment 4a - due Tuesday @ 7am
  - Assignment 4b - due Friday @ 5 pm



Find the tangent line to  $f(x)$   
at  $(a, f(a))$ .

$$\frac{y - f(a)}{x - a} = f'(a)$$

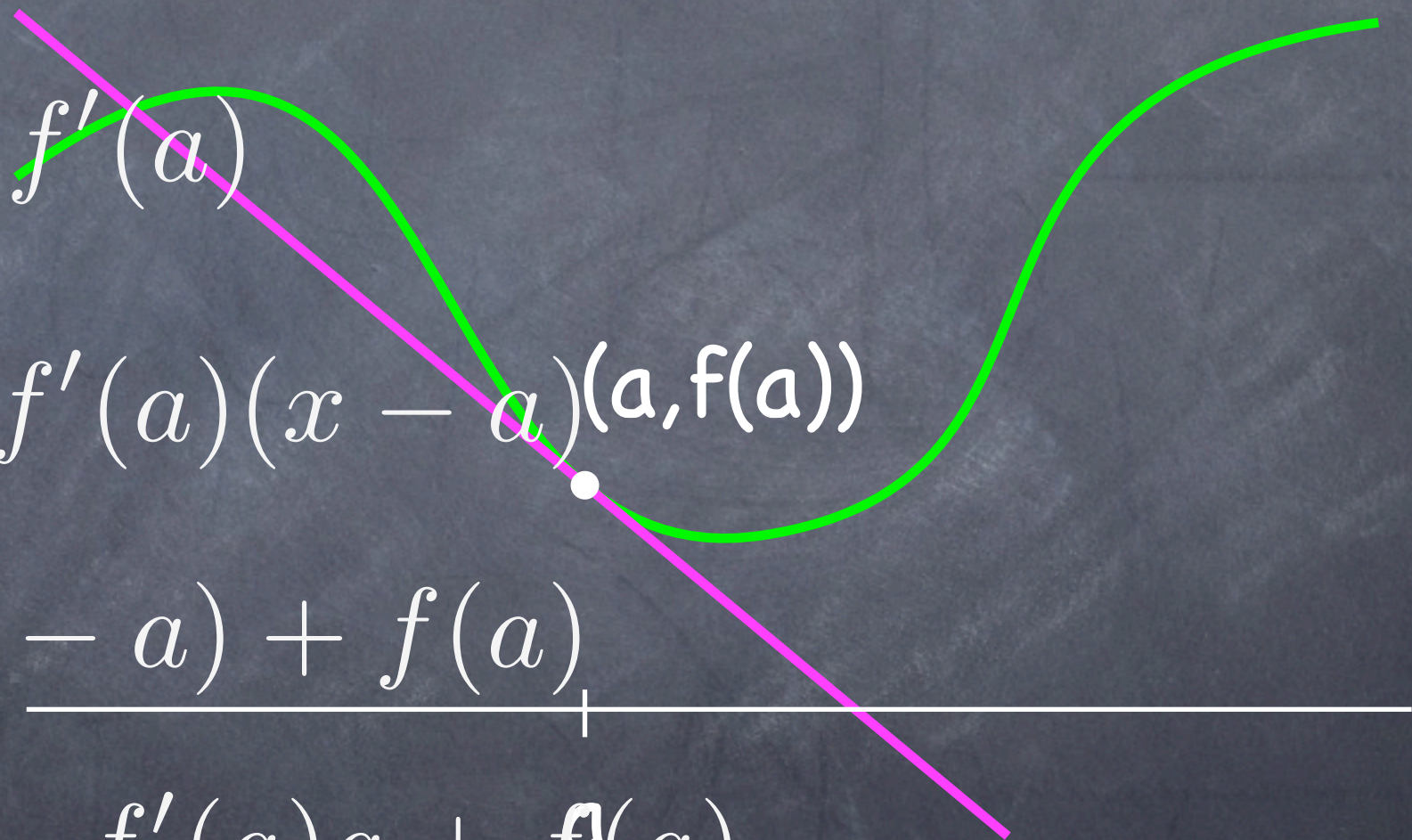
$$y - f(a) = f'(a)(x - a) \quad (a, f(a))$$

$$y = f'(a)(x - a) + f(a)$$

$$y = f'(a)x - f'(a)a + f(a)$$

$\underbrace{\hspace{1.5cm}}$   
m

$\underbrace{\hspace{3.5cm}}$   
b





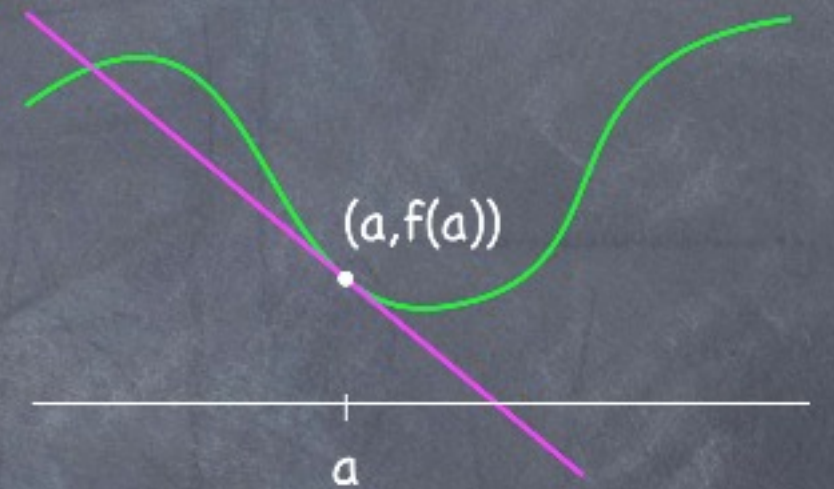
Find the tangent line to  $f(x)$   
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$$\frac{y - f(a)}{x - a} = f'(a)$$

$$y - f(a) = f'(a)(x - a)$$

$$y = f'(a)(x - a) + f(a)$$

If  $x = a$  then  $y = f(a)$ , so the line goes  
through  $(a, f(a))$ . It also has slope  $f'(a)$ .





# Tangent line to $\sin(x)$ at $x=0$

- Slope of  $\sin(x)$  at  $x=0$  is 1 (from last week).
- In general, tangent line:  $y = f'(a)(x-a) + f(a)$ .
- In this case, . . .
  - (A)  $y = \cos(x)x + \sin(x)$
  - (B)  $y = x$
  - (C)  $y = x - \pi/2$
  - (D)  $y = \cos(x_0)(x-x_0) + \sin(x_0)$
  - (E) Don't know



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(D)  $y = \cos(x_0)(x-x_0) + \sin(x_0)$

(E) Don't know

$$a=0,$$

$$f(a)=0,$$

$$f'(a) = 1.$$



# From midterm 1, 2013

4. **Tangent lines:** As shown in the figure below, the tangent line to the graph of  $f(x)$  at  $x = a$  intersects the x-axis at  $x = b$ . Which of the following expressions gives the value of  $b$ ?

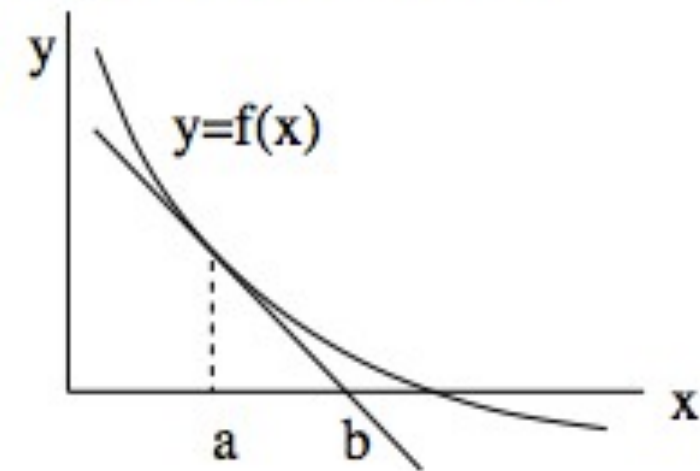
(A)  $b = a - \frac{f(a)}{f'(a)}$ ,

(B)  $b = a + \frac{f(a)}{f'(a)}$ ,

(C)  $b = a + \frac{f'(b)}{f(b)}$ ,

(D)  $b = f(a) - f'(a)a$ ,

(E)  $b = f(a) + f'(a)(x - a)$ .



•  $y = f'(a)(x-a) + f(a)$

•  $0 = f'(a)(b-a) + f(a)$

•  $b = a - f(a)/f'(a)$



# General case

- Objects involved:

- (i) a function  $f(x)$

- (ii) a point of tangency  $(a, f(a))$

- (iii) slope at point of tangency  $f'(a)$

- (iv) a tangent line  $y = f'(a)(x-a) + f(a)$

- Some piece of information is missing – could be from any of these.



# Example – simplest case

- Let  $f(x) = x^3 + 2x^2 - x + 2$ .
- Find tangent line at  $x=3$ .
- Need equation of line
  - slope is  $m=f'(3)$ , point on line is  $(3, f(3))$
  - Either  $y = mx + b$  or  $y = m(x-a) + f(a)$ ...

(A)  $y = 3x + 44$

(C)  $y = 38(x-3) + 44$ .

(B)  $y = 38x + 44$

(D)  $y = 44$



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- Find tangent line at  $x=3$ .
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(C)  $y = 38(x-3) + 44$ .

(B)  $y = 38x + 44$

(D)  $y = 44$



# Example - slightly harder

- Let  $f(x) = x^3 + 2x^2 - x + 2$ .
- Find a tangent line parallel to  $y = -x + 3$ .
- Need: a point of tangency, a slope  $\rightarrow$  line

We need to...

- (A) Find a <sup>(i)</sup> such that  $f(a) = -a + 3$ .  
<sup>(ii)</sup> a point of tangency  $(a, f(a))$
- (B) Find a <sup>(iii)</sup> such that  $f'(a) = -1$ .  
slope at point of tangency  $f'(a)$
- (C) Solve  $x^3 + 2x^2 - x + 2 = -1(x - a) + f(a)$



# Example – slightly harder

- Let  $f(x) = x^3 + 2x^2 - x + 2$ .
- Find a tangent line parallel to  $y = -x + 3$ .
- Need: a point of tangency, a slope  $\rightarrow$  line

We need to...

(A) Find  $a$  such that  $f(a) = -a + 3$ .

(B) Find  $a$  such that  $f'(a) = -1$ .

(C) Solve  $x^3 + 2x^2 - x + 2 = -x + 3$ .

$$3a^2 + 4a - 1 = -1$$

$$3a^2 + 4a = 0$$

$$a = 0, -4/3$$

$$y = -x + 2$$



# Example - even harder

- Find tangent line to  $f(x)=x^2$  that goes through  $(1,-1)$ . Note:  $f(1)\neq -1!!$

(i) yes, (ii) no, (iii) if we had (ii), (iv) if we had (ii).

- Name unknown point  $(a, f(a))$ . Pretend you know  $a$ . Means you also know  $f(a)$ ,  $f'(a)$ .

- What can we now write down?

(i) a function  $f(x)$

- $y = f'(a)(x-a) + f(a)$  (ii) a point of tangency  $(a, f(a))$

- $(1, -1)$  must be on the line so (iii) slope at point of tangency  $f'(a)$

(iv) a tangent line  $y = f'(a)(x-a) + f(a)$   
 $-1 = 2a(1-a) + a^2$ . Solve for  $a$ .



Find tangent line to  $f(x)=x^2$   
that goes through  $(1,-1)$ .

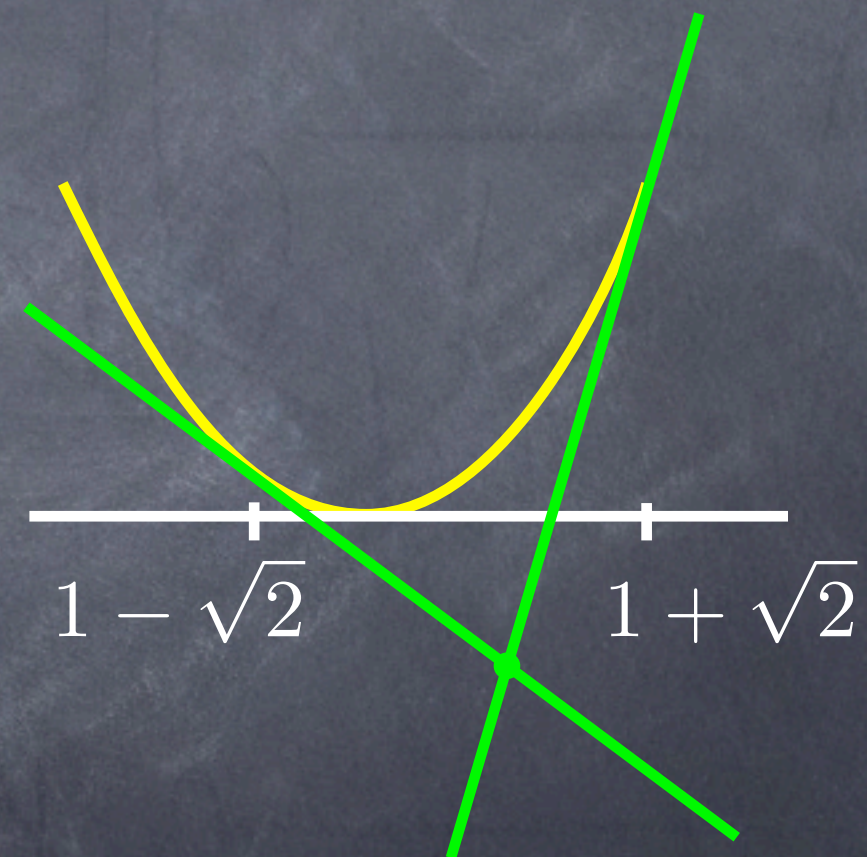
Point of tangency is at

(A)  $(1 + \sqrt{2}, 3 - 2\sqrt{2})$

(B)  $(1 + \sqrt{2}, 3 + 2\sqrt{2})$

(C)  $(1, -1)$

(D)  $(1 - \sqrt{2}, 3 - 2\sqrt{2})$



Work through it on your own!