Today

- Checking that a function satisfies a DE.
- Solving an Initial Value Problem.
- Solving Newton's Law of Cooling (NLC).
- Solving equations similar to NLC.

Checking that a function solves a differential equation (DE)

Which function solves the equation $p'(t) = kp(t)$?

(A) $p(t) = t^k$

(B) $p(t) = kt^{k-1}$

(C) $p(t) = ke^t$

(D) $p(t) = e^{kt}$

(E) $p(t) = -1/(kt+2)$

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Checking that a function solves a DE

Which function solves the equation $p'(t) = p(t)^2$?

(A) $p(t) = t^3$

(B) $p(t) = t^3/3$

(C) $p(t) = (e^t)^2$

(D) $p(t) = 1/(t+3)$

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In fact,
$p(t)=-1/(t+C)$
solves it for any C!

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  - Solution: $p(t) = 1+2e^{-t}$.

Which function solves the equation $p'(t)=p(t)^2$ subject to the initial condition $p(0)=1/2$.

(A) $p(t) = 3/2 \cdot 1/(t+3)$

(B) $p(t) = 1/(t-2)$

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(A) $T'(t) = E\,(\,k - T(t)\,)$

(C) $T'(t) = k\,(\,E - T(t)\,)$

(B) $T'(t) = k\,(\,T(t) - E\,)$

(D) $T'(t) = k\,(\,T(t) - E/k\,)$

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What do you expect

$\lim_{t\to\infty} T(t)$ to be?

(A) E

(B) kE

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- $D'(t) = -T'(t)$
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- New equation: $D'(t) = -kD(t)$

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What is the solution to $D'(t) = -kD(t)$, subject to initial condition $D(0) = E - T_0$?

(A) $D(t) = E - T_0 e^{-kt}$

(B) $D(t) = -(E - T_0)\, e^{kt}$

(C) $D(t) = (E - T_0) + e^{-kt}$

(D) $D(t) = (E - T_0)\, e^{-kt}$

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(D) $T(t) = T_0 + (E - T_0)\, e^{kt}$

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How does our expectation match up?

As $t \to \infty$, $T(t) \to$???