Today

- Related rate example (water in cone)
- Implicit differentiation
    - Tangent line example
    - Power rule for fractional powers (next week)

Studying what you don't know how to do

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  - get used spending a few minutes totally confused before something clicks.

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

Which of the following matches your intuition for how these rates are related?

(A) h(t) decreases quickly at first and then slows down.

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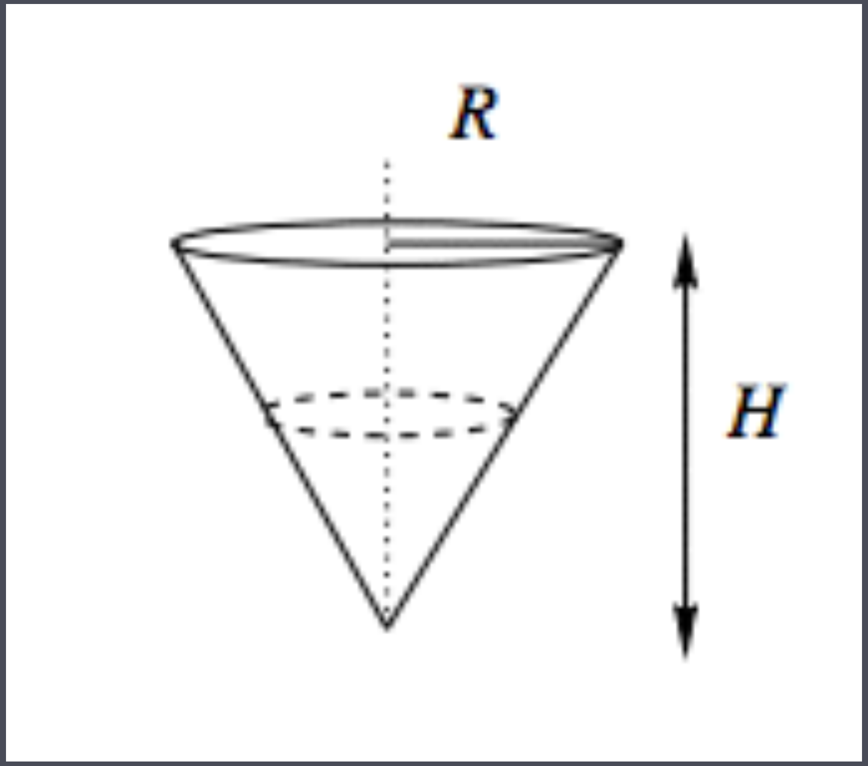

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Which is the relevant equation relating the quantities when the water is at height h (not rates of change yet)?

(A) $V = 1/3\ \pi\ R^2 H$

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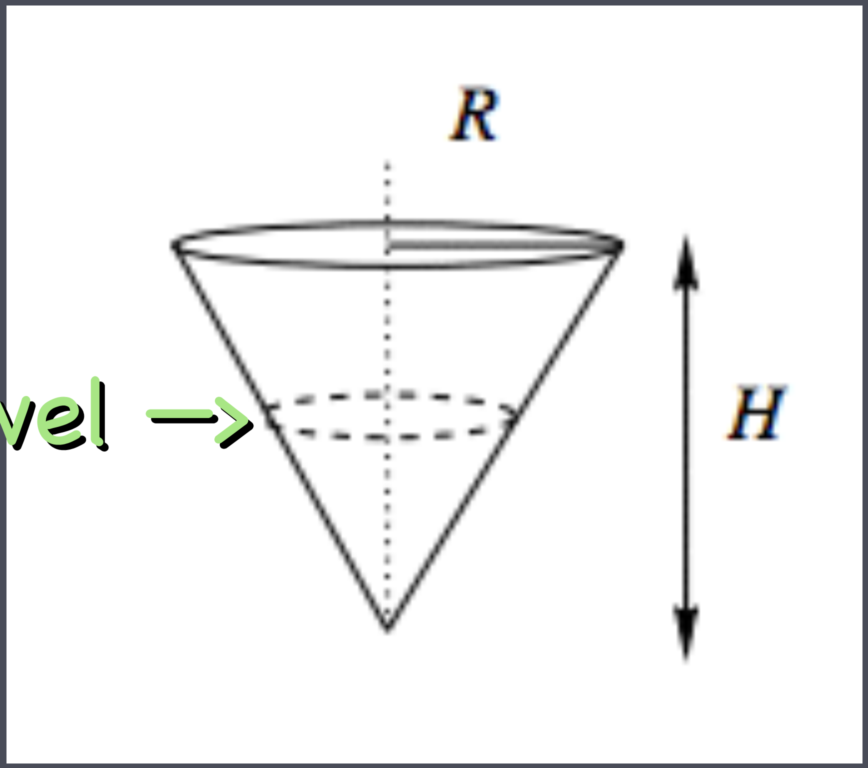

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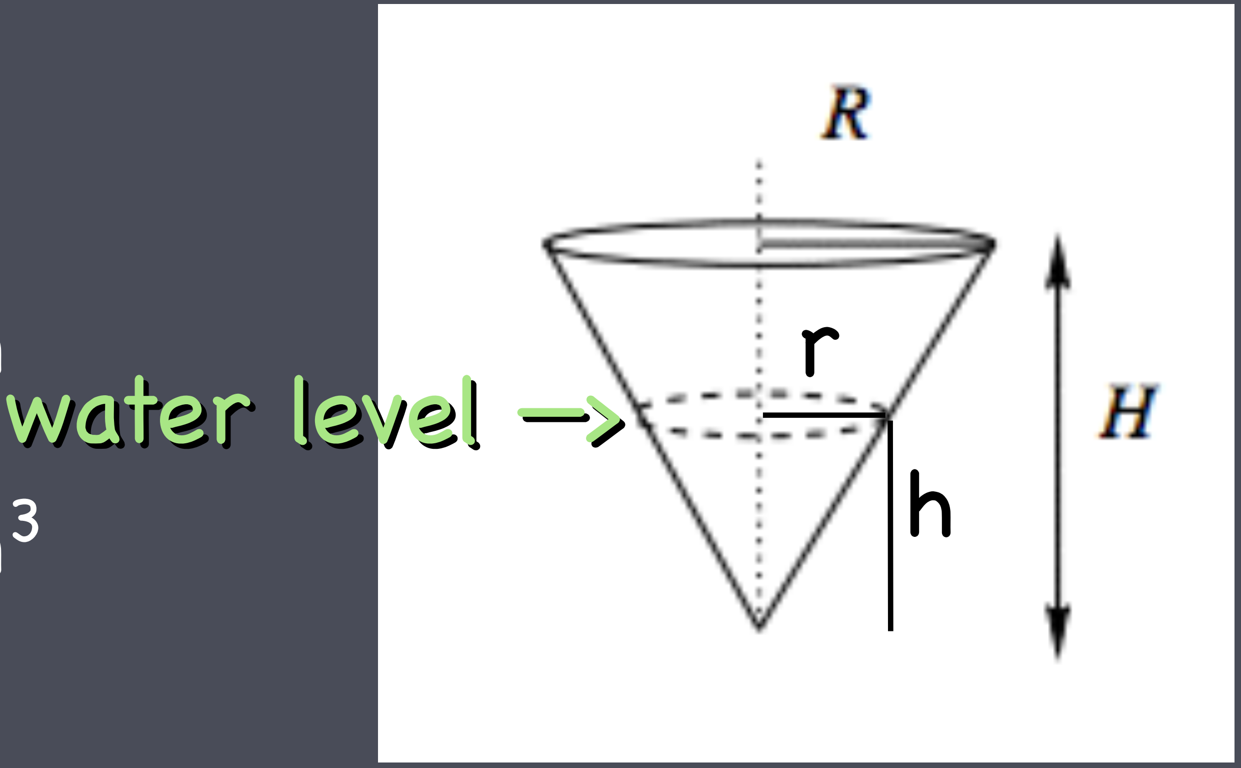

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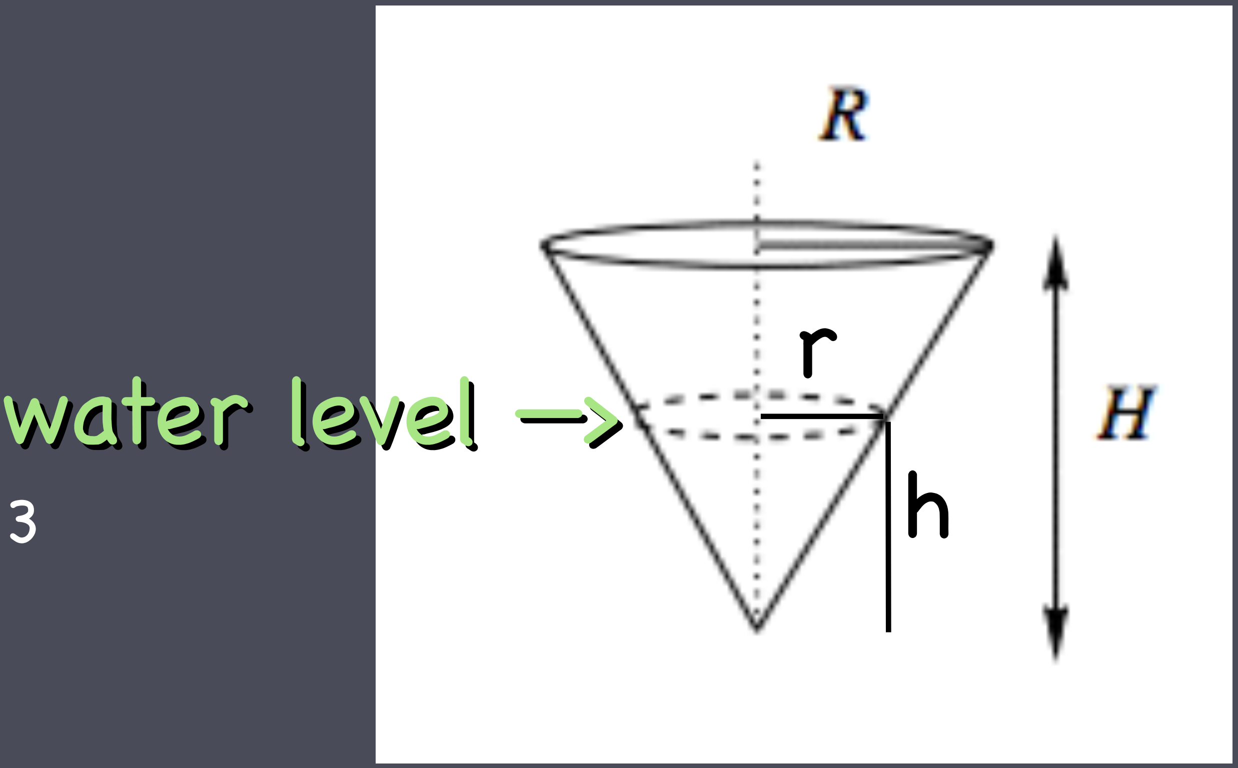

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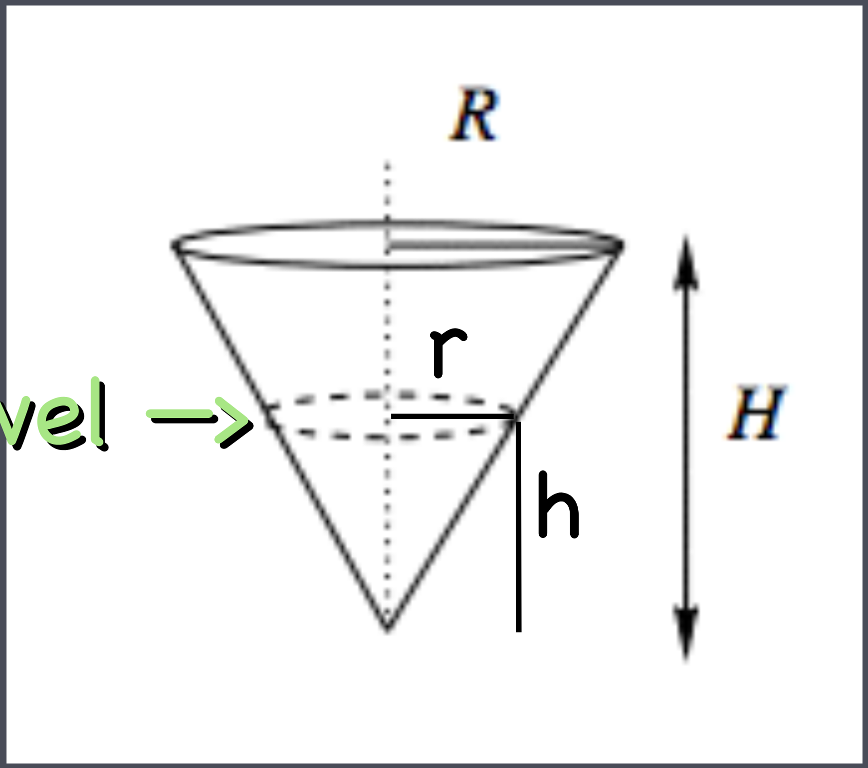

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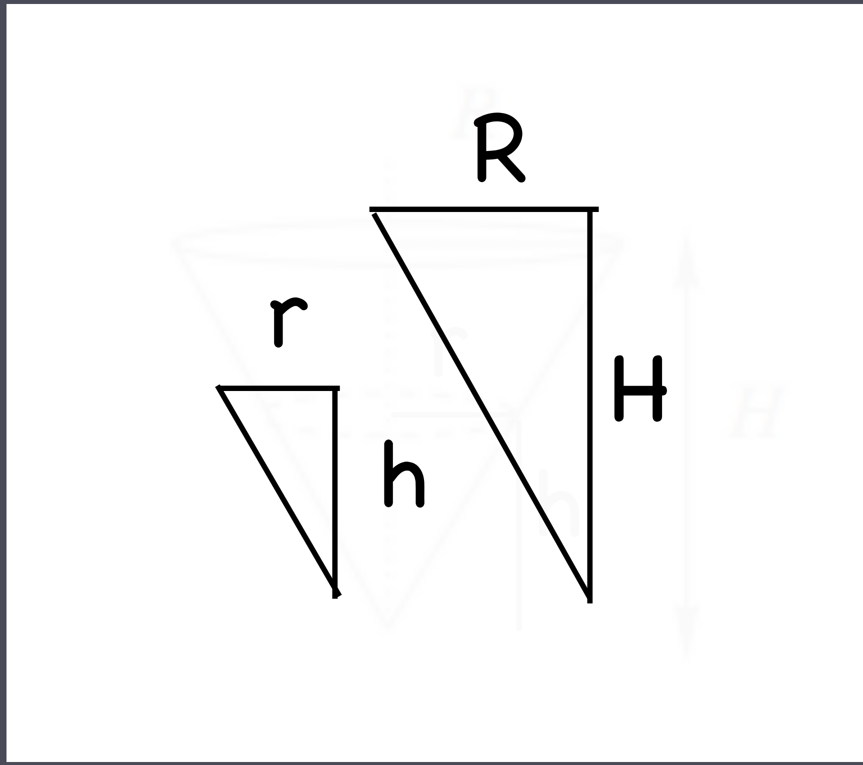

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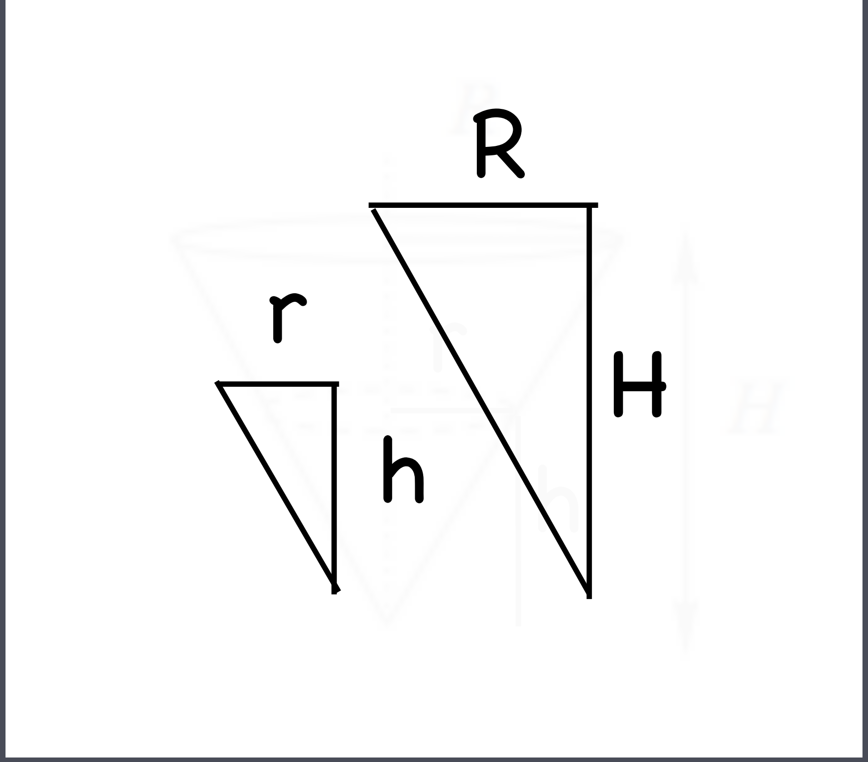

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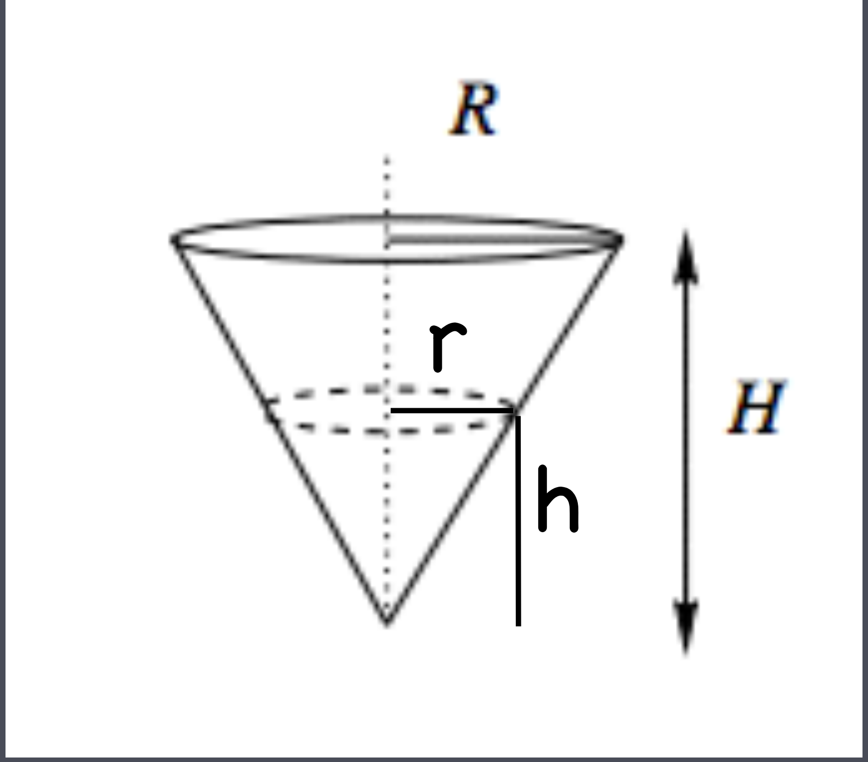

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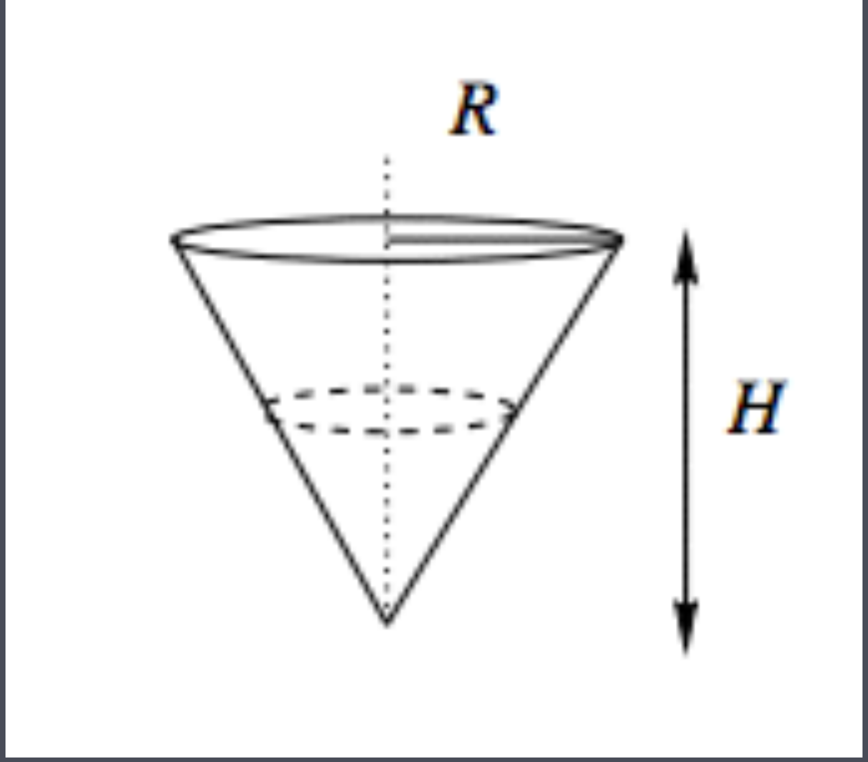

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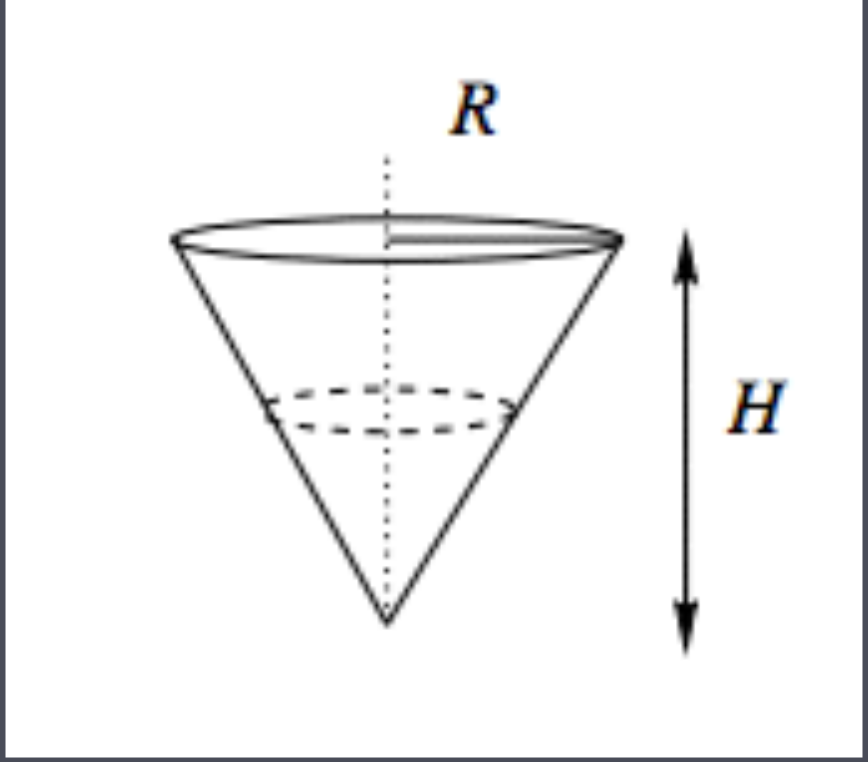

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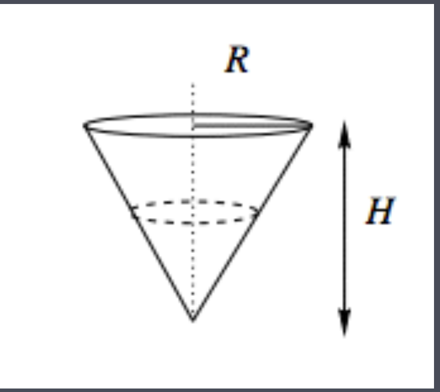

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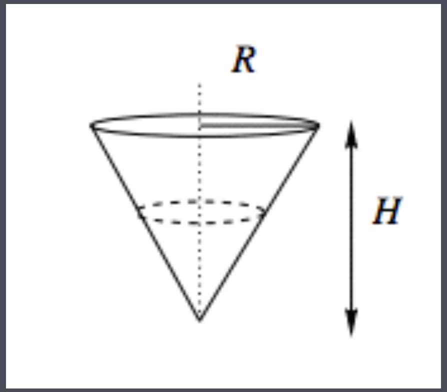

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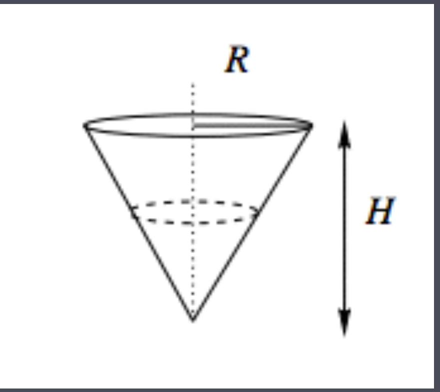

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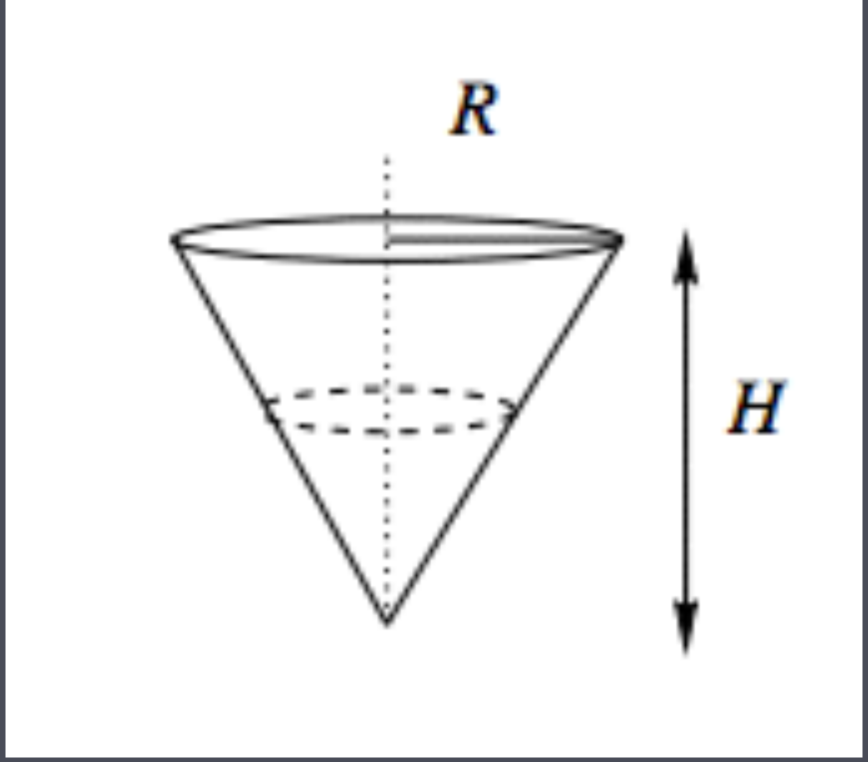

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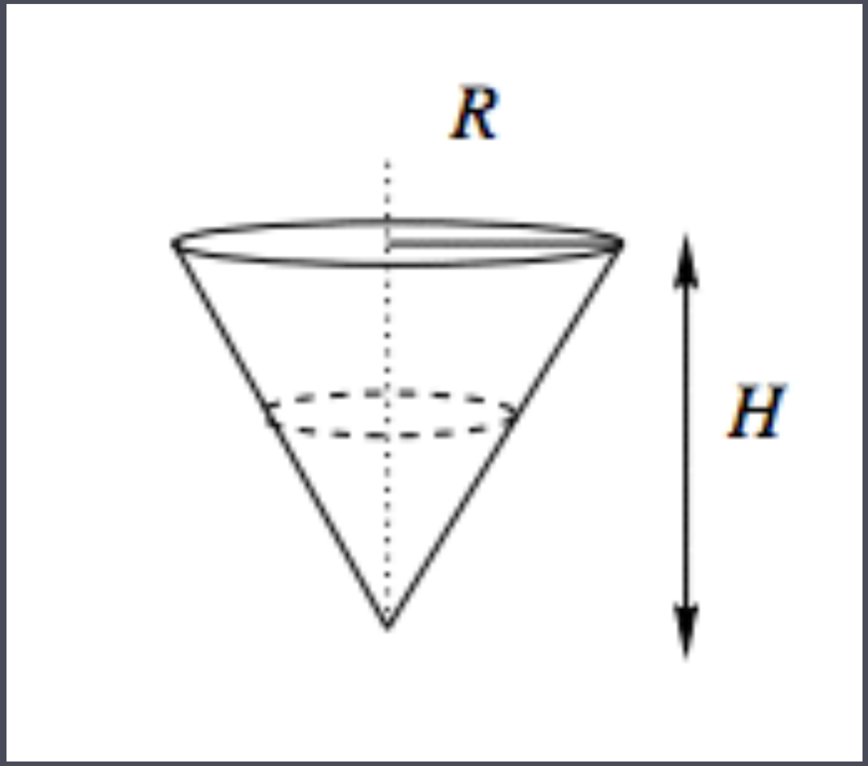

Plug h=H into $h' = -k / (\pi (R^2/H^2)\, h^2)$

How many times do you think you should have to read a problem before finding the answer?

(A) 1-2 times.

(B) 3-4 times.

(C) 5-6 times.

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- 2 x number of steps.

Procedure

- Establish expectation(s) based on sketch or otherwise.
- Find equation relating Q_1 and Q_2.
- Take derivatives on both sides.
- Finally, plug in specific values.
- Reality check - compare answer against expectation.

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- e.g. What is the highest point on the ellipse $x^2 + 3y^2 - xy = 1$?
- Let $y=y(x)$ and take "implicit derivative" of e.g. $x^2+y(x)^2=25$ ----->

Find the tangent line to the curve defined by $x^2+y^2=25$ at $(3,-4)$.

What can you predict about the answer without calculus?

(A) The slope of the tangent line will be positive.

(B) The slope of the tangent line will be negative.

(C) The slope of the tangent line will be 4/3.

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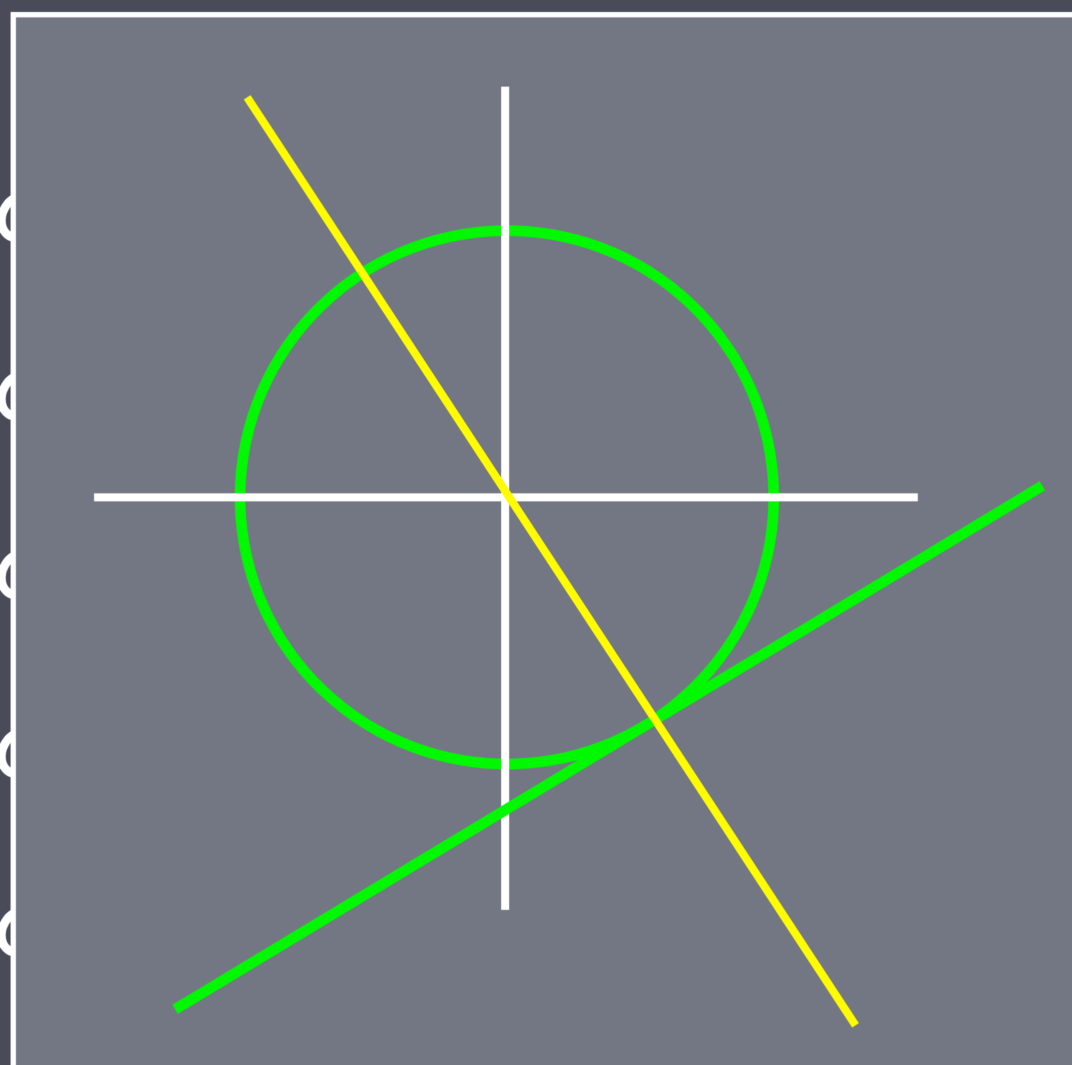

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The derivative of each side of this equation must also be equal. That means...

(A) $2xx' + 2yy' = 25$.

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Assume that $y=y(x)$. If by $'$ we mean d/dx, then (C) is technically ok but $dx/dx=1$.

Find the tangent line to the curve defined by $x^2+y^2=25$ at $(3,-4)$.

(A) $y' = -2x / 2y$

(B) $y = 3/4 (x-3) - 4$

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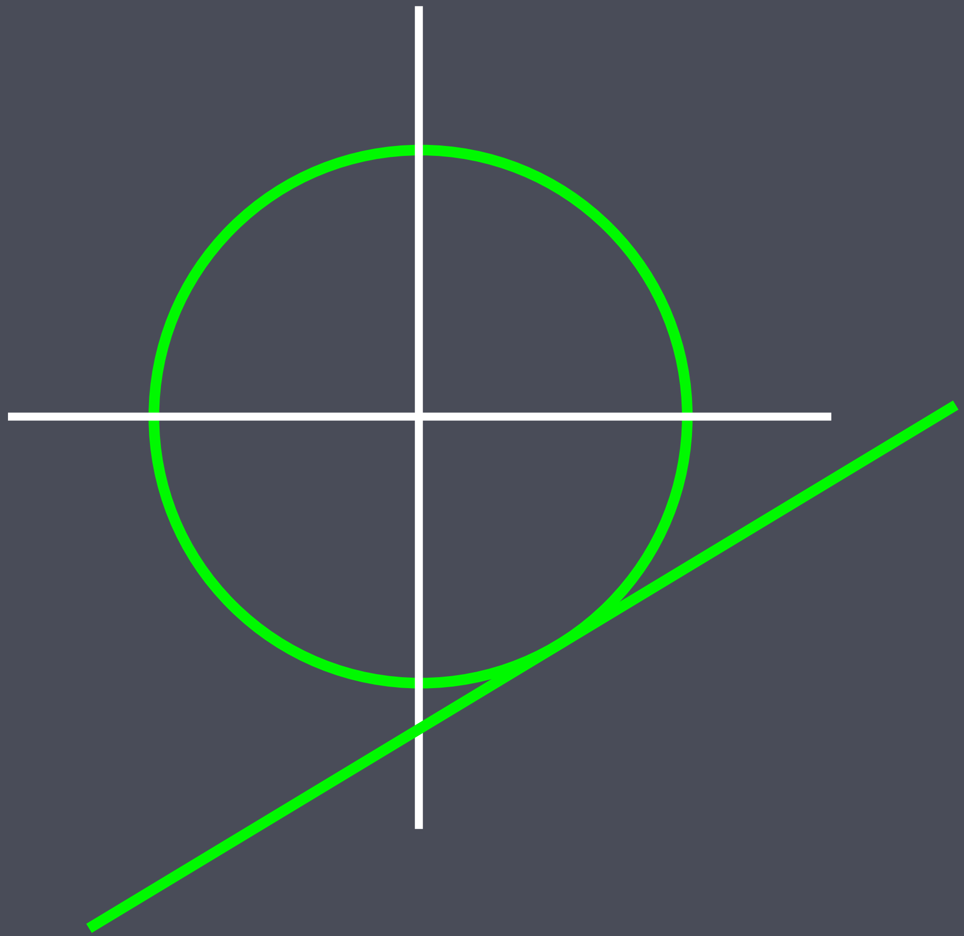

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(A) $y' = -2x / 2y$ <--- correct formula for slope at the point (x,y).

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(A) $y' = -2x / 2y$ <--- correct formula for slope at the point (x,y).

(B) $y = 3/4\ (x-3) - 4$

(C) $2x + 2yy' = 0$ <--- correct ans from previous slide.

(D) $y' = 3/4$ <--- correct slope at $(3,-4)$

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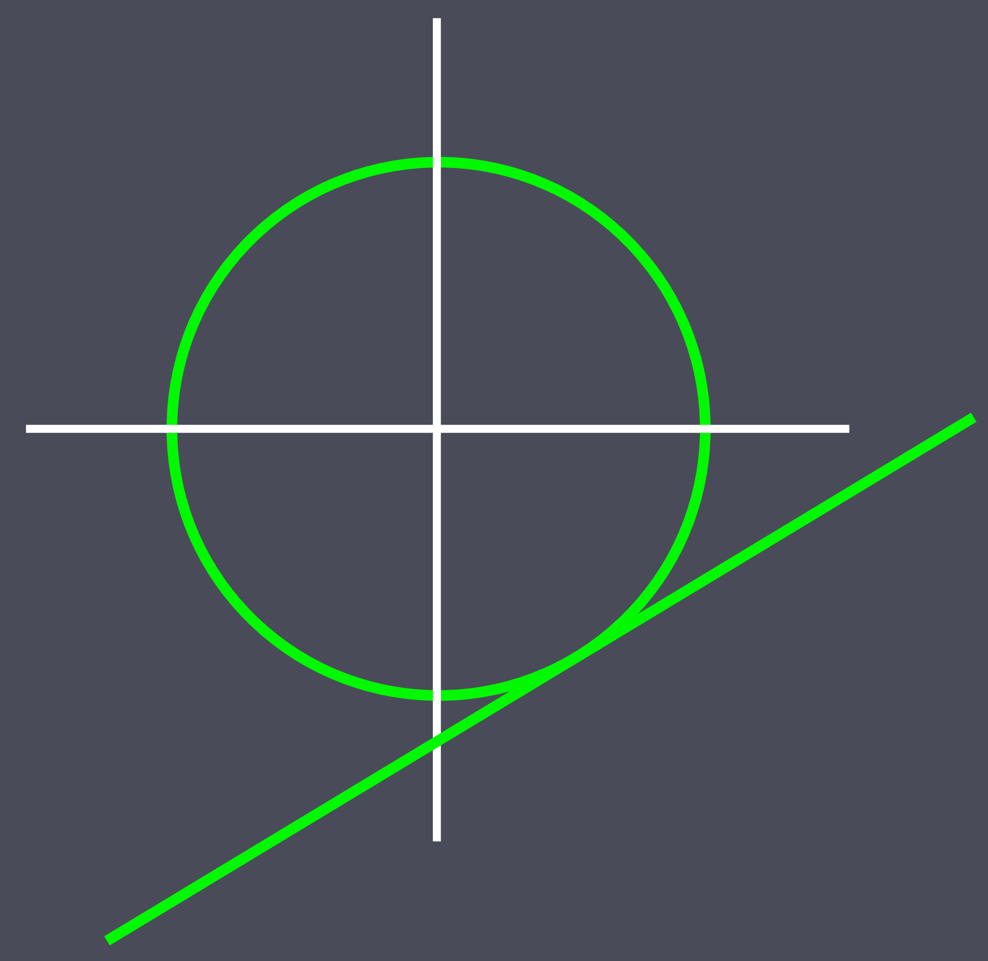
Comments on this problem:



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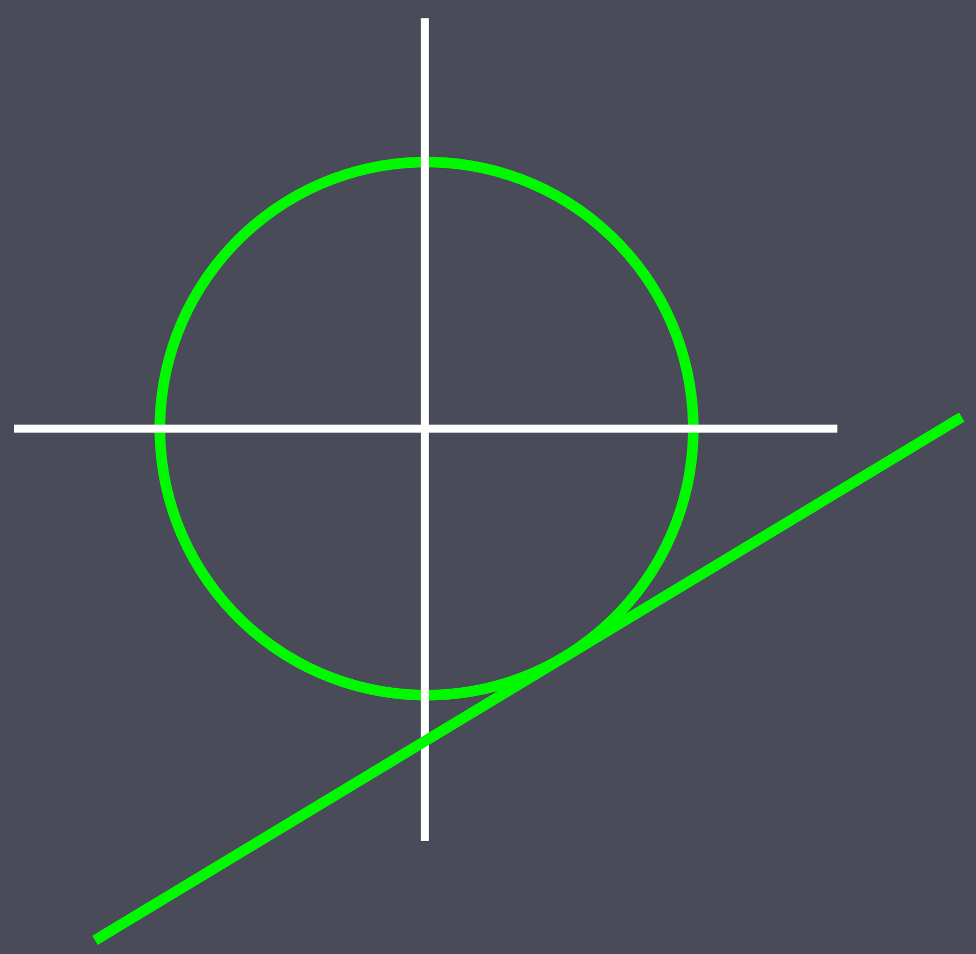

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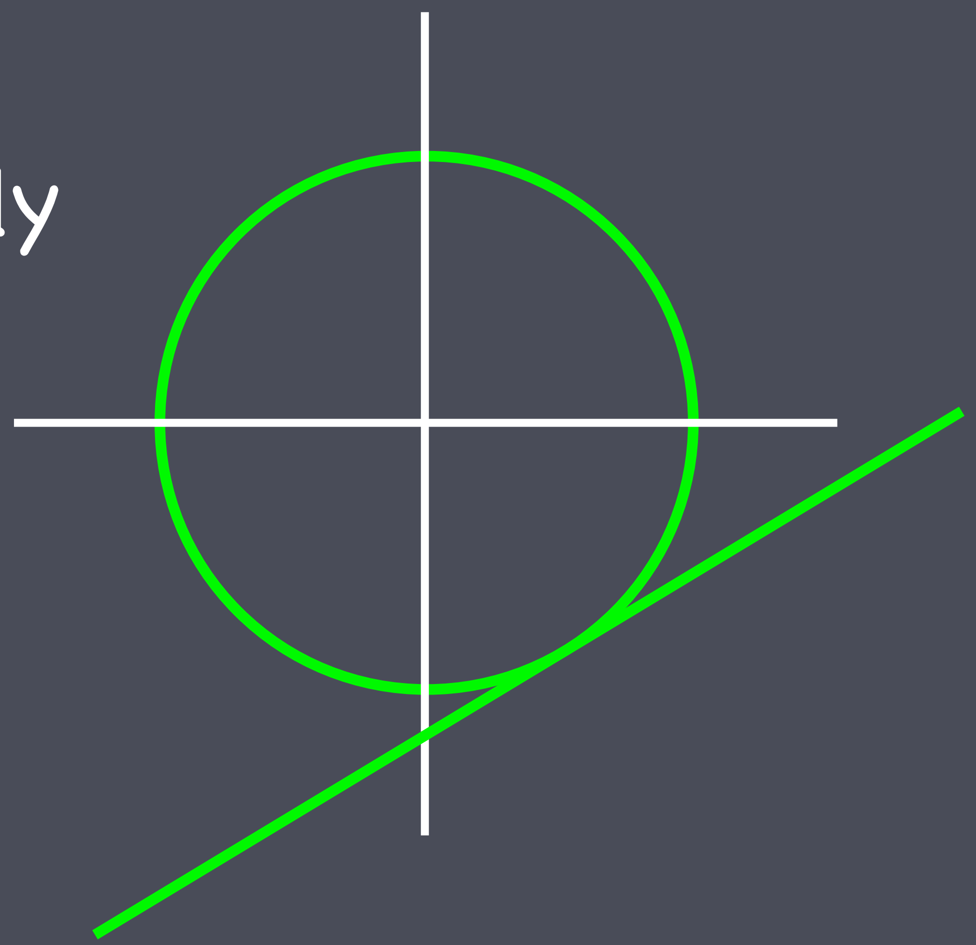

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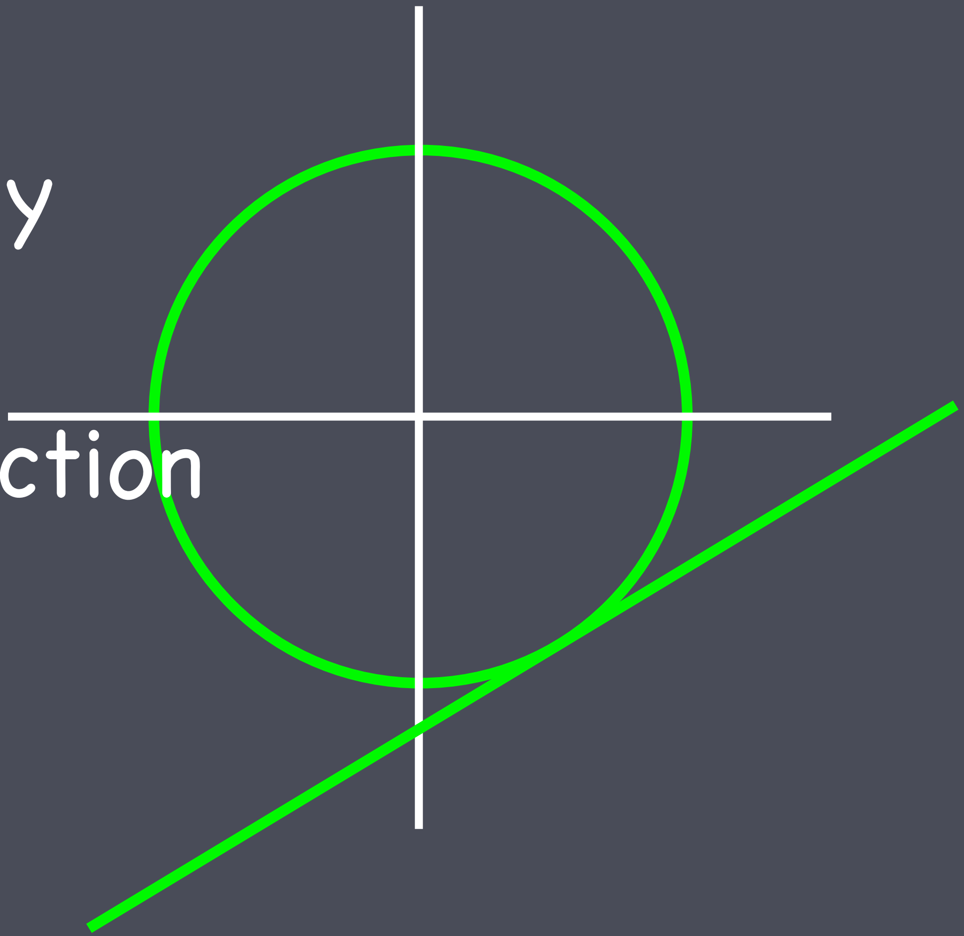

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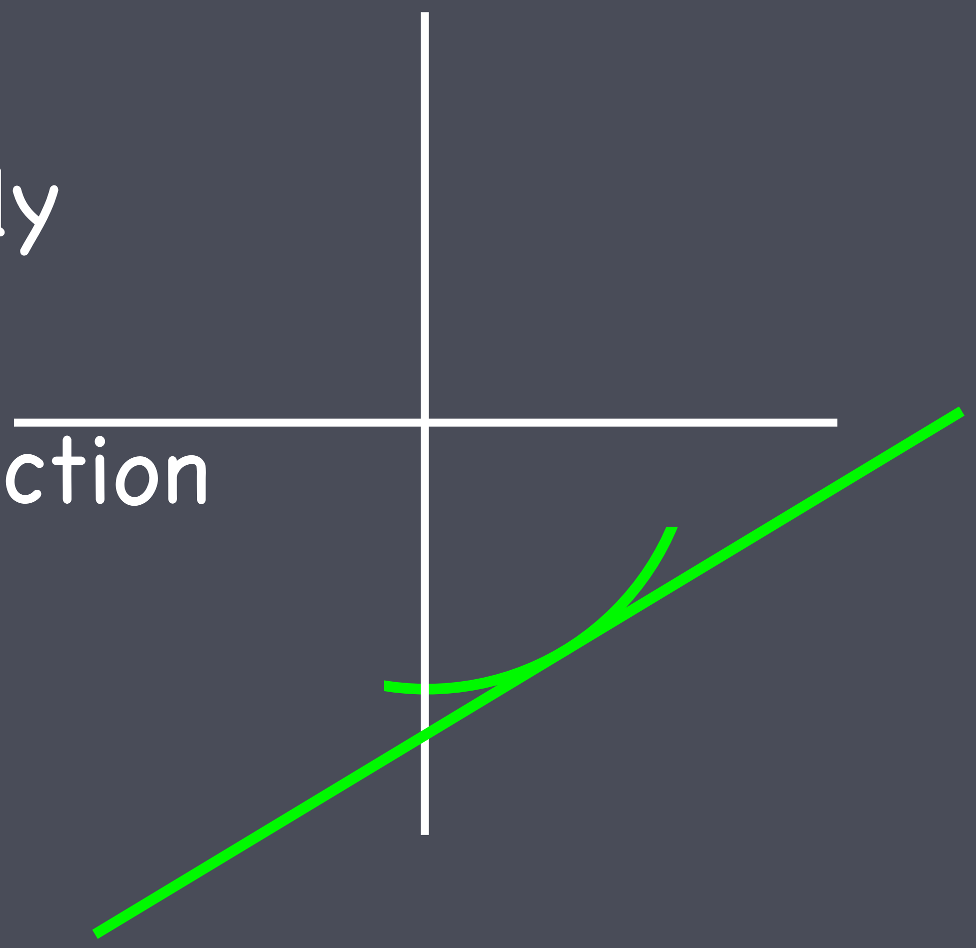

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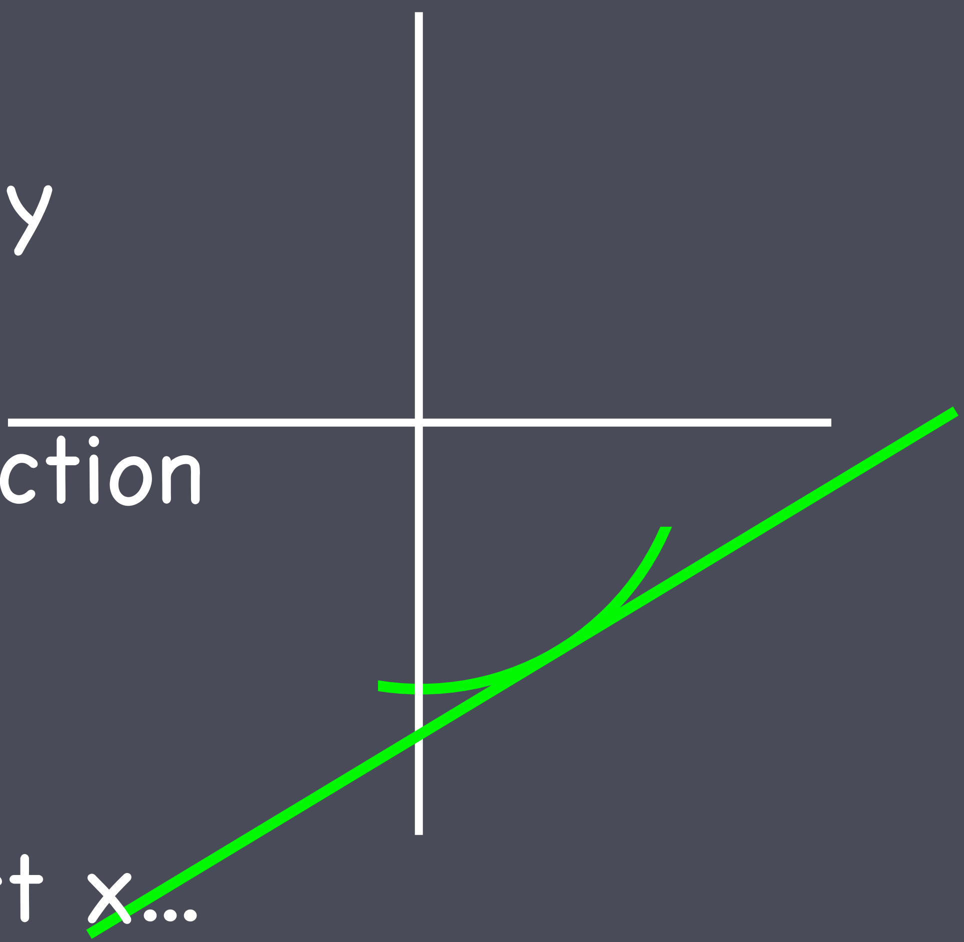

- Take derivative of both sides wrt x...

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- Take derivative of both sides wrt x...
- This doesn't work at (1,0)! (how might you deal with this?)