Math 102 exam review

Use two steps of Euler's method to approximate $y(1)$ where $y' = y^2$ and $y(0) = 1$.

(A) 6/5

(B) 21/8

(C) 4/3

(D) 3/2

(E) 9/8

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Which equation gives the point on the graph of $f(x)=x^2+b$ whose tangent line goes through the origin?

(A) $a^2 = b$

(B) $y = 2a\,(x-a) + a^2 + b$

(C) $x^2 + b = 0$

(D) There is no such point.

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(A) 0

(B) 20

(C) 100

(D) 40 + 20 sqrt(7/3)

(E) It grows without bound (“infinity”)

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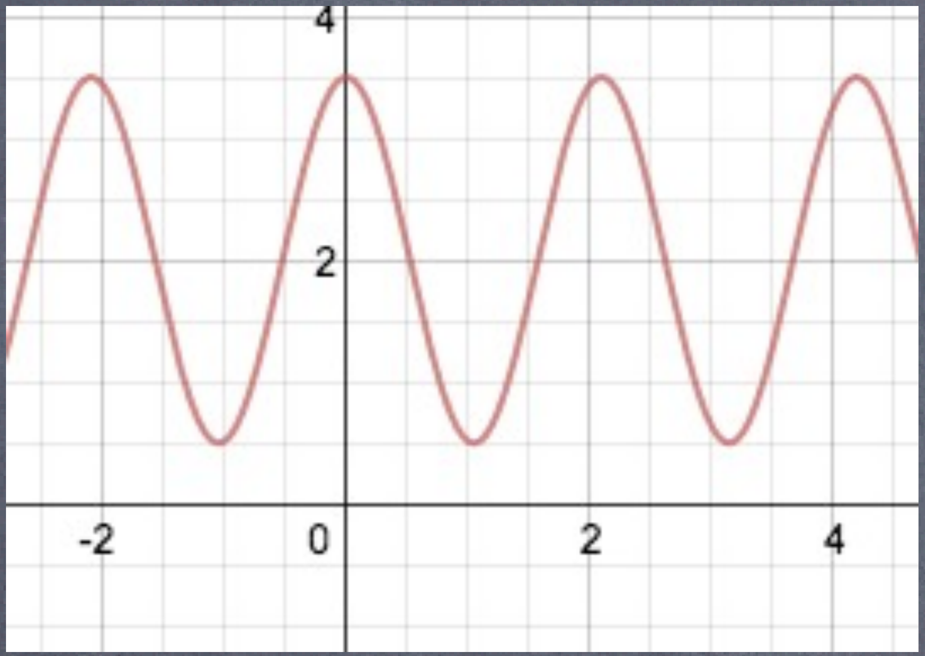
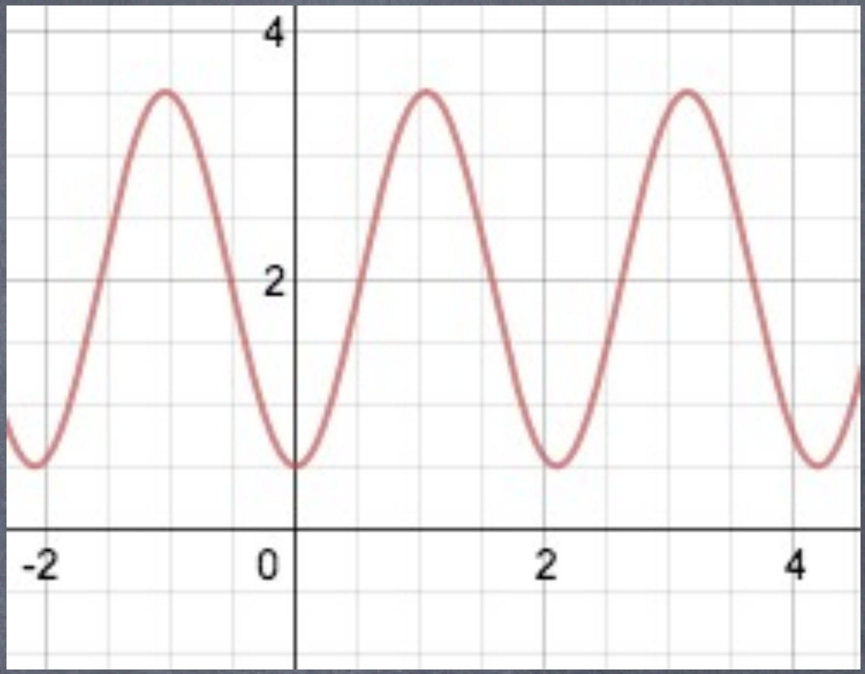
Which is the graph of $y = 2 + 1.5 \sin(3x - \pi/2)$?

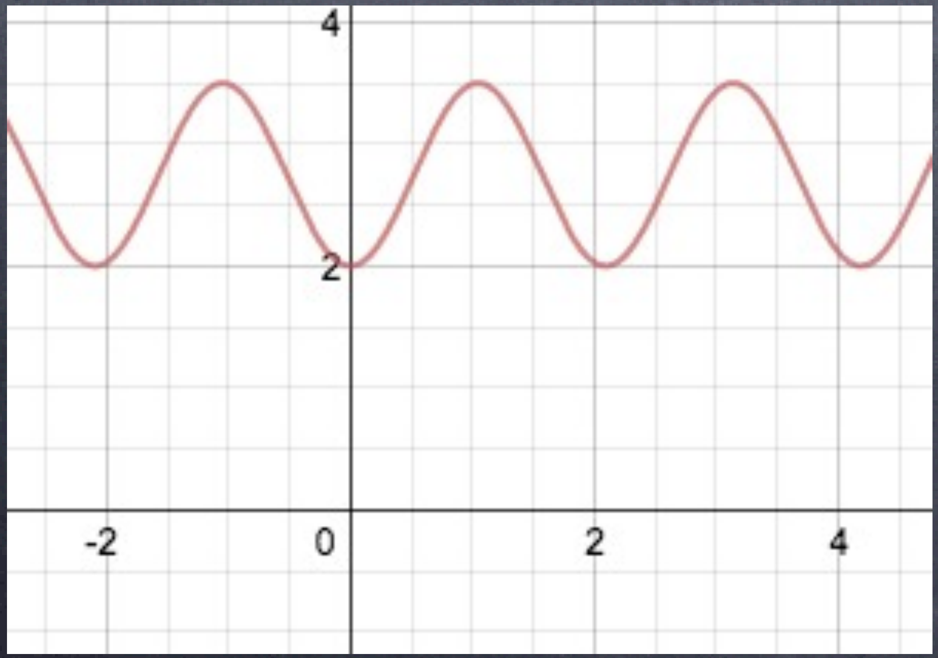
(A)



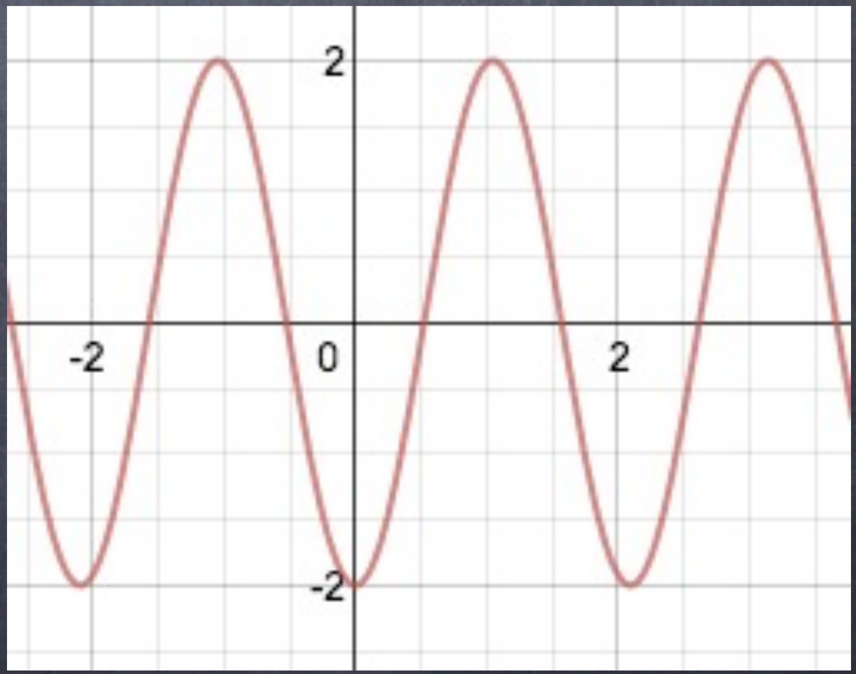
(B)



(C)

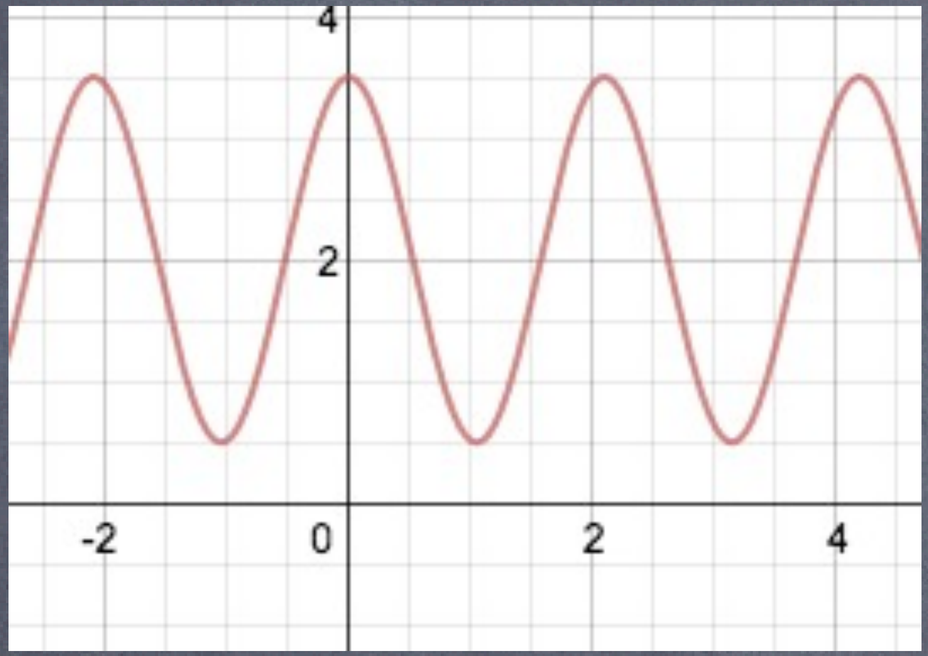


(D)

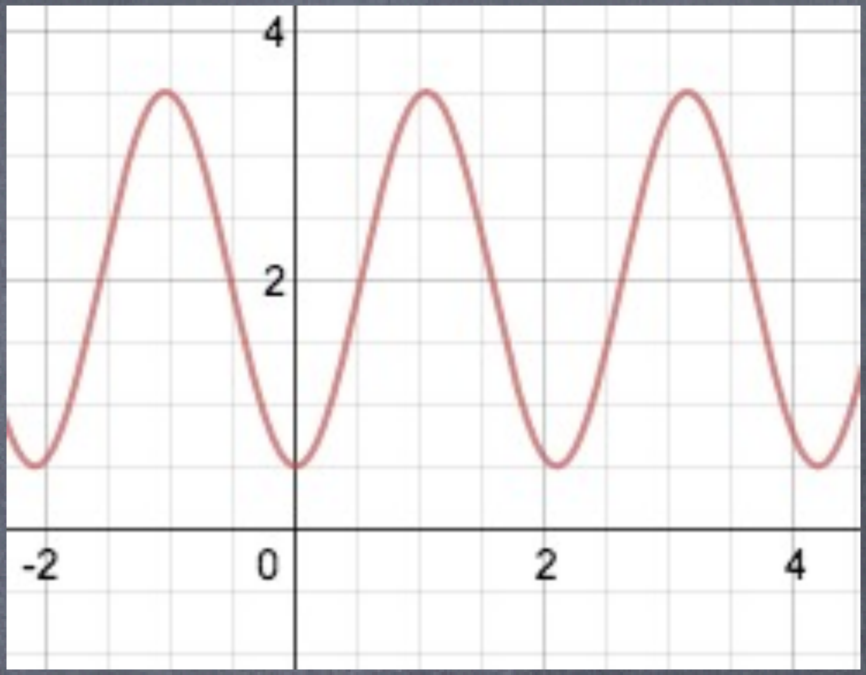


Which is the graph of $y = 2 + 1.5 \sin(3x-\pi/2)$?

(A)



(B)

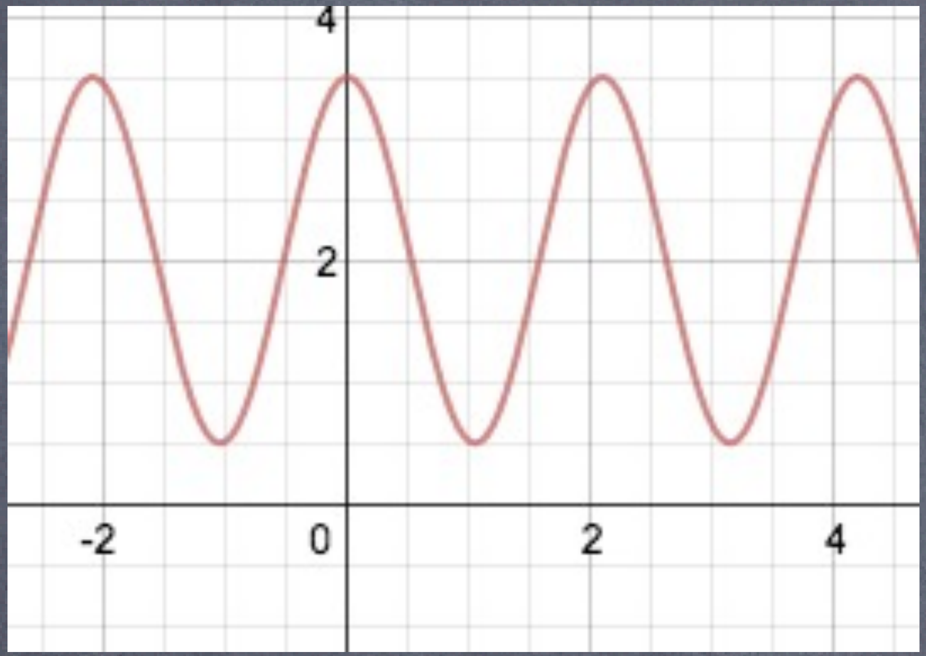


$y=2+1.5 \sin(3(x-\pi/6))$

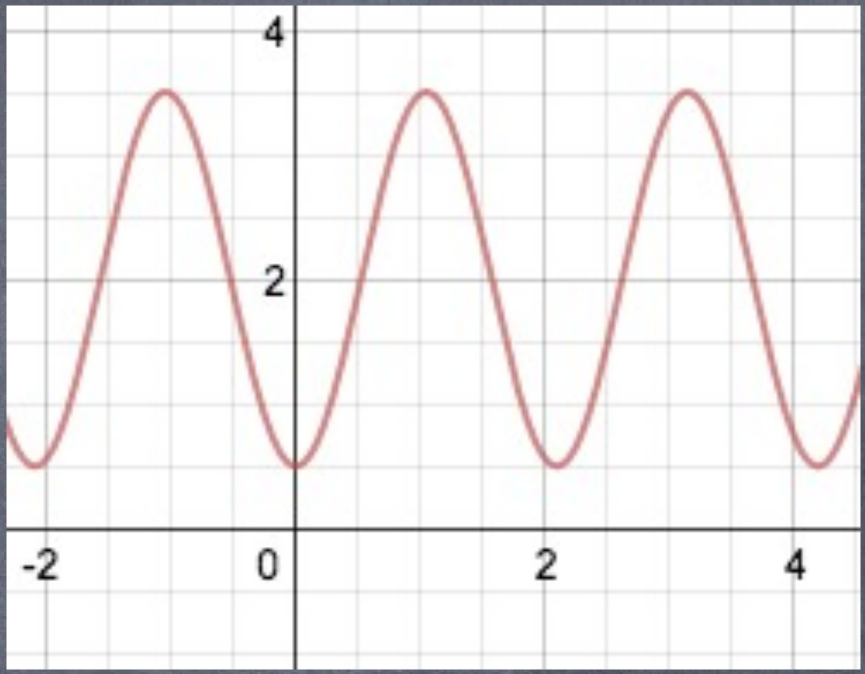
so this is like $\sin(3x)$ shifted right by $\pi/6$.

Which is the graph of $y = 2 + 1.5 \sin(3x - \pi/2)$?

(A)



(B)



Or just plug in x=0...

Choose the correct one

(A) If $f'(0) = 0$ then $x=0$ must be a local max or min of f.

(B) If $f''(0) = 0$ then $x=0$ must be an inflection pt of f.

(C) If $f'(0) = 0$ and $f''(0) < 0$ then $x=0$ must be a local min.

(D) If $f(-1) = 0$ and $f'(x)<0$ for $-1 < x < 1$ then $f(0)<0$.

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Use the definition of the derivative to calculate $f'(x)$ where $f(x)=x^2$.

(should take < 2min... click when you're done)

What are the stable steady states of $y' = \sin(y)$?

(A) $-3\pi/2, -\pi/2, \pi/2, 3\pi/2$

(B) $-3\pi, -\pi, \pi, 3\pi, 5\pi$

(C) $-2\pi, -\pi, 0, \pi, 2\pi$

(D) $-7\pi/2, -3\pi/2, 3\pi/2, 7\pi/2$

(E) $-4\pi, -2\pi, 0, 2\pi, 4\pi$

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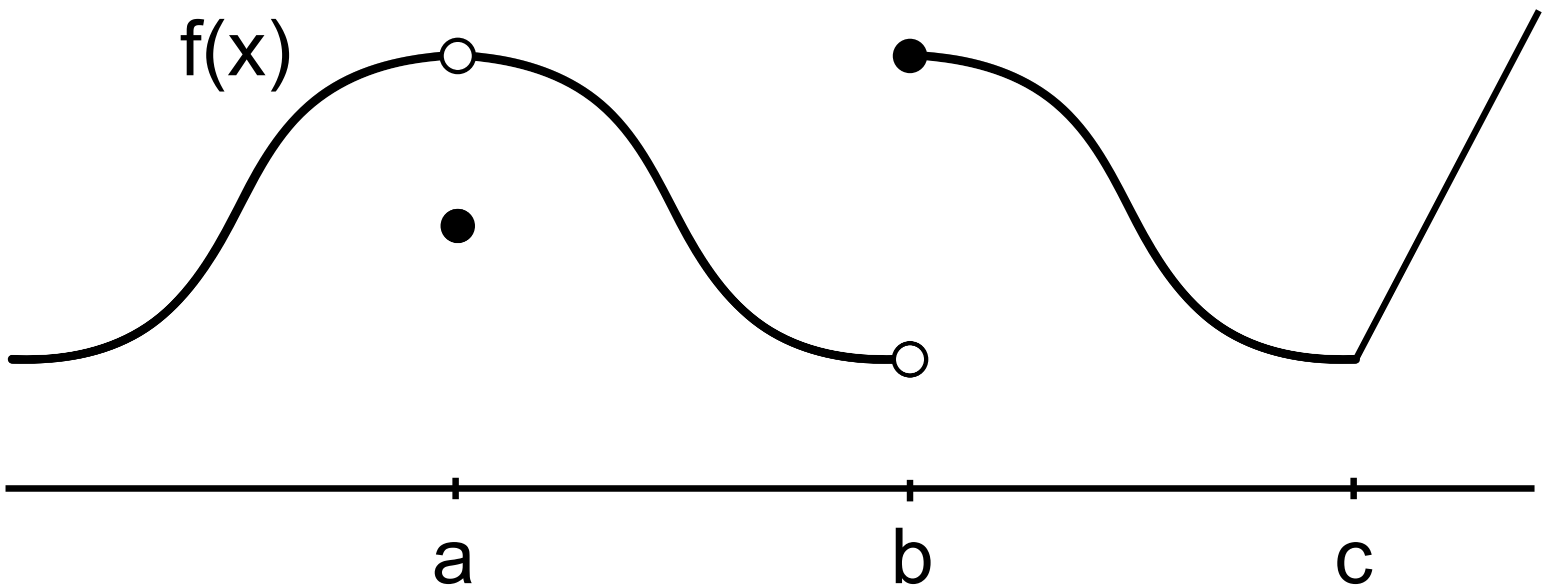
Limits



Which of the following are true?

1. $\lim_{x \to a} f(x) = f(a)$
2. $\lim_{x \to b} f(x) = f(b)$
3. $\lim_{x \to c} f(x)$ does not exist.
4. $\lim_{x \to a} f(x)$ exists.
5. $\lim_{x \to b} f(x)$ exists.

(A) 1, 4

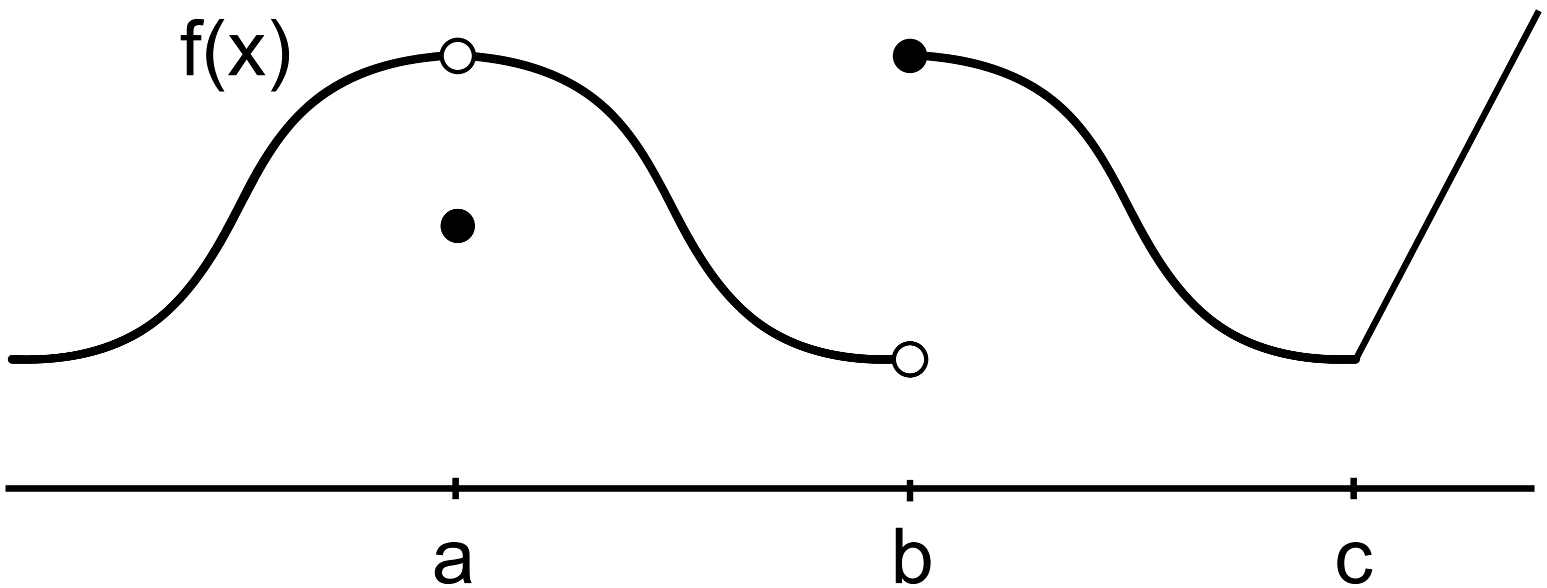
(B) 2, 5

(C) 3

(D) 4

(E) 5

Limits



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(C) 3

(D) 4

(E) 5

m_1 is the slope of $f(x)$ at $x=0$.
m_2 is the slope of $f^{-1}(x)$ at $x=0$.

(A) $m_1 = m_2$

(B) $m_1 = -m_2$

(C) $m_1 = 1/m_2$

(D) $m_1 = -1/m_2$

(E) None of the above.

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(E) None of the above.

$$g(x) = f^{-1}(x)$$

$$f(g(x)) = x$$

$$f'(g(x))\, g'(x) = 1$$

$$g'(x) = 1/f'(g(x))$$

$$g'(0) = 1/f'(g(0))$$

$$g'(0) \neq 1/f'(0) \text{ unless } g(0)=0.$$

Requested problem 1...

A.4 [2 pts] The transmembrane potential in a neuron is well-described by the equation

$$\frac{dv}{dt} = -v^3 + 20v^2 + 3500v.$$

If the transmembrane potential starts at $v(0) = 10$, what value (v_∞) does it approach as $t \to \infty$?

(a) $v_\infty = -70$, (b) $v_\infty = -50$, (c) $v_\infty = 0$, (d) $v_\infty = 50$, (e) $v_\infty = 70$.

Followup Q - solving DEs

- "Solve" $y'=ky$ (state $y(t)=Ce^{kt}$).
- Use substitution to solve $y'=a+by$.
- Pull $y(t)=-b/a+(y_0+b/a)e^{bt}$ out of a hat and use it for calculations.
- Check that a given function solves any other DE.
- Find steady states to any DE ($y'=f(y)$).
- Note: "solving a DE" means coming up with a solution. There are other DE things you need to know.

Requested problem 2

8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s^2.

   i. (4 points) Write down a function $d(t)$ that is the distance from your car to the moose where $t = 0$ indicates the moment when you start backing away.

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d(0)=?

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$d(0)$=?

$v_{car}'(t) = 2\ \text{m/s}^2$ so $v_{car}(t)$ = ?

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$d'(t) = v_{car}(t) - v_{moose}(t) = ?$

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so $d(t) = ?$

Requested problem 3

C.5.2 [8 pts] An architect is designing a house in the form of a cylinder covered by a roof in the shape of half a sphere (extending above the cylinder). Suppose the material used to build the cylindrical wall is half the price of the material that is used to build the roof per unit area. If the total volume of the house is fixed, what ratio between the height of the wall and the radius of the roof will minimize the cost?

$f(x) = \ln(x) - e^x$ has a critical point at x_{crit} where $1/2 < x_{crit} < 1$.

(A) x_{crit} is a local min.

(B) x_{crit} is a local max.

(C) x_{crit} is an inflection point.

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List all mins and maxes of $g(x) = x^4 - 8/3\ x^3 + 2x^2$.

(A) 0

(B) 1

(C) –1

(D) 0, 1

(E) –1, 0

List all mins and maxes of $g(x) = x^4 - 8/3\ x^3 + 2x^2$.

(A) 0 (min)

(B) 1 (inflection pt)

(C) -1

(D) 0, 1

(E) -1, 0

$\cos(\arcsin(x)) = \ldots$

(A) $\text{sqrt}(1-x^2)$

(B) $1/\text{sqrt}(1-x^2)$

(C) $x/\text{sqrt}(1-x^2)$

(D) $\text{sqrt}(1-x^2)/x$

(E) $1/x$

cos(arcsin(x)) = . . .

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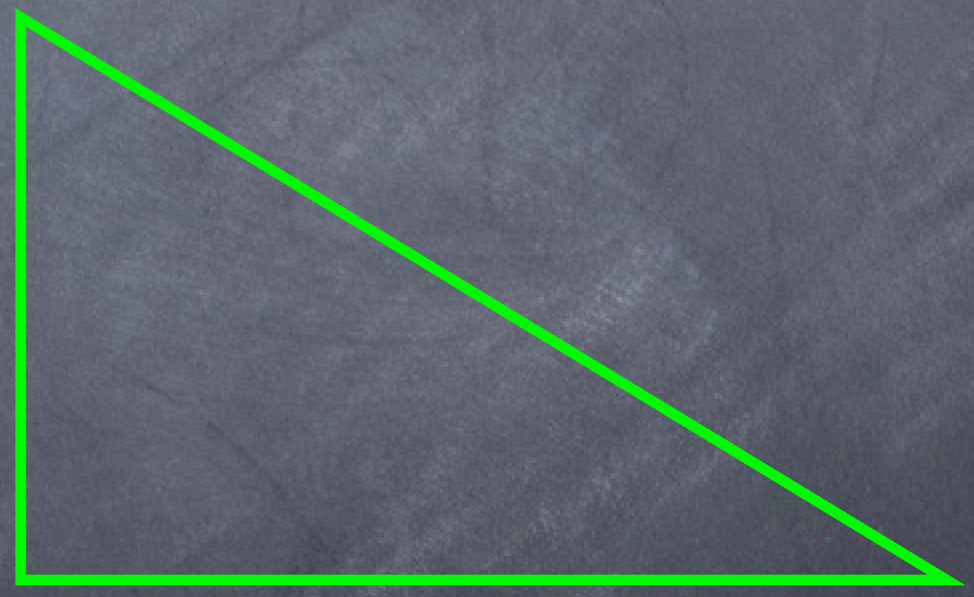

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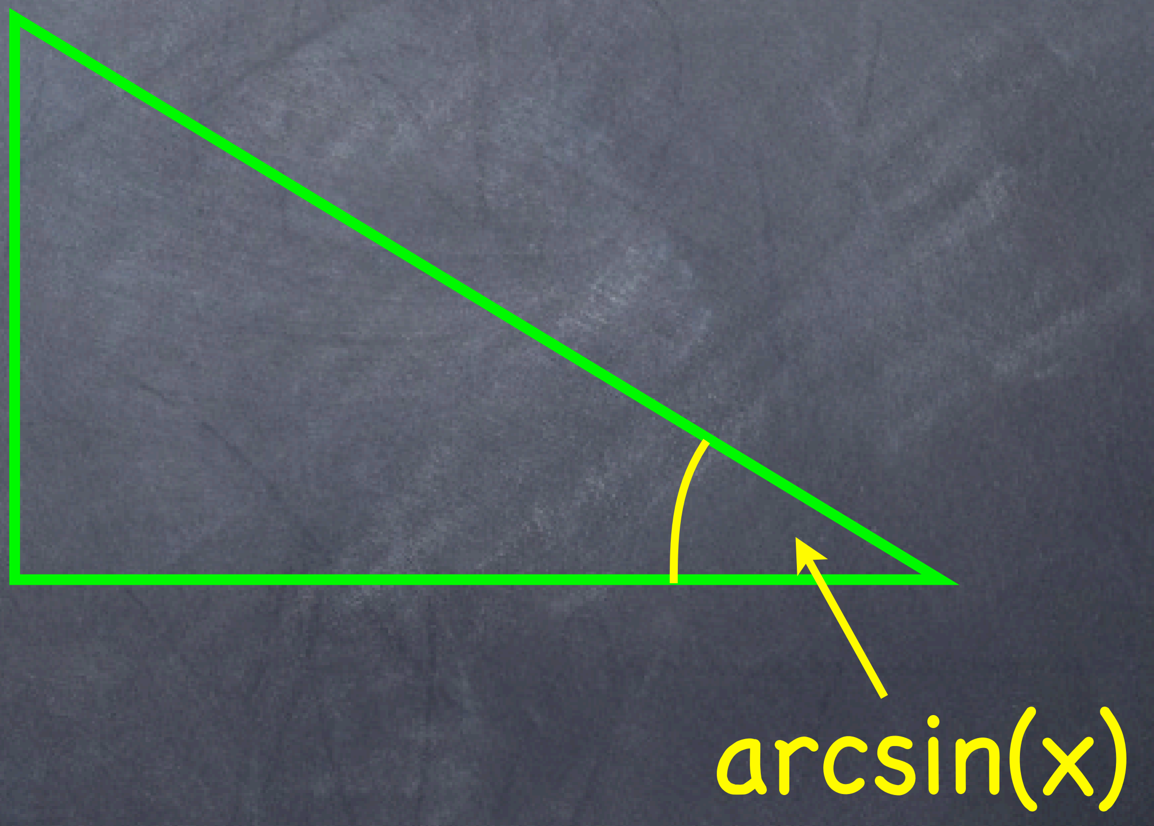

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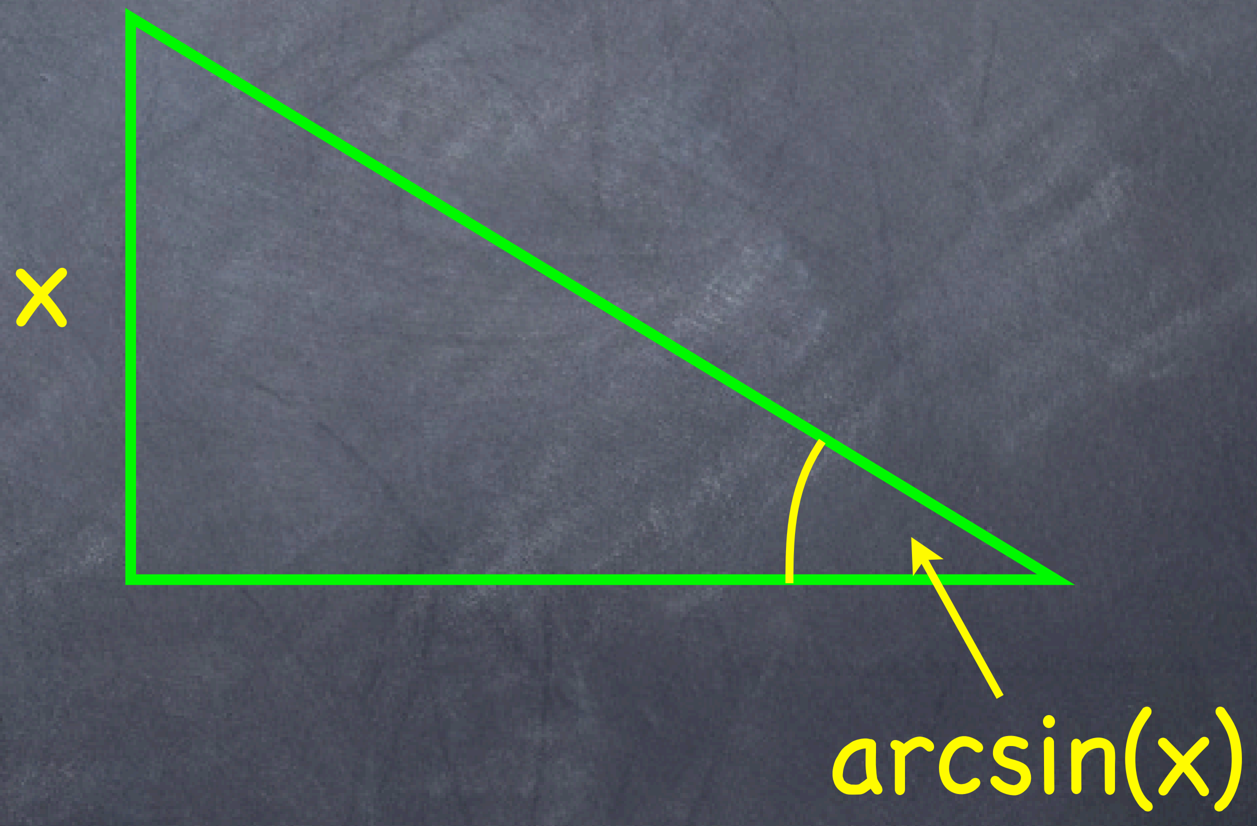

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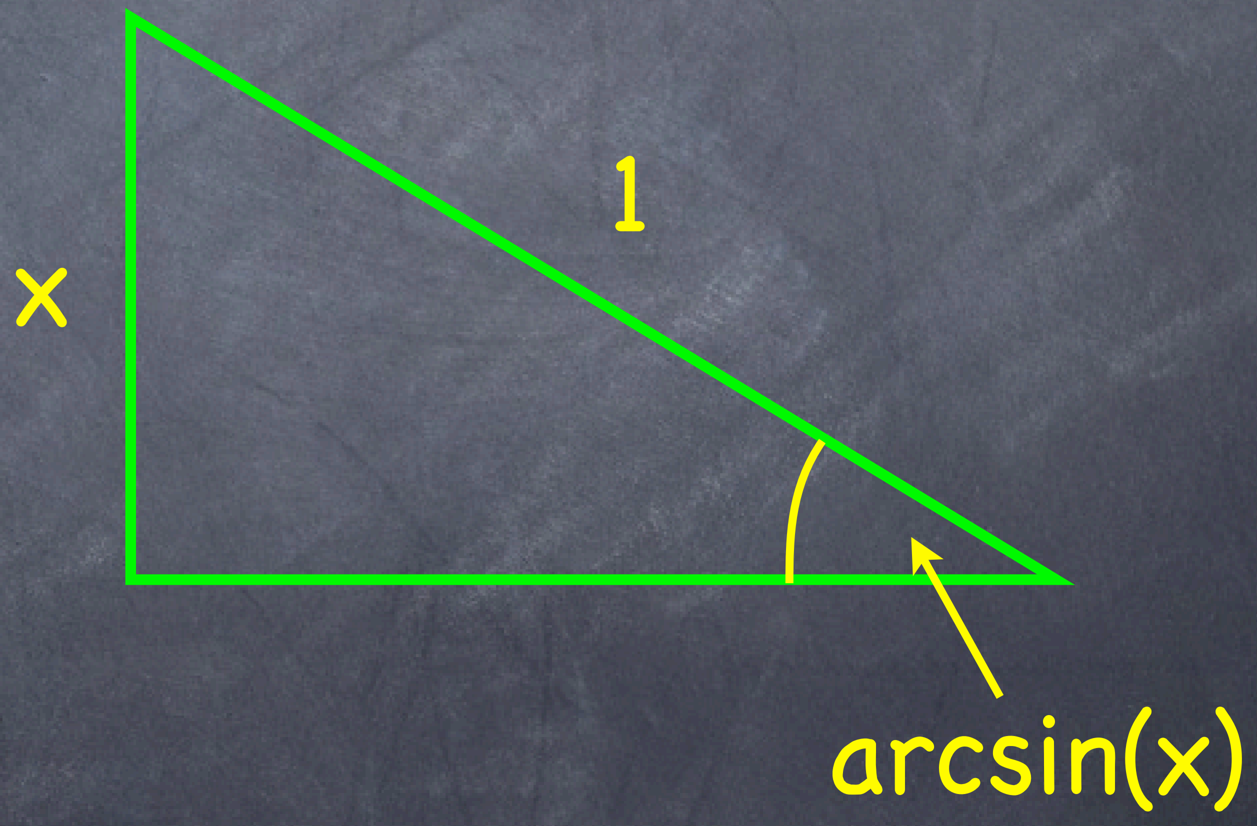

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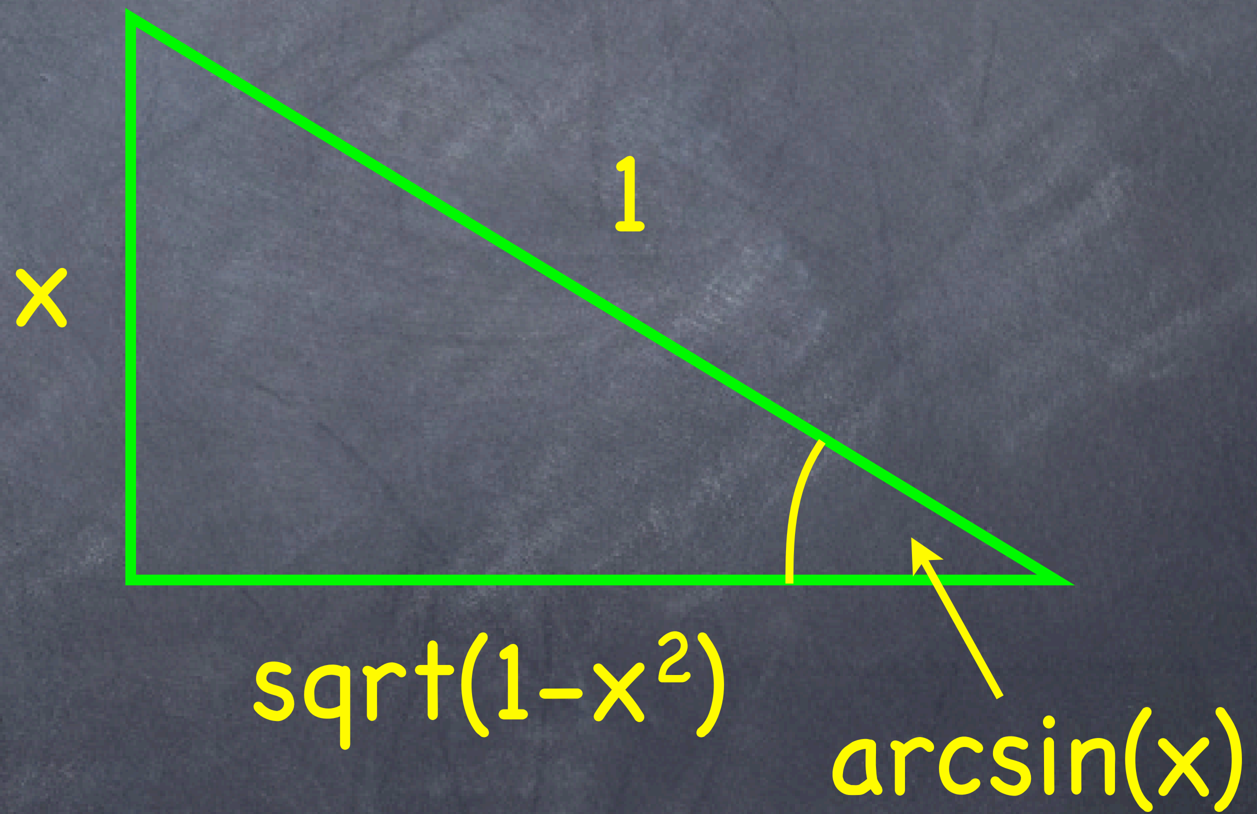

Ensuring continuity

- For what value of a is the following function continuous at all points x?

(A) a=2

(B) a=-2

(C) a=0

(D) a=1

$$f(x) = \begin{cases} 4 - a^2 + 3x & x < 1 \\ x^2 + ax & x \geq 1 \end{cases}$$

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$$f(x) = \begin{cases} 4 - a^2 + 3x & x < 1 \\ x^2 + ax & x \geq 1 \end{cases}$$

$7\text{-}a^2\text{=}1\text{+}a$ --> $a^2\text{+}a\text{-}6\text{=}0$ --> a=2,-3.

Find tangent line to $f(x)=x^2$ that goes through (1,-1).

Point of tangency is at

(A) $(1+\sqrt{2}, 3-2\sqrt{2})$

(B) $(1+\sqrt{2}, 3+2\sqrt{2})$

(C) (1, -1)

(D) $(1-\sqrt{2}, 3-2\sqrt{2})$

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