1. Population Growth

Let $N(t)$ is the number of individuals in a population at time $t$.

Over a small time interval $\Delta t$:

$$N(t+\Delta t) - N(t) = b \cdot N(t) \Delta t - d \cdot N(t) \Delta t$$  (1)

- **births**
- **deaths**

Where $b =$ per capita birth rate.

$$\# \text{births/person/time}$$

$d =$ per capita death rate

$$\# \text{deaths/person/time}$$

*Units work*: $b \cdot N \cdot h$ has units $\frac{\#}{N \cdot T}$
Take (1):

\[
\frac{N(t+h) - N(t)}{h} = bN(t) + dN(t) = N(t)(b-d)
\]

\[
\frac{N(t+h) - N(t)}{h} = KN(t) \quad K = b-d.
\]

Take the limit as \( h \) goes to 0;

\[
\frac{dN}{dt} = KN.
\]

* Keeping track of births + deaths

\[
\Rightarrow \text{D.E. for population growth}.
\]

If there are \( N_0 \) people at time \( t=0 \), then the size of the population is

\[
N(t) = N_0 e^{kt}, \quad k = b-d
\]
Notes

* If $k > 0$, $b - d > 0$, $b > d$, then the population should grow.

* If $k < 0$, $b - d < 0$, $b < d$, then the population should shrink.

Yes! This works

if $k > 0$, $N(t) = N_0 e^{kt}$:

if $k < 0$, $|N(t)| = N_0 e^{-kt}$.

Problems:

- We've ignored immigration, emigration, or competition.
- Population growth isn't unlimited.
- There exists a time $t$ when $N(t) < 1$. What does a pop. size of $N(t) = 0.5$ mean?