Today

- Please email requests for problems to discuss on Monday.
- Midterm Tuesday!
- Office hours M 11-12:30, M2-2:45, T10:30-12.
- Translating from word problem to IVP.
- Solving a linear ODE: $y' = a - by$.

A population grows proportional to the size of the population itself. Which of the following is a differential equation for the population size $P(t)$?

(A) $P'(t) = kt$

(B) $P(t) = e^{kt}$

(C) $P'(t) = kP(t)$

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(B) $P(t) = e^{kt}$ <-- This is a solution. So is $P(t)=P_0e^{kt}$ for any value P_0.

(C) $P'(t) = kP(t)$

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Doubling time

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- This is called the doubling time.

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- If $k<0$, $c(t)$ is decreasing and reaches $1/e$ its original value when $c_0e^{kt} = c_0/e$.
- That is when $t=-1/k$.
- This is called the characteristic time or mean life. Just like half-life but replace 2 with e (could be called 1/e-life).

The rate of change of an object's temperature is proportional to the difference between the objects temperature and the surrounding environment.

(A) $T'(t) = k\,(\,T(t) - E\,)$

(B) $T'(t) = k\,(\,E - T(t)\,)$

(C) $T'(t) = E - kT(t)$

(D) $T'(t) = kT(t) - E$

Assume $k>0$ and E is the temperature of the environment.

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Make sure eq matches physical intuition.

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(A) $T'(t) = k (T(t) - E)$

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If the coefficient on T(t) is +ive, the solution ---> ∞.

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Units have to match!

Assume $k>0$ and E is the temperature of the environment.

An object dropped in water will accelerate under the influence of the constant downward force of gravity and an upward drag force proportional to the velocity.

(A) $v'(t) = \delta (v(t) - g)$

(B) $v'(t) = \delta (g - v(t))$

(C) $v'(t) = \delta v(t) - g$

(D) $v'(t) = g - \delta v(t)$

Assume $\delta > 0$ and $g = 9.8 \text{ m/s}^2$.

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Newton's 2^{nd} Law

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$$ma = F_{grav} + F_{drag}$$
$$= mg - \gamma v(t)$$

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- Sign on δv is independent of reference frame.
- Sign on g depends on ref. fr.

Assume $\delta > 0$ and $g = 9.8\ m/s^2$.

A drug delivered by IV accumulates at a constant rate k_{IV}. The body metabolizes the drug proportional to the amount of the drug.

(A) $d'(t) = k_{IV} - k_m d(t)$

(B) $d'(t) = (k_{IV} - k_m)\, d(t)$

(C) $d'(t) = k_{IV} d(t) - k_m$

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$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

(A) $d(t) = k_{IV}/k_m\,(1-\exp(k_m t))$

(B) $d(t) = k_{IV}/k_m\,(1-\exp(-k_m t))$

(C) $d(t) = k_{IV}/k_m - \exp(k_m t)$

(D) $d(t) = k_{IV}/k_m - \exp(-k_m t)$

(E) Not sure how to do this one.

Note: $\exp(x)=e^x$.

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