MATH 102 - MIDTERM TEST 1

University of British Columbia

Last name (print): Solution key First name (print):

ID number:

Section number:

Date: *September 30, 2014.*
Time: *6:00 p.m. to 7:00 p.m.*
Number of pages: *9 (including cover page)*
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/record-ers/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

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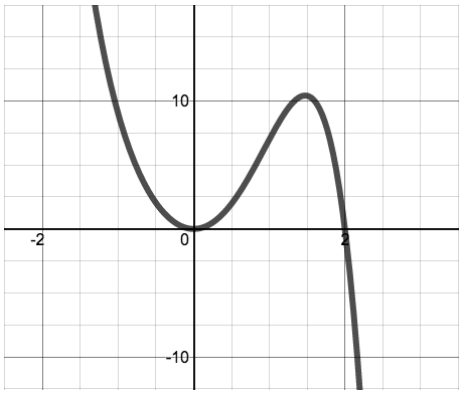
Section	Mark	Possible marks
MC		10
SAP		12
LAP 1		7
LAP 2		5
LAP 3		6
Total		40

Multiple choice (MC)

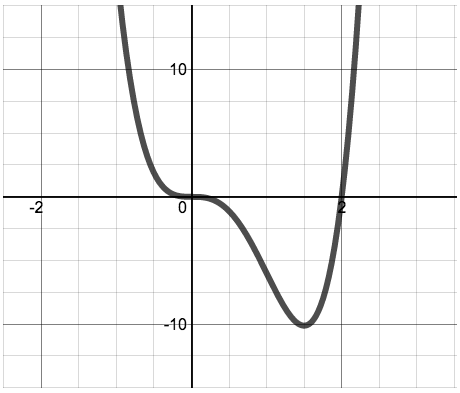
No partial points will be given for work shown.

1. Which of the following graphs is the the graph of $f(x) = 8x^2 - x^5$? This can be done without using derivatives.

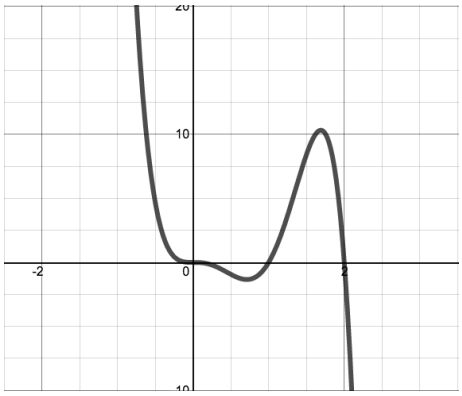
A



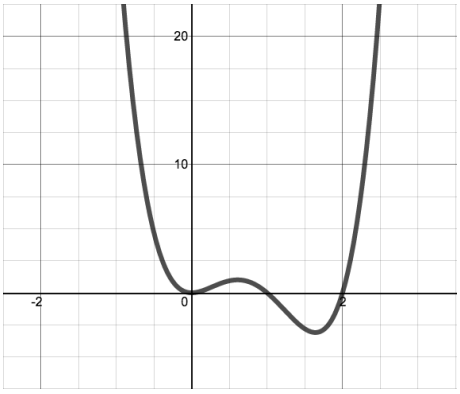
B



C



D



2. When $x = 1000$ the function $g(x) = \dfrac{6x^4 + 12x^2 + 64x - 87}{2x^3 - 6x^2 + x}$ is closest to

(A) 0.003 (B) 3000 (C) 1000000 (D) 6 (E) 3

Enter your answers to these questions here:

MC.1 [**2 pts**]	MC.2 [**2 pts**]
A	B

Multiple choice (continued)

3. Which of the following describes the derivative of a function $f(x)$?

   (a) It is the line we see when we zoom into the graph of $f(x)$.

   (b) It is the average rate of change of $f(x)$ over the interval $0 < x < h$.

   (c) It is defined as $\dfrac{f(x+h) - f(x)}{h}$.

   (d) More than one of the above answers are correct.

   (e) None of the above are correct.

4. Is the following statement true or false about ANY function $f(x)$: if $f(x)$ is discontinuous at the point x_0 then it has no derivative at x_0.

   (A) True (B) False

5. Is the following statement true or false about ANY function $f(x)$: if $f(x)$ is continuous at the point x_0 then it has a derivative at x_0.

   (A) True (B) False

Enter your answers to these questions here:

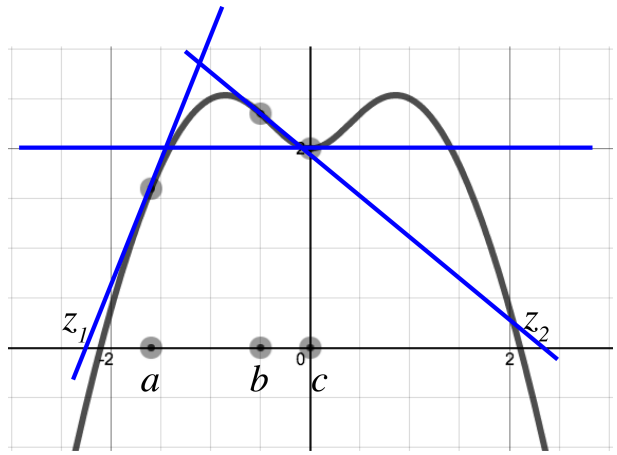
MC.3 [**2 pts**]	MC.4 [**2 pts**]	MC.5 [**2 pts**]
E	A	B

Short-answer problems

A correct answer in the box will get full points. Partial marks might be given for work shown.

1. **[3 pt]** If you were to use Newton's method to find the zeros of the function shown in the figure below (z_1 and z_2), to which zero would Newton's method converge (or neither) if you started with the specified initial value?

Initial value x_0	Zero (z_1, z_2 or neither)
$x_0 = a$	z_1
$x_0 = b$	z_2
$x_0 = c = 0$	Neither



2. **[3 pt]** Suppose that $f(x) = (3 + x^2)g(x)$ and that $g(2) = 2$, $g'(2) = -1$. Compute $f'(2)$.

$f'(x) = 2xg(x) + (3+x^2)g'(x)$ 1

$f'(2) = 4g(2) + 7g'(2)$ 1

$= 8 - 7 = 1$ 1

$f'(2) = \boxed{1}$

3. **[2 pt]** Let $f(x) = \sqrt{x+1}$. Then according to the definition of the derivative (no need to simplify),

$$f'(x) = \lim_{h \to 0} \boxed{\frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}}$$

4. **[2 pt]** We wish to find a good approximation to $7^{2/3}$ using a linear approximation.

(a) What function $f(x)$ could we use to help us to compute this value.

$f(x) =$ $x^{2/3}$

(b) At what point $(a, f(a))$ would you construct a tangent line to carry out the approximation?

$a =$ 8

5. **[2 pt]** We wish to find a good approximation to $7^{2/3}$ using Newton's method.

(a) What function $f(x)$ could we use to help us to compute this value.

$x = 7^{2/3}$

$x^3 = 7^2$

$x^3 - 49 = 0$

$f(x) = x^3 - 49$ 1

$f(x) = x^{3/2} - 7$ also ok

(but not as good for doing by hand)

$f(x) =$ $x^3 - 49$

(b) What value of x_0 would you choose as your starting estimate if you were planning to do the calculations by hand?

$x_0 = 8^{2/3} = (8^{1/3})^2 = 2^2 = 4$

1

$x_0 =$ 4

Long-Answer Problem #1

[7 pt] Consider the function $f(x) = \dfrac{3}{x-2}$.

(a) At which points $(a, f(a))$ does the graph of this function have tangent lines parallel to the line $y = -x$.

$$f'(x) = \frac{0\cdot(x-2) - 3}{(x-2)^2} = \frac{-3}{(x-2)^2} \quad 2$$

$$f'(a) = \frac{-3}{(a-2)^2} = -1 \quad 1$$

$$a^2 - 4a + 4 = 3$$

$$a^2 - 4a + 1 = 0$$

$$a = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3} \quad 1$$

$$f(a) = \frac{3}{a-2} = \frac{3}{\pm\sqrt{3}} = \pm\sqrt{3} \quad 1$$

$$\boxed{\begin{array}{l} 1 \ (2+\sqrt{3}, \sqrt{3}) \\ 2 \ (2-\sqrt{3}, -\sqrt{3}) \end{array}} \quad 1$$

This point is for associating the right a with the right f(a).

(b) What is the equation of the tangent lines at each of these points.

1 ① $y = \sqrt{3} - (x - (2+\sqrt{3})) = \boxed{-x + 2 + 2\sqrt{3} = y}$

② $\boxed{y = -x + 2 - 2\sqrt{3}}$

Long-Answer Problem #2

[5 pt]

$$f(x) = \begin{cases} x^2 - a & \text{for } x \le 2, \\ x^3 + bx^2 - 5 & \text{for } x > 2. \end{cases}$$

(a) What equation must a and b satisfy so that $f(x)$ is continuous at $x = 2$.

1 (Doing or stating)

$\lim_{x \to 2^+} f(x) = f(2) = \lim_{x \to 2^-} f(x)$ so

$4 - a = 8 + 4b - 5 = 3 + 4b$

$\boxed{4 - a = 3 + 4b}$ 1 (correct eq)

On this page, just answers in the box get full points.

(b) What additional equation must b satisfy so that $f'(2)$ exists?

1 (Doing or stating)

$\lim_{x \to 2^+} f'(x) = \lim_{x \to 2^-} f'(x)$

so $2 \cdot 2 = 3 \cdot 2^2 + 2b \cdot 2$

$\boxed{4 = 12 + 4b}$ 1

(c) What values of a and b guarantee that $f'(2)$ exists?

$4 = 12 + 4b$

$-8 = 4b$

$\boxed{b = -2}$

$4 - a = 3 + 4b$

$4 - a = -5$

$\boxed{a = 9}$

1

Long-Answer Problem #3

[6 pt]
At an all-you-can-eat buffet, the total calories you gain can be represented by the function

$$E(t) = \frac{At}{b+t}$$

where $t \geq 0$ is the time in minutes you spend at the restaurant and A and b are positive constants.

(a) If you stayed for a long time, what asymptote would your total caloric gain approach?

$E(t) \to \boxed{A}$ ② as $t \to \infty$

(b) After how much time do you gain exactly half of that asymptotic caloric amount?

At $\boxed{t=b}$ ②, $E(b) = \frac{Ab}{b+b} = \frac{A\cancel{b}}{2\cancel{b}} = \frac{A}{2}$.

(c) At time t, what is the instantaneous rate at which your caloric gain changes?

$E'(t) = \frac{A(b+t) - At}{(b+t)^2} = \boxed{\frac{Ab}{(b+t)^2}}$ ① ①

This page may be used for rough work. It will not be marked.