1.
$$f(x) = \frac{3x^3 + 2x^2}{x^3 + x}$$

(a) Determine $\lim_{x\to\infty} f(x)$.

(b) Determine $\lim_{x\to 1^+} f(x)$.

(c) Determine $\lim_{x\to 0} f(x)$.

2. From the definition of the derivative, determine
$$\frac{d}{dx}(3x^2)$$

3. $f''(x) = 2x$. Determine $f(x)$, such that $f'(0) = 1$ and $f(0) = 3$.

4. Determine
$$\frac{d}{dx}\left[\sin\left(\ln(x)x^2\right)\right]$$

5. Using linear approximation, determine an approximate solution to the equation
$$e^{3x} = x$$

6.
$$f(x) = xe^x \text{ for } -3 \le x \le 3$$

(a) Determine local extrema of $f(x)$.

(b) Determine global extrema of $f(x)$.

(c) Determine inflection points of $f(x)$.

(d) Sketch $f(x)$.

7. The (x, y) coordinates of a projectile are given by the equations
$$x(t) = v_0 t\cos(\alpha),\ y(t) = v_0 t\sin(\alpha) - 4.9t^2,\ \text{for } t \text{ such that } y(t) \ge 0,$$
where v_0 is the initial velocity of the projectile, α is the angle at which the projectile is fired, and t is time. Determine α that maximizes the distance that the projectile is fired.

8. A searchlight 10m from a road is tracking a car that is moving 5m/s. At what rate is the searchlight rotating when the car is 12m from the searchlight?

9. For $0 \leq x \leq \frac{\pi}{2}$ and $y \geq 0$, determine $\frac{dy}{dx}$ for

$$\sin(x) = e^{-y^2}$$

10. A patient in a hospital is receiving an intravenous treatment at a constant rate of 3 potent units per day. The patient's body breaks down the treatment at a rate proportional to the total amount of potent units in the body, with proportionality rate constant of 2/day. Time is measured in days from the beginning of treatment.
    (a) Write down a differential equation that describes the amount of treatment in the patient's body.
    (b) Determine steady state treatment amount(s) in the patient's body.
    (c) Sketch a state-space diagram for the differential equation.
    (d) Determine stability of steady state treatment amount(s) in the patient's body.
    (e) Solve the differential equation.
    (f) How long does it take for the amount of treatment in the patients's body to reach half of the value of the stable steady state amount?