

# Today...

- Points for 3 cases on OSH 1.
- Limits at infinity



# Limits at infinity

- Polynomials
- Rational functions
  - power tricks (approx or divide by  $x^m$ )



# Polynomials first...

• ex.  $\lim_{x \rightarrow \infty} -x^3 + x^2 = ? -\infty$   
     $\Rightarrow$  Highest power wins

• ex.  $\lim_{x \rightarrow -\infty} x^6 + 10x^5 = ? \infty$   
     $\Rightarrow$  Even powers kill minus signs.

• ex.  $\lim_{x \rightarrow -\infty} 10x^5 + x^4 = ? -\infty$   
     $\Rightarrow$  Odd powers don't.

On the board - technique for rational functions...



# Two ways to find limits at infinity

$$\lim_{x \rightarrow -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$$



For  $|x|$  large,  $4x^7 - 2x^4 + 4$

(A)  $\approx 4$

(B)  $\approx 4x^7$

(C)  $\rightarrow \infty$

(D)  $\rightarrow -\infty$



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For  $|x|$  large,  $10x^5 - 3x^3 + 4x$

(A)  $\approx 4x$

(B)  $\approx 10x^5$

(C)  $\rightarrow \infty$

(D)  $\rightarrow -\infty$



For  $|x|$  large,  $10x^5 - 3x^3 + 4x$

(A)  $\approx 4x$

(B)  $\approx 10x^5$

(C)  $\rightarrow \infty$

(D)  $\rightarrow -\infty$



For  $x$  large,  $\frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$

(A)  $\approx 4 / 10$

(B)  $\approx 2 / 5$

(C)  $\approx 2 / (5x^2)$

(D)  $\approx 2x^2 / 5$



For  $|x|$  large,  $\frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x}$

(A)  $\approx 4 / 10$

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(D)  $\approx 2x^2 / 5$

$$\lim_{x \rightarrow -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x} = \infty$$



Or divide top/bottom by highest power in denom.

$$\lim_{x \rightarrow -\infty} \frac{4x^7 - 2x^4 + 4}{10x^5 - 3x^3 + 4x} =$$

$$\lim_{x \rightarrow -\infty} \frac{4x^2 - \frac{2}{x} + \frac{4}{x^5}}{10 - \frac{3}{x^2} + \frac{4}{x^4}} = \infty$$



What is  $\lim_{x \rightarrow -\infty} \frac{3x^n + x^2 - 1}{x^3 + 4}$ ?

- (A) If  $n=2$ , the limit is  $-\infty$ .
- (B) If  $n=3$ , the limit is  $\infty$ .
- (C) If  $n>3$  and even, the limit is  $-\infty$ .
- (D) If  $n>3$  and odd, the limit is  $-\infty$ .



Suppose  $\lim_{x \rightarrow \infty} f(x) = 3$ .

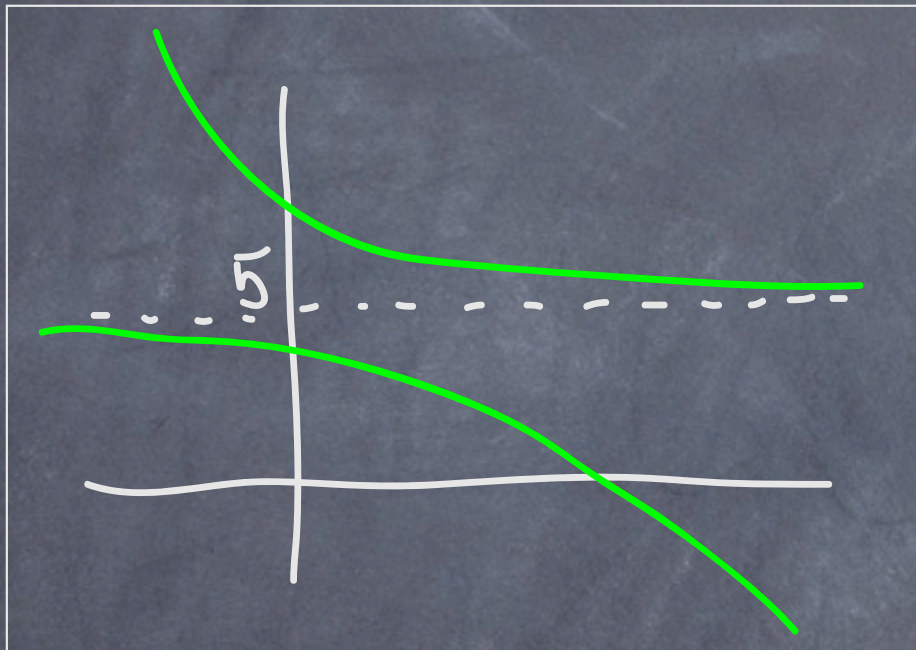
What is  $\lim_{x \rightarrow -\infty} f(x)$  ?

- (A) 3
- (B) -3
- (C) If  $f(x)$  is even, then the limit is 3.
- (D) If  $f(x)$  is odd, then the limit is 3.

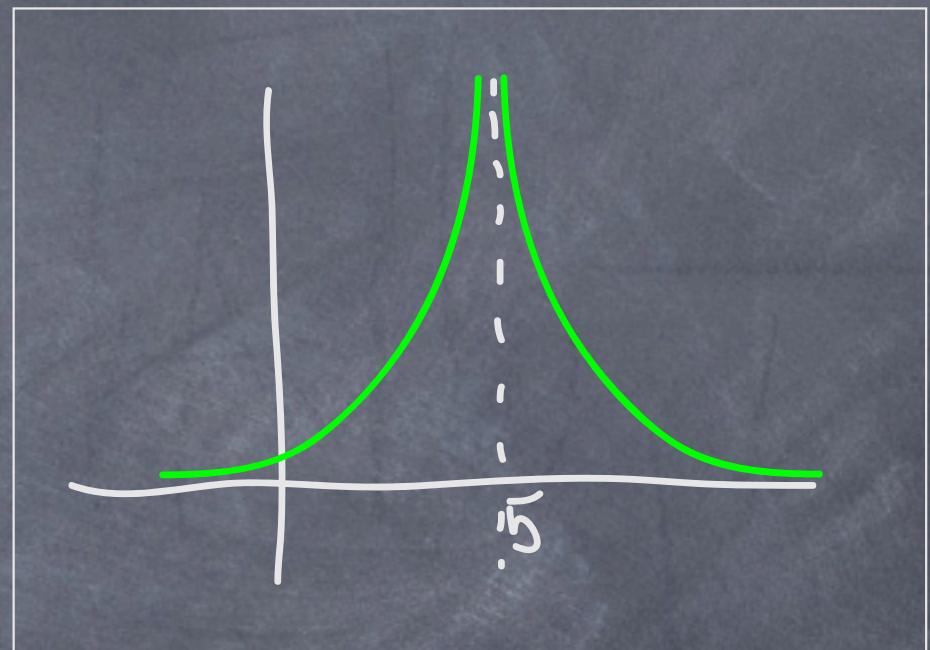


Which of the following could be  $f(x)$  where  $\lim_{x \rightarrow \infty} f(x) = 5$ ?

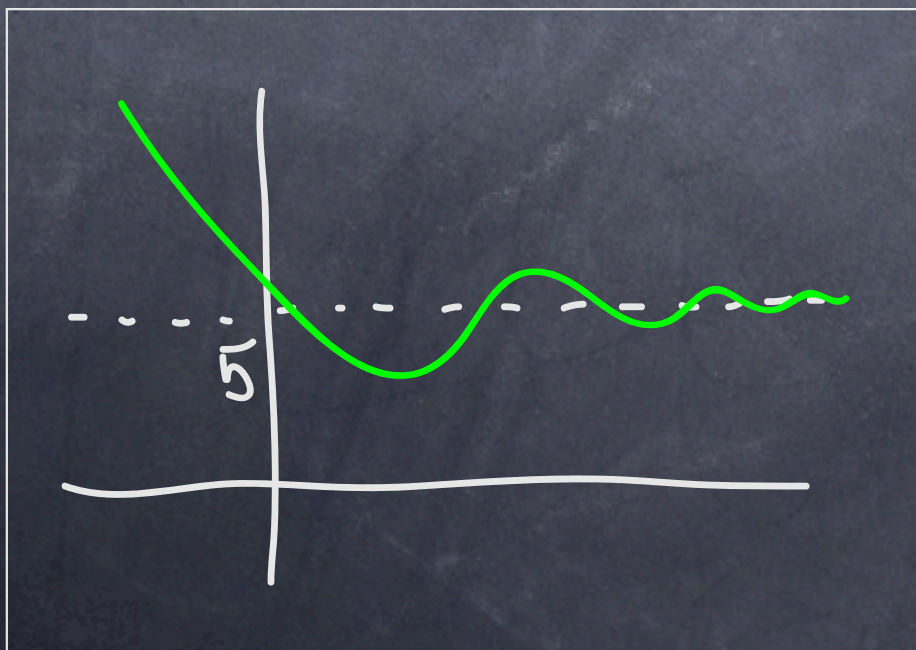
(A)



(C)



(B)



(D)

