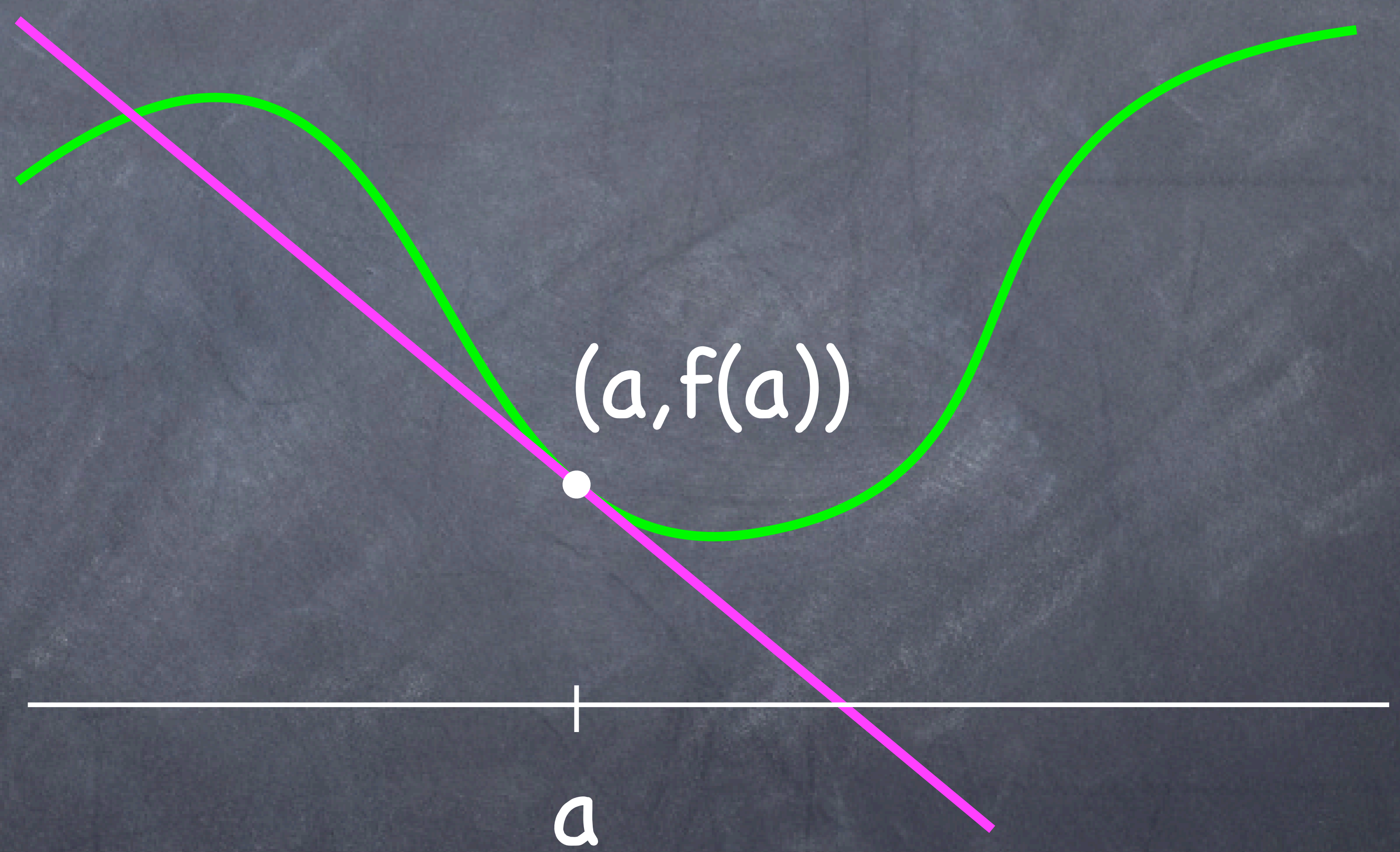
Today…

- Tangent lines
- Power rule

Find the tangent line to f(x) at (a,f(a)).



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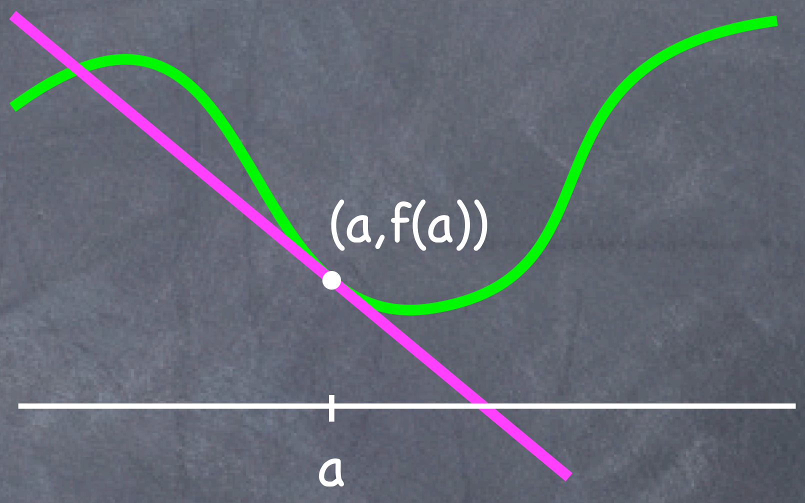
$$\frac{y - f(a)}{x - a} = f'(a)$$



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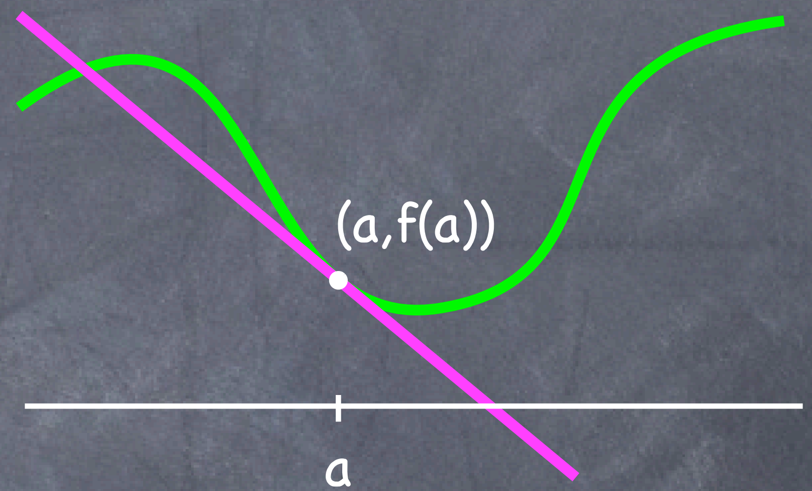

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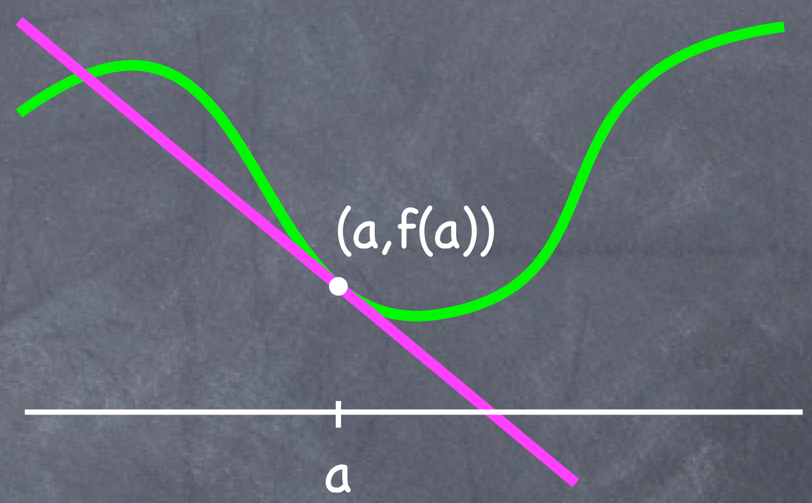

$$\frac{y - f(a)}{x - a} = f'(a)$$

$$y - f(a) = f'(a)(x - a)$$

$$y = f'(a)(x - a) + f(a)$$

Tangent line to sin(x) at x=0

- Slope of sin(x) at x=0 is 1 (spreadsheet last class).
- In general, tangent line: $y = f'(x_0)\,(x-x_0) + f(x_0)$.
- In this case, . . .

(A) $y = \cos(x)\,x + \sin(x)$

(B) $y = x$

(C) $y = x - \pi/2$

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General case

- Objects involved:

  (i) a function $f(x)$

  (ii) a point of tangency $(a,f(a))$

  (iii) slope at point of tangency $f'(a)$

  (iv) a tangent line $y = f'(a)(x-a) + f(a)$

- Some piece of information is missing - could be from any of these.

Example - simplest case

- Let $f(x) = x^3 + 2x^2 - x + 2$.
- Find tangent line at $x=3$.
- Need equation of line
  - slope is $f'(3)$, point on line is $(3,f(3))$
  - $y = f'(3)(x-3) + f(3) = 38(x-3) + 44$.

Example - slightly harder

- Let $f(x) = x^3 + 2x^2 - x + 2$.
- Find a tangent line parallel to $y=-x+3$.
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We need to...

(A) Find a such that $f(a) = -a+3$.

(B) Find a such that $f'(a)=-1$.

(C) Solve $x^3 + 2x^2 - x + 2 = -x + 3$.

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Example - even harder

- Find tangent line to $f(x)=x^2$ that goes through (1,-1).

- Name unknown point (a,f(a)). Pretend you know a. Means you also know f(a), f'(a).
- What can we now write down?
- $y = f'(a)(x-a) + f(a) = 2a\,(x-a) + a^2$.
- (1,-1) must be on this line so

$-1 = 2a\,(1-a) + a^2$. Solve for a.

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