Disease Dynamics

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Math 102 Section 106

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Today...

1. Teaching Evaluations
2. Disease Dynamics

- Reminder: Office Hours today from 4-5 pm in Math Annex 1118
- WebWorK due Thursday
- OSH 6 due Friday
Teaching Evaluations

- Please use the next 10 minutes to fill out the Teaching Evaluation (e.g. on your phone or laptop or tablet)
- You already know that I use your feedback to adjust my teaching
- The Department of Math and Faculty of Science takes these surveys very seriously
- https://eval.ctlt.ubc.ca/science
Quick Question

What are you thinking about majoring in?

A. Arts
B. Business
C. Medicine or related
D. Biology (not medicine or related)
E. Other sciences
Q1. A drug delivered by IV accumulates at constant rate $k$. The body metabolizes the drug proportional to the amount of the drug.

A. $\frac{dc}{dt} = (k - m)c$
B. $\frac{dc}{dt} = kd - m$
C. $\frac{dc}{dt} = -k + mc$
D. $\frac{dc}{dt} = k - mc$
Q2. A drug delivered by IV accumulates at constant rate $k$. The body metabolizes the drug proportional to the amount of the drug.

$$\frac{dc}{dt} = k - mc, \quad c(0) = 0$$

A. $c(t) = \frac{k}{m} (1 - e^{mt})$
B. $c(t) = \frac{k}{m} (1 - e^{-mt})$
C. $c(t) = \frac{k}{m} - e^{mt}$
D. $c(t) = \frac{k}{m} - e^{-mt}$
E. Send help
Epidemiology

- How does a disease spread within a population?
- Suppose that 1 infected individual is introduced to a healthy population.
  1. How many people will become infected?
  2. Will the disease persist or not?
Mathematical Epidemiology

- $t = \text{time}$
- $S(t) = \text{number of healthy people (called susceptibles)}$
- $I(t) = \text{number of infected people}$
- $N(t) = I(t) + S(t) = \text{total population}$
Initial Conditions

Discussion: If there is initially one infected person in the population, what are the correct initial conditions?
Disease transmission

Discussion: The disease is transmitted between infected individuals and susceptible individuals.

What is the rate of transmission of the disease?
Keeping track of individuals

Discussion: What differential equation should govern the rate of change of the number of infected individuals?
Susceptible Population

\[ \frac{dI}{dt} = \beta SI - \mu I \]

Q3. What differential equation should \( S \) satisfy?

A. \( \frac{dS}{dt} = \beta SI - \mu I \)
B. \( \frac{dS}{dt} = -\beta SI + \mu I \)
C. \( \frac{dS}{dt} = \beta SI - \mu S \)
D. \( \frac{dS}{dt} = -\beta SI + \mu S \)
Kermack and McKendrick Model


14,000 hits this year!
Our model is a slight variant of the original model studied by Kermack and McKendrick in 1927

\[
\frac{dS}{dt} = -\beta SI + \mu I \\
\frac{dI}{dt} = \beta SI - \mu I \\
N = I + S
\]

This type of model is called a system of differential equations
Choose your own class:

A. Qualitative Analysis of the $SI$ model
B. Euler’s Method to approximate the solution to the $SI$ model
Summary

- Law of Mass Action can be extended to “rate of change that are proportional to two things”
  \[ \beta SI \]

- A system of differential equations describes how quantities can change in response to each other

- Euler’s Method and/or Qualitative Analysis can sometimes be used to understand the behaviour of a system of differential equations

- The basic reproduction number, \( R_0 \), quantifies whether an epidemic will occur.
Disease Dynamics

\[ \frac{dS}{dt} = -\beta SI + \mu I \]
\[ \frac{dI}{dt} = \beta SI - \mu I \]
\[ N = I + S \]

Discussion of model limitations.
What about vaccination?
What about immunity to the disease?
What about virus dynamics?
There are many interesting questions:
  - If a virus is too deadly, it may kill its host before transmission. If a virus is not virulent enough, it may be wiped out by the immune system. How does vaccination influence the evolution of virulence?
  - If you’re curious, come talk to me!
Answers

1. D
2. B
5. The model given below on the left has been suggested for the spread of HIV within the immune system of an infected person. $C(t)$ is the density of healthy immune cells, $I(t)$ is the density of HIV-infected immune cells and $V(t)$ is the density of virus in the blood of a patient. Which of the options on the right gives a correct interpretation of some part of the model?

\[
\frac{dC}{dt} = P - \alpha CV - \gamma_1 C \\
\frac{dI}{dt} = \alpha CV - \gamma_2 I \\
\frac{dV}{dt} = \beta I - \gamma_3 V
\]

(a) Healthy cells can become infected when they encounter infected cells.
(b) Healthy cells can become infected when they encounter virus.
(c) Virus is produced at a rate proportional to the current viral density.
(d) Infected cells die at a rate proportional to the viral density.
(e) Virus is killed/removed at a rate proportional to the density of healthy immune cells.