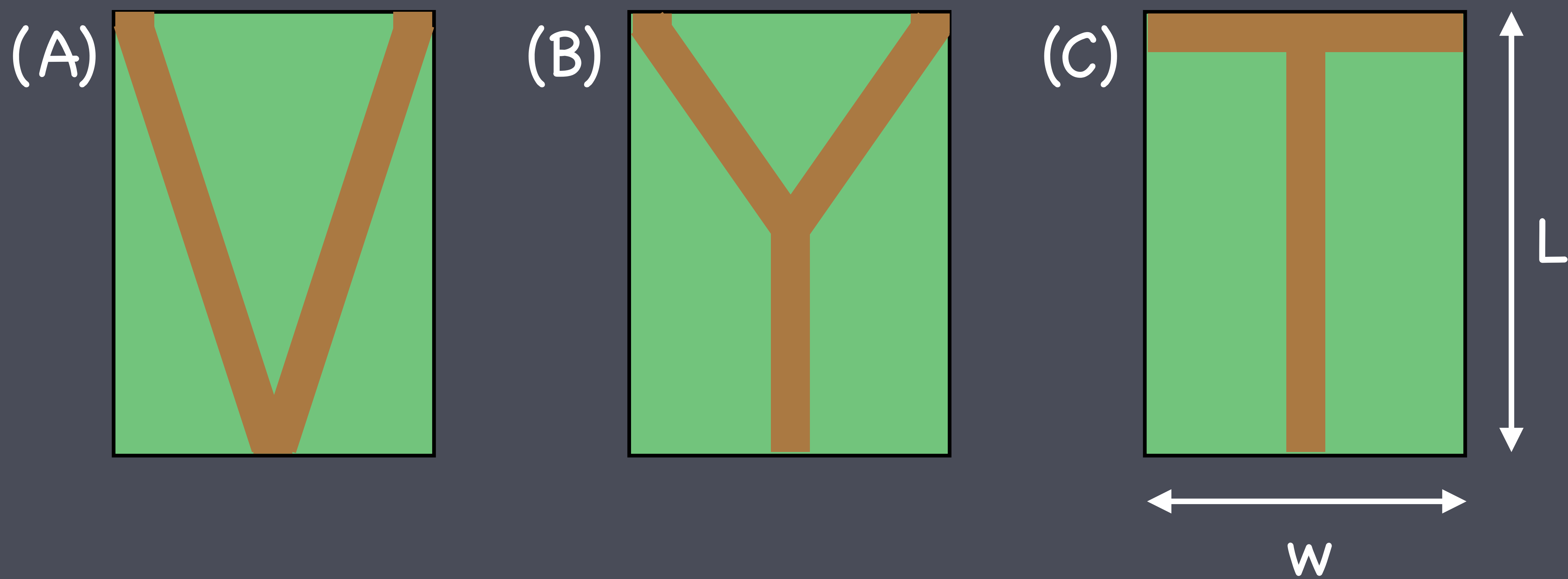
Today

- An optimization example
- Chain rule refresher for...
- Related rates examples

You need to dig a trench for pipes starting at the midpoint of the south edge of your property to bring water to two locations on either side of the north edge. Which arrangement requires the least digging?



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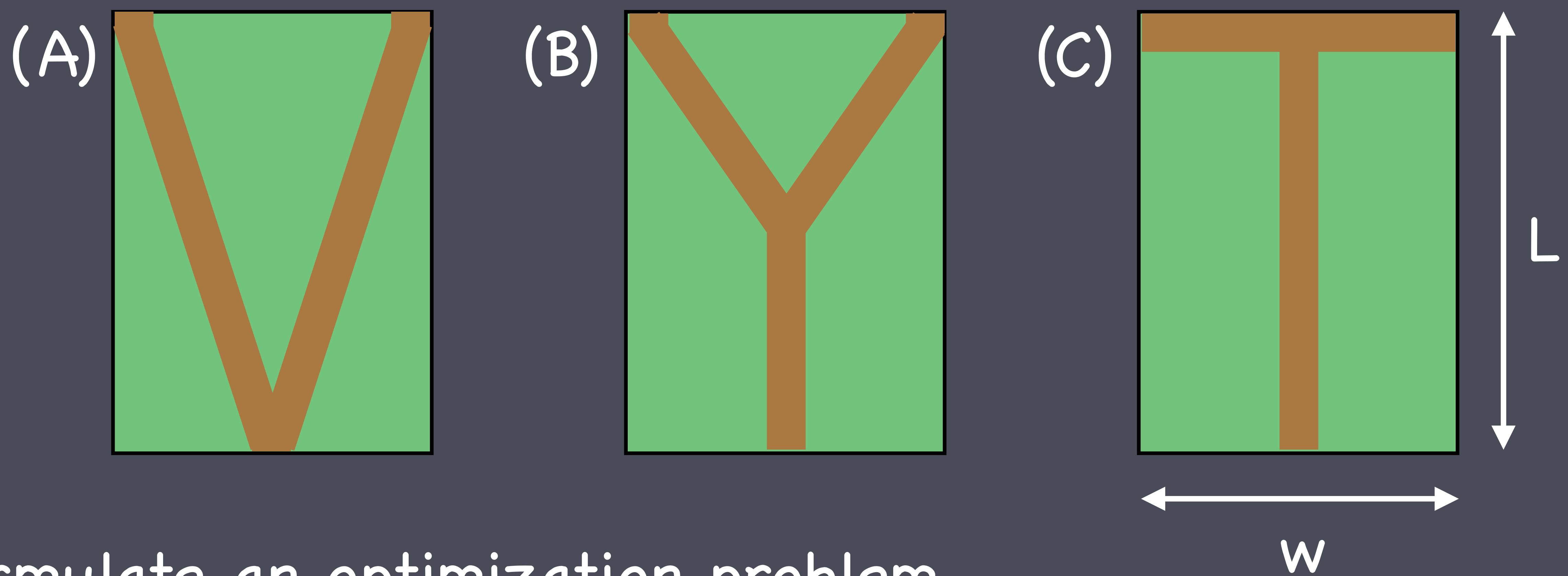

Formulate an optimization problem whose solution is the arrangement that requires the least digging.

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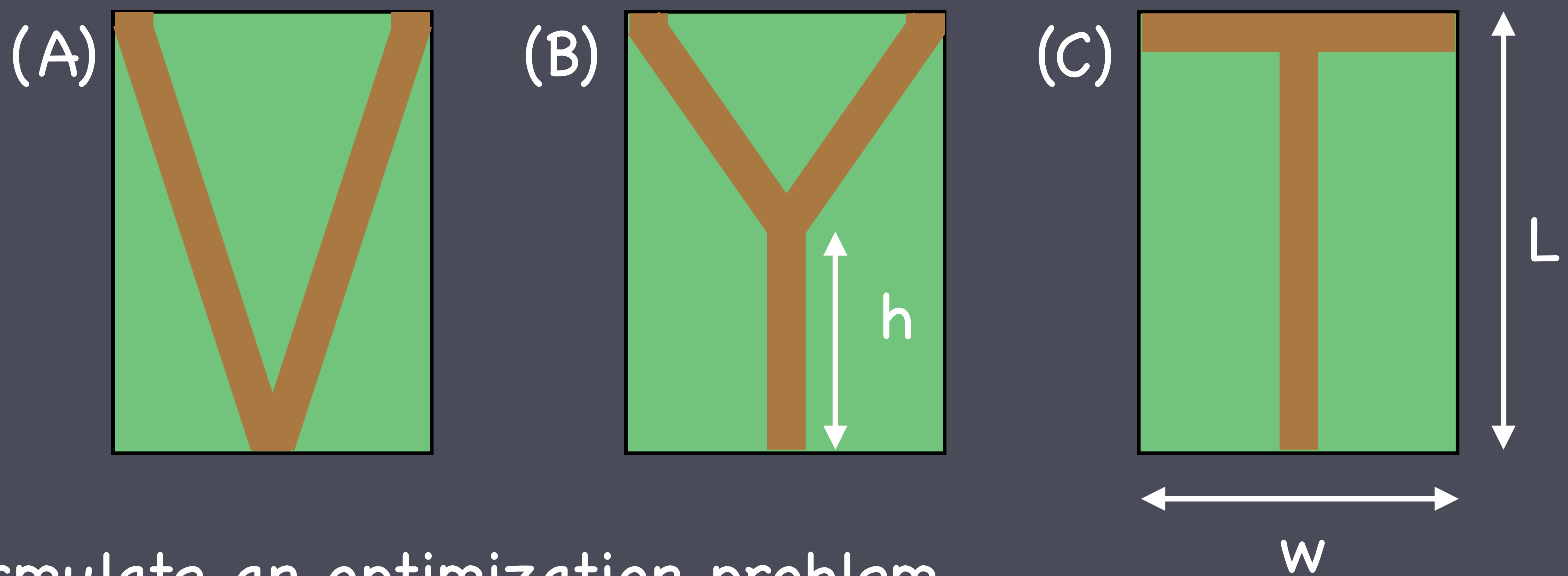

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Ans: h=L-w/sqrt(12)

Geometry of function composition

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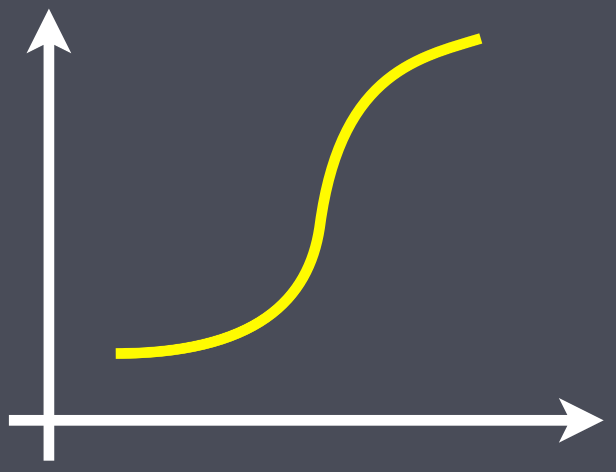

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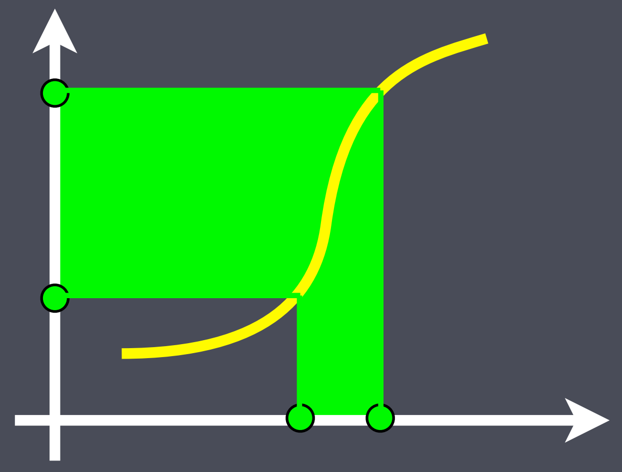

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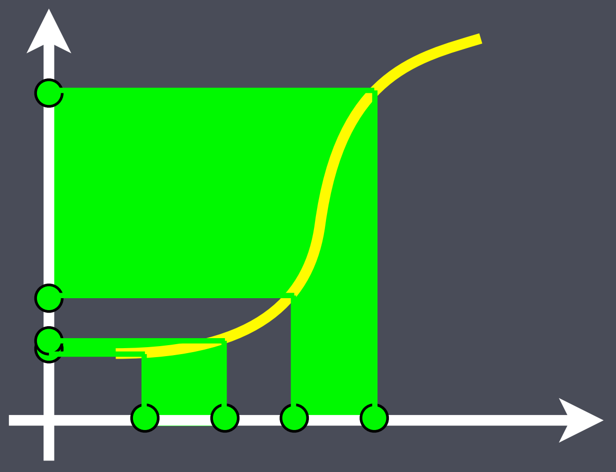

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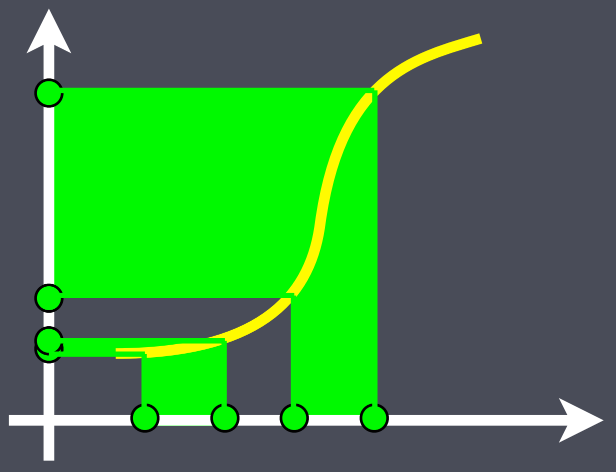

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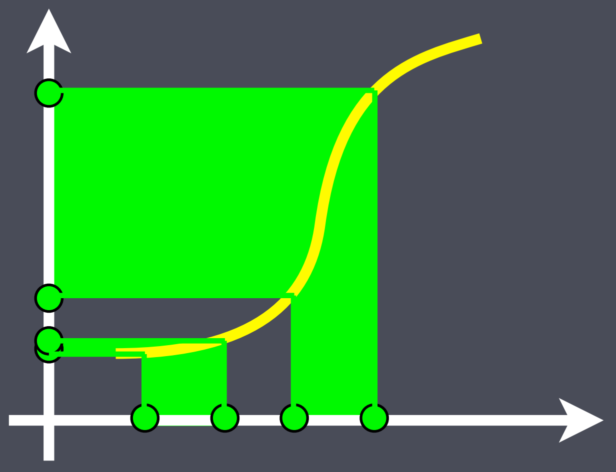
- So $f'(x)$ is the stretch factor near x.
- Where you are matters!



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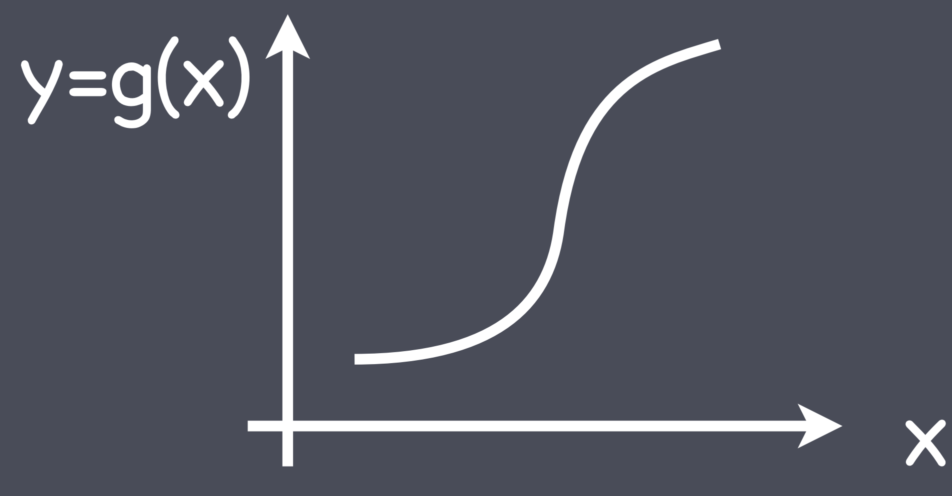

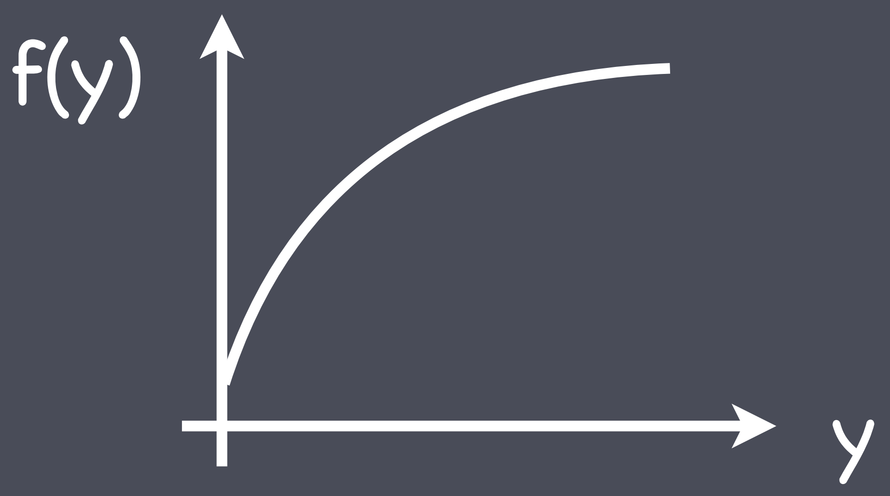

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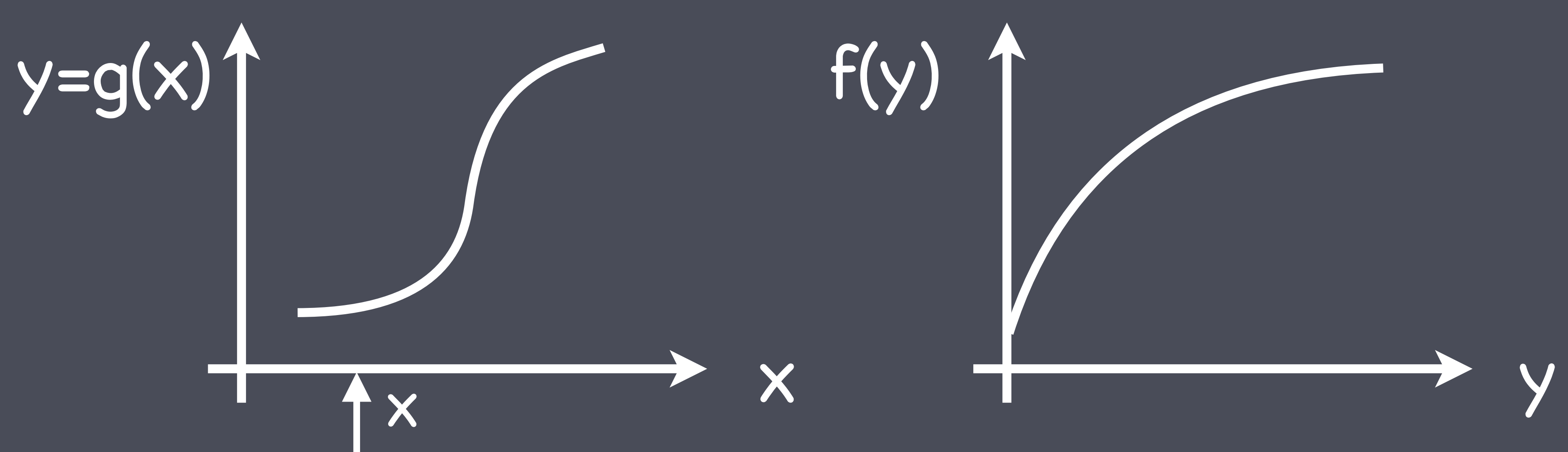

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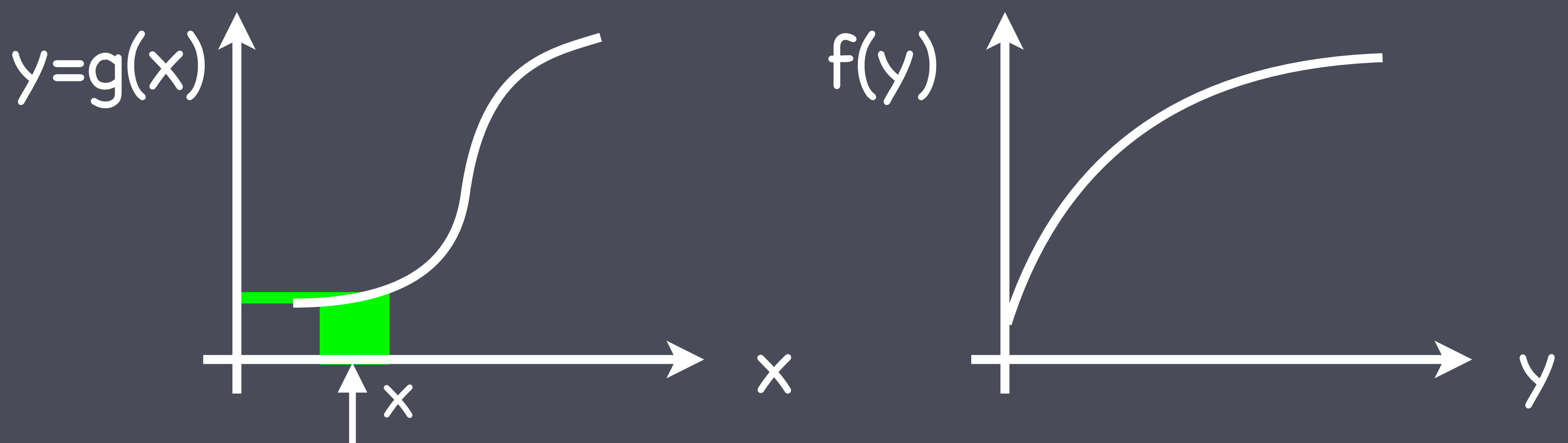

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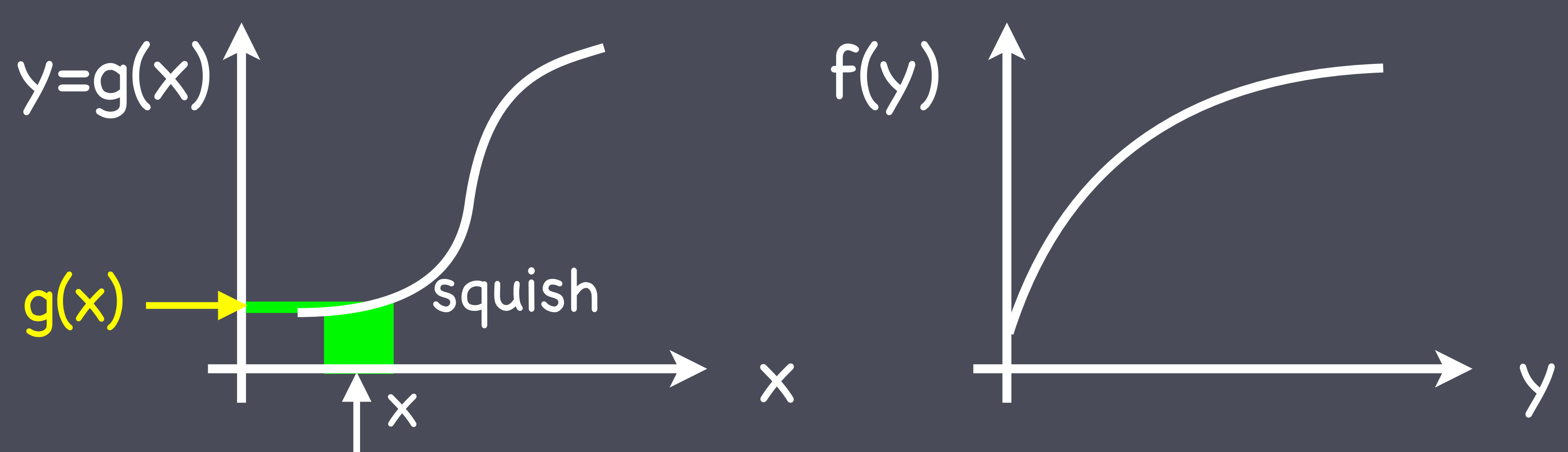

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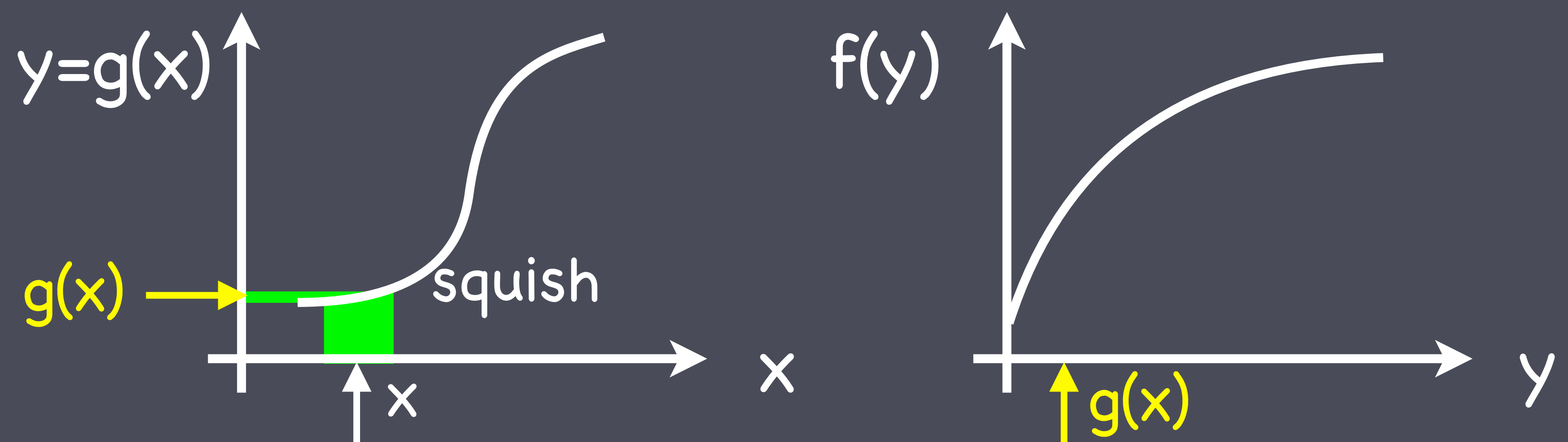

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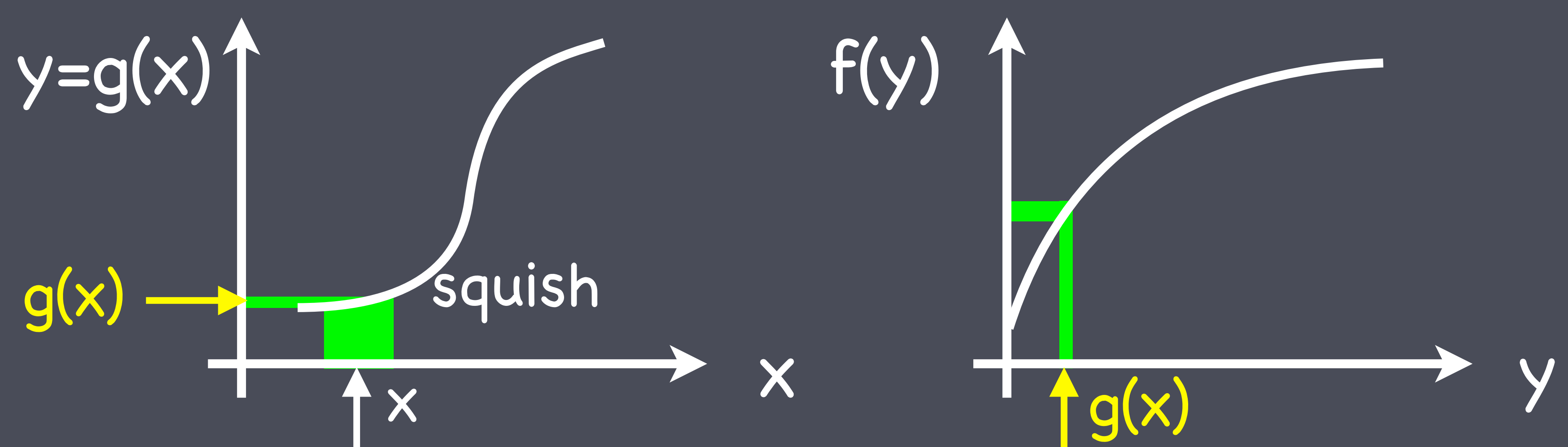

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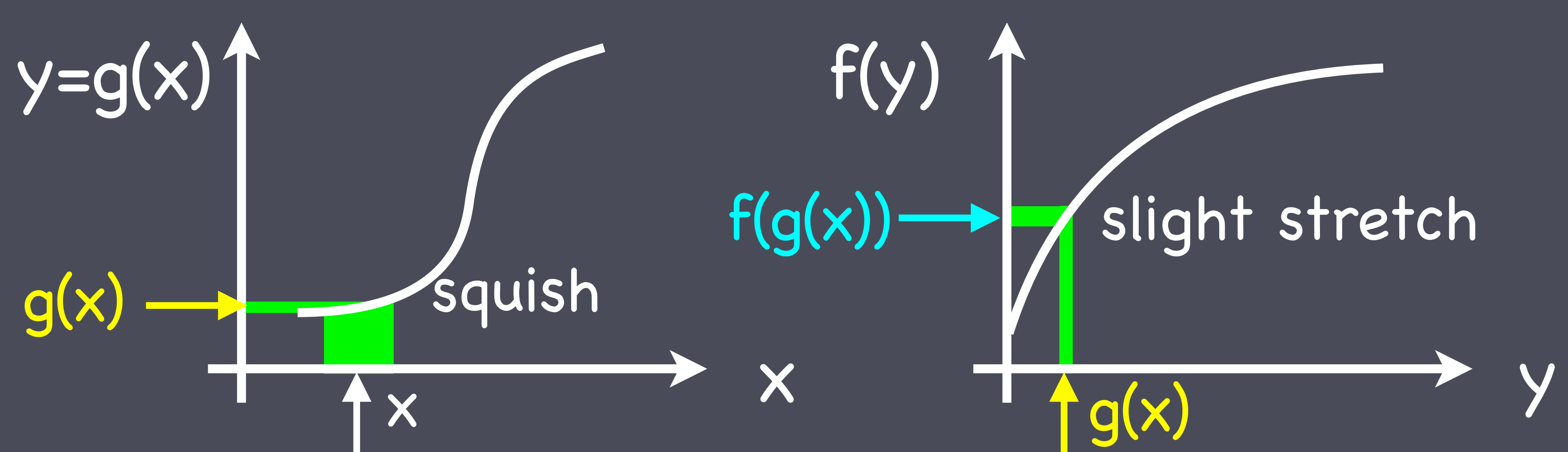

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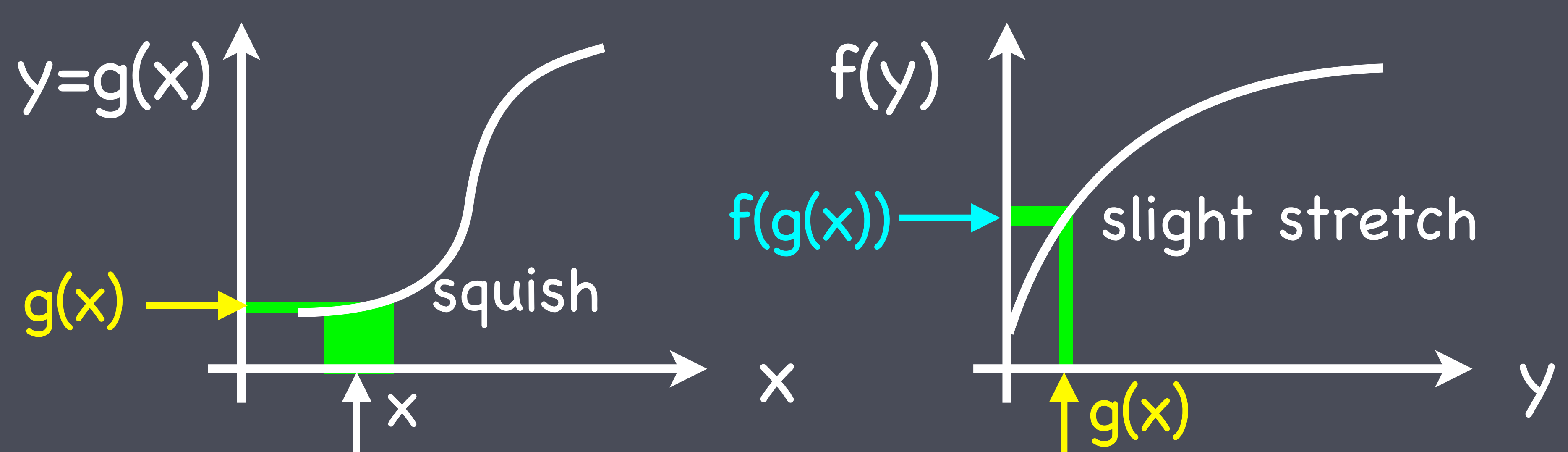

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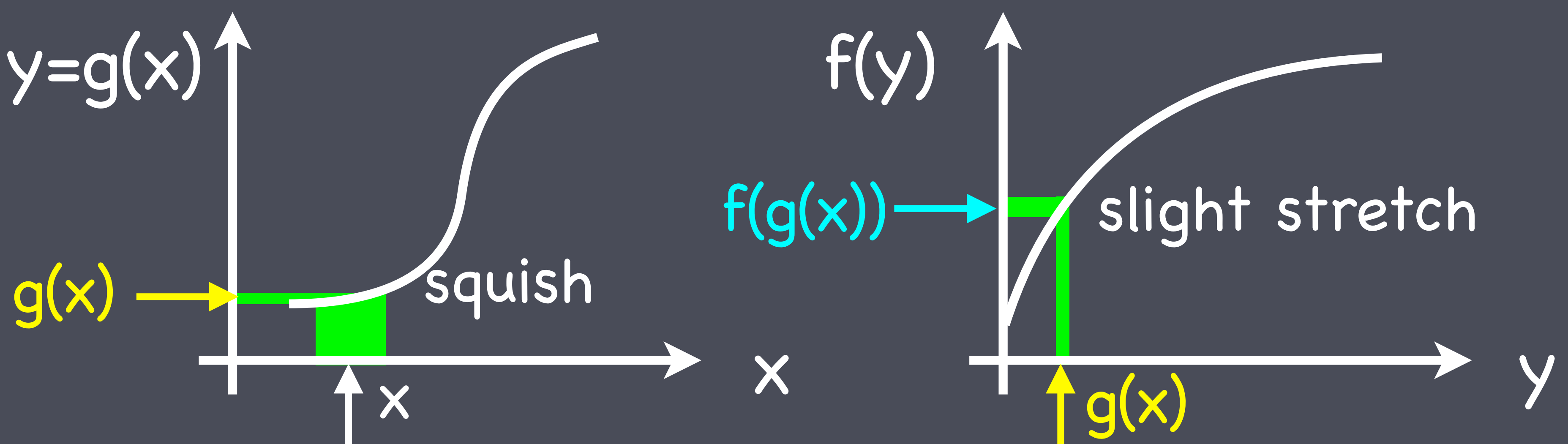
- Where you are on f matters - multiply the stretch factor of g near x: g'(x), by the stretch factor of f near g(x): f'(g(x)).



Related rates

- When two quantities (e.g. Q_1 and Q_2) are related to each other, if one changes in time so will the other.
- Knowing the relationship between Q_1 and Q_2 gives you the relationship between Q_1' and Q_2'.

The radius of a spherical tumor grows at a constant rate, k. Determine the rate of growth of the volume of the tumor when the radius is one centimeter.

Which is the relevant equation relating the quantities (not rates of change yet)?

(A) $V = 4/3\ \pi\ r^3$

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Now we can plug in r=1.

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  - get used spending a few minutes totally confused before something clicks.

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

Which of the following matches your intuition for how these rates are related?

(A) h(t) decreases quickly at first and then slows down.

(B) h(t) increases quickly at first and then slows down.

(C) h(t) decreases slowly at first and then speeds up.

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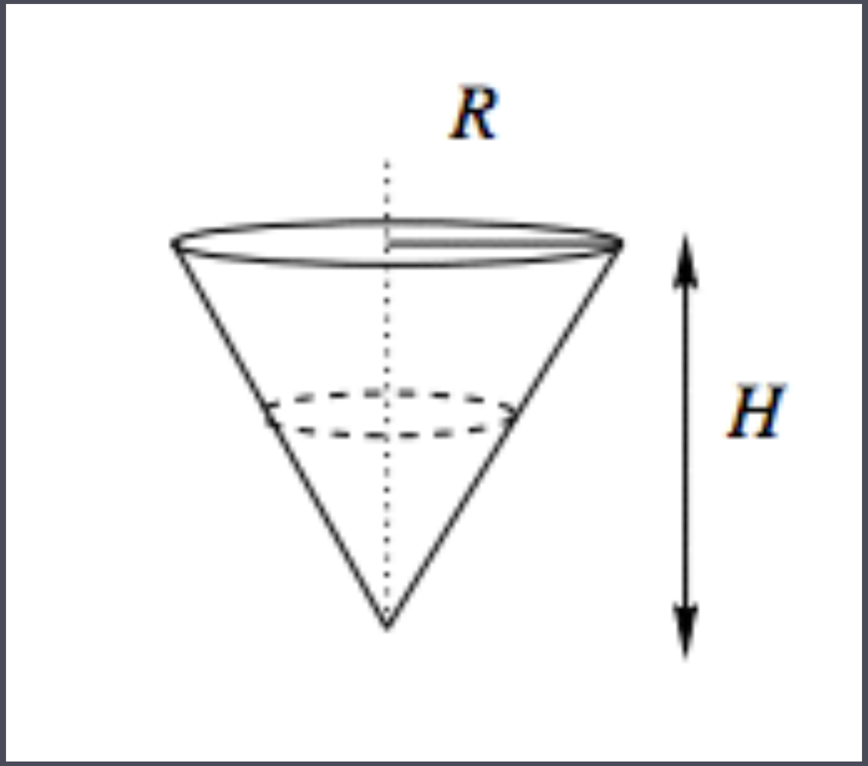

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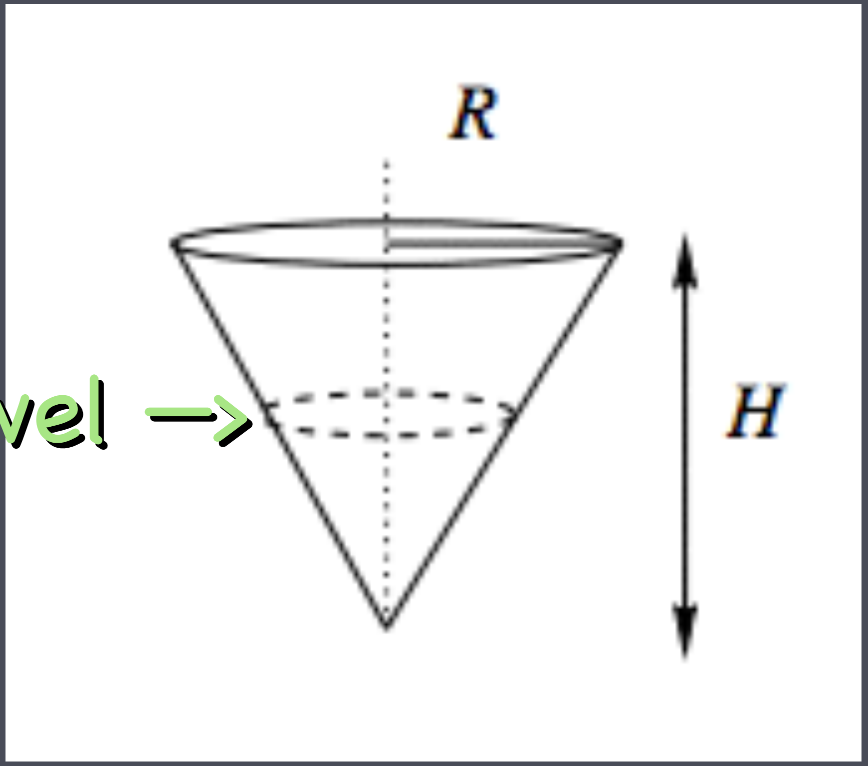

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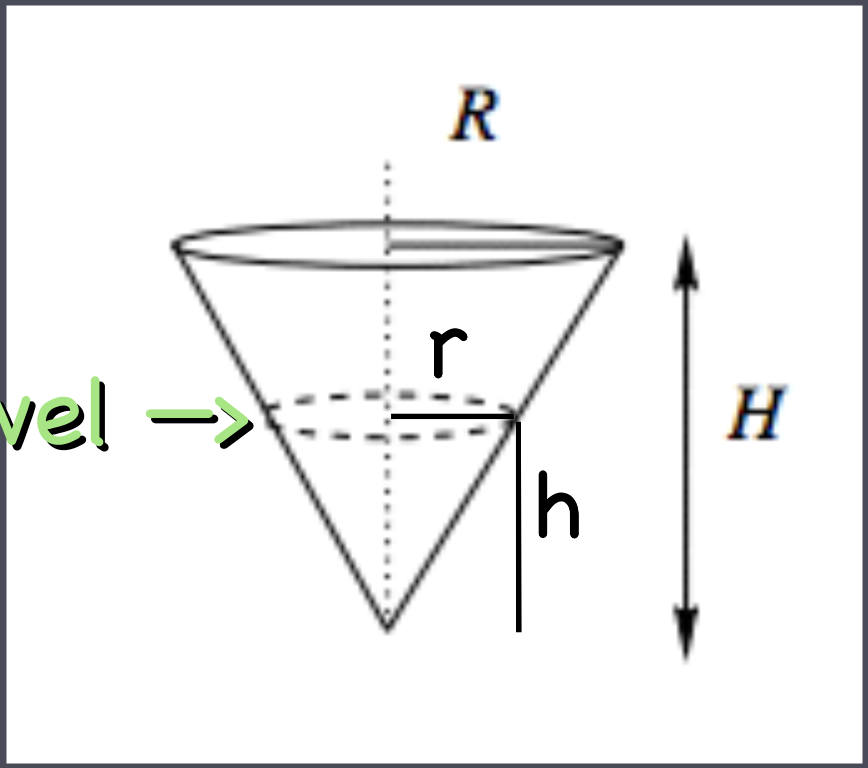

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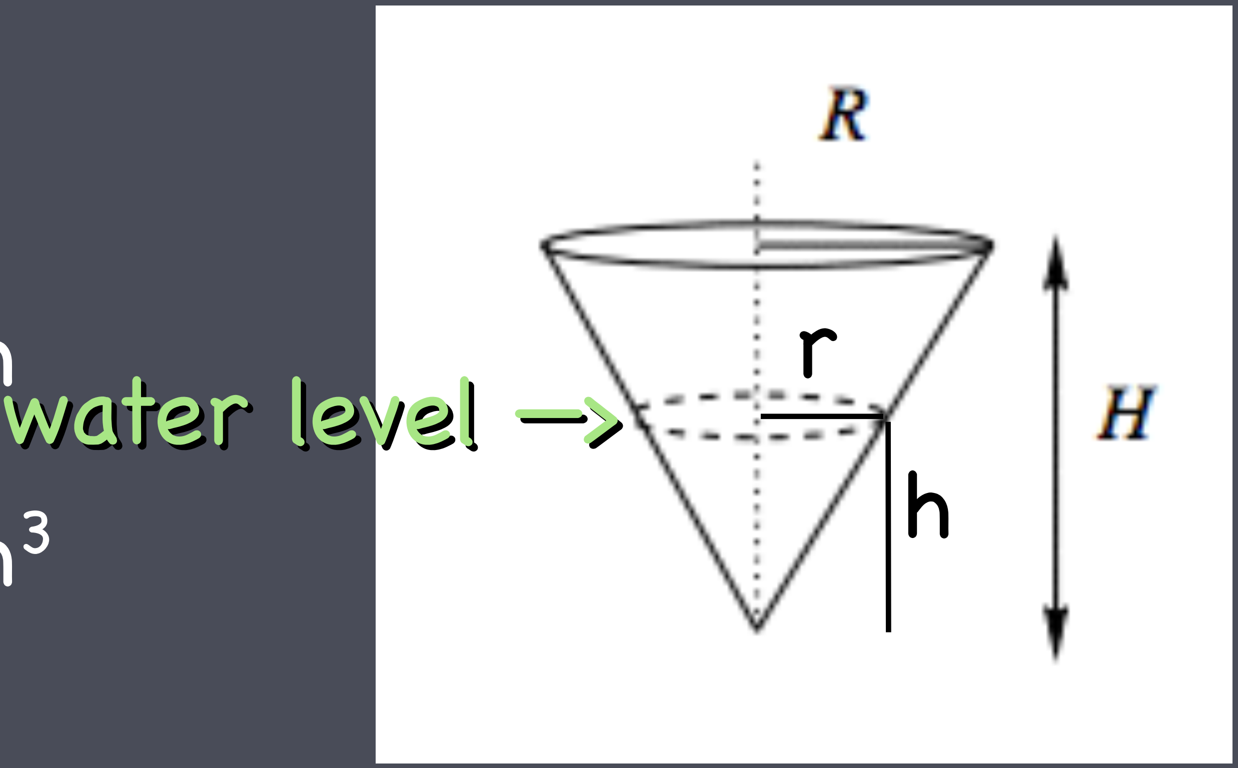

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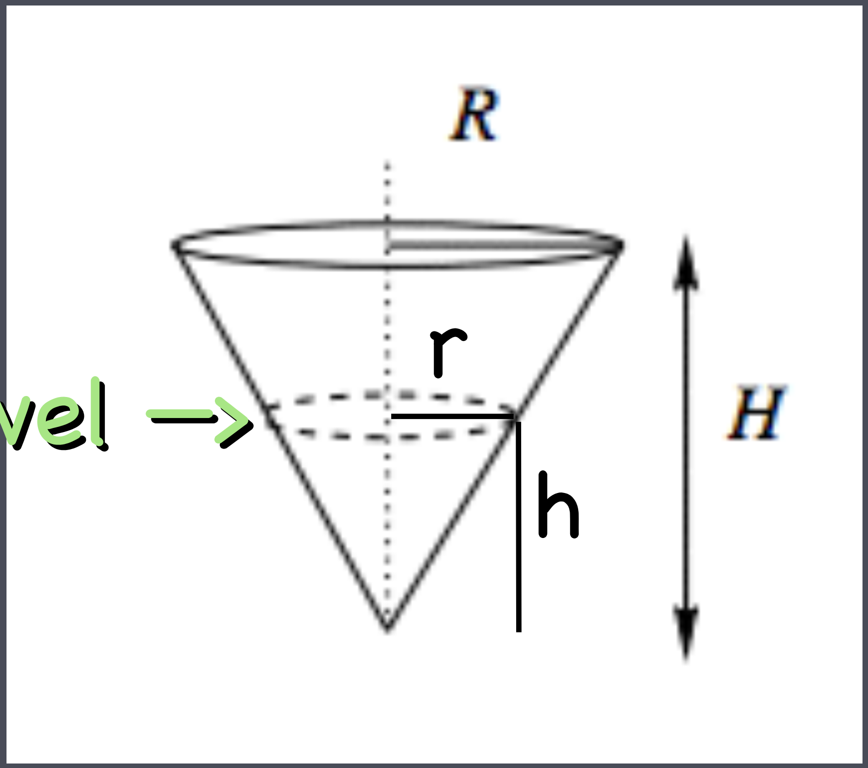

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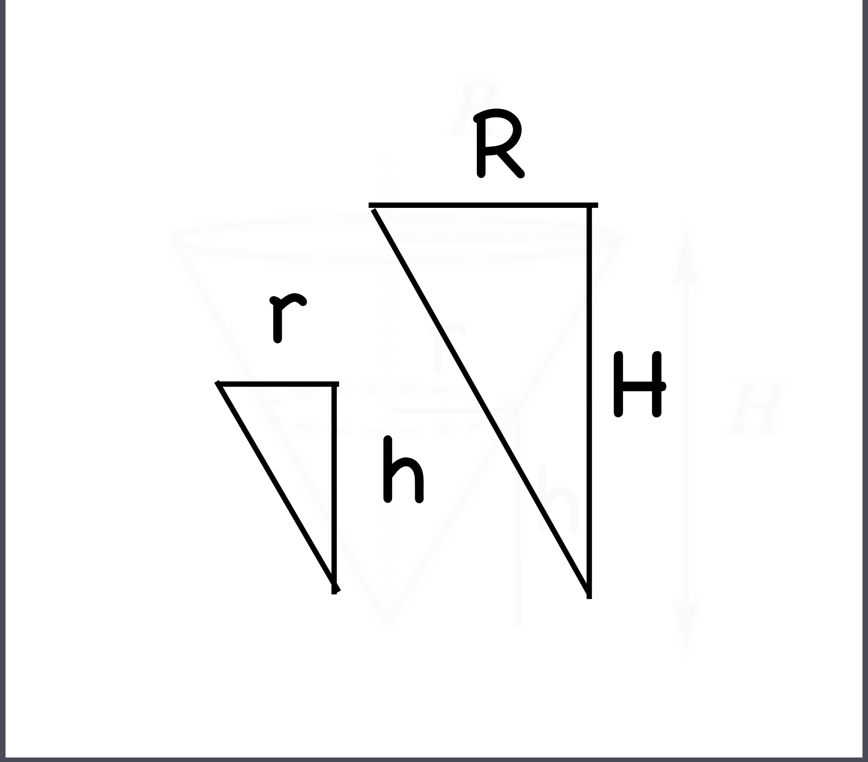

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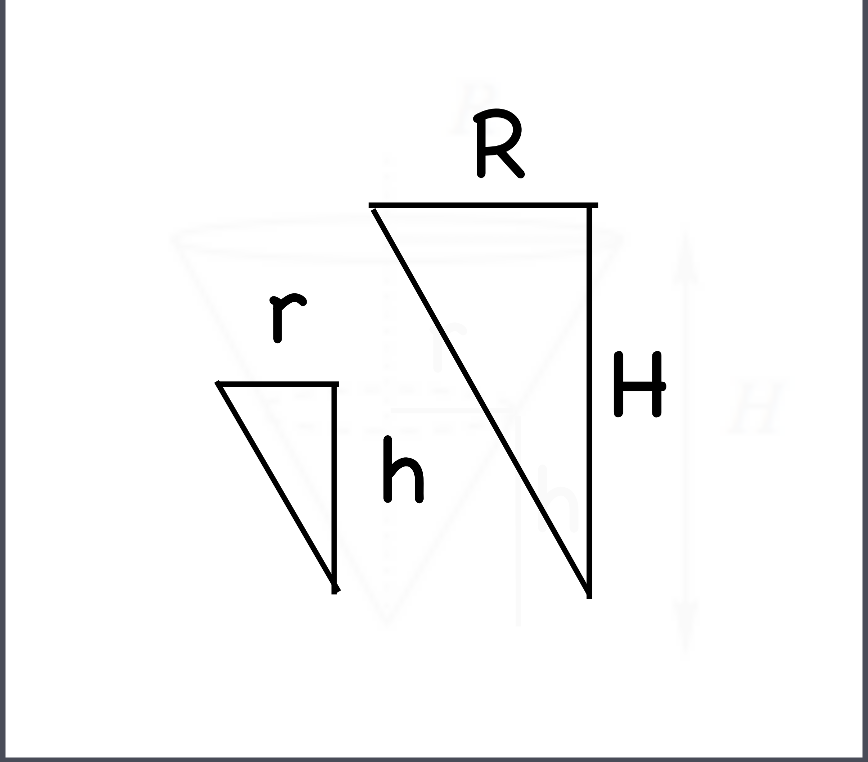

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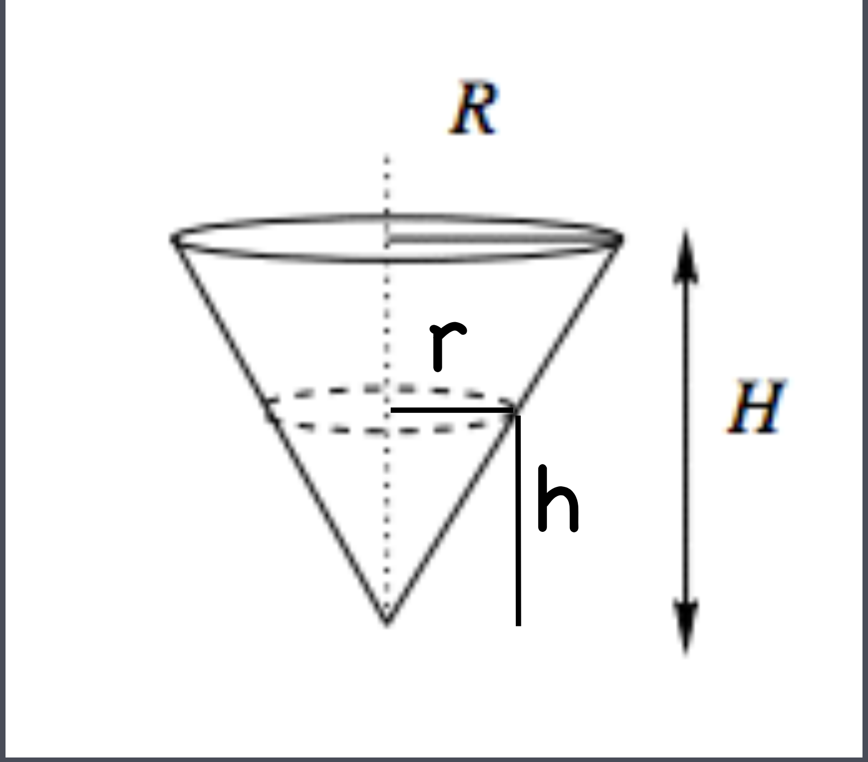

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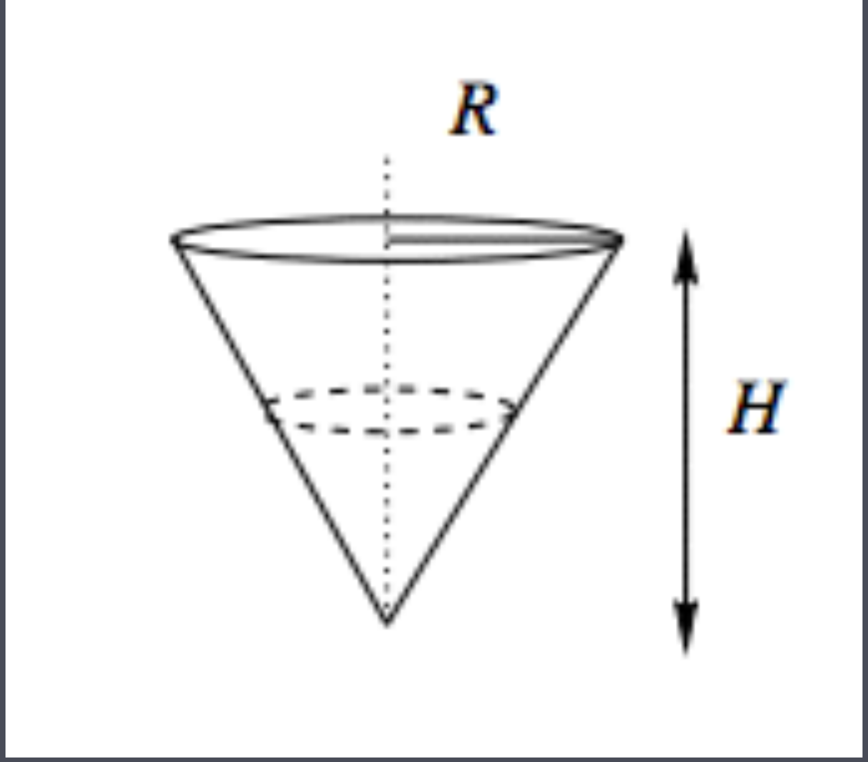

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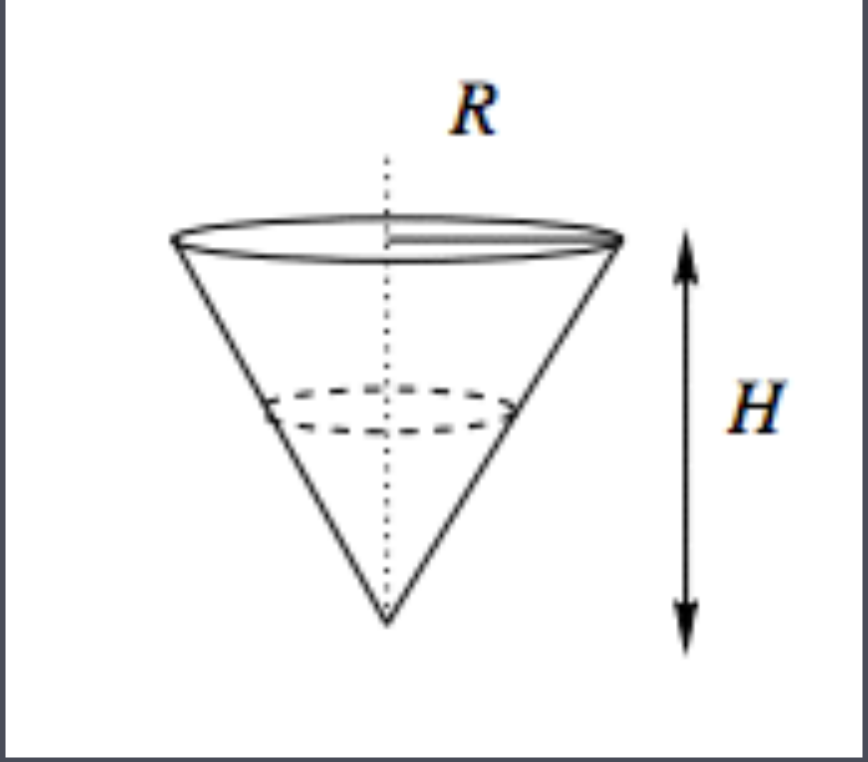

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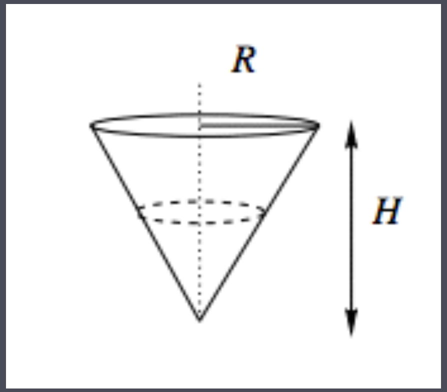

Let's check this answer against our intuition:

(A) h(t) decreases quickly at first and then slows down.

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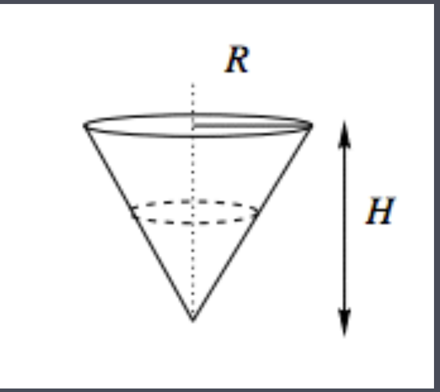

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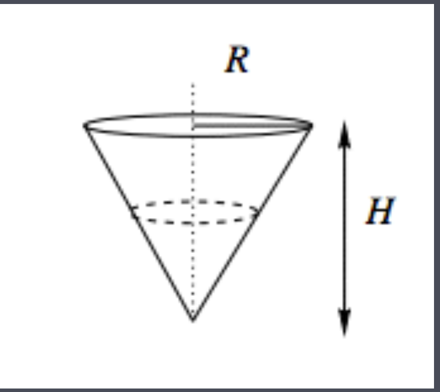

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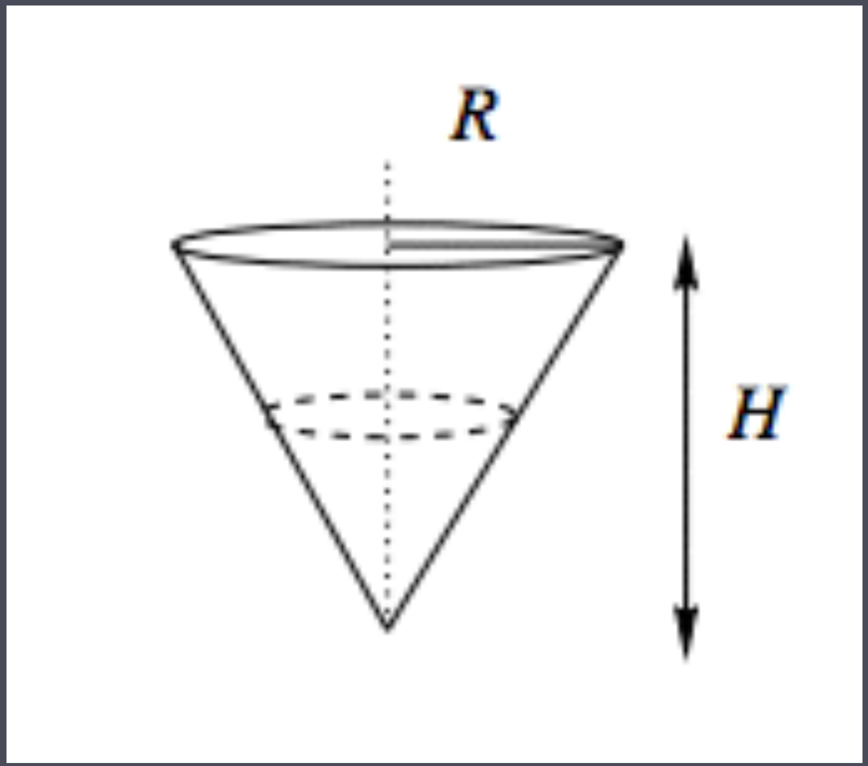

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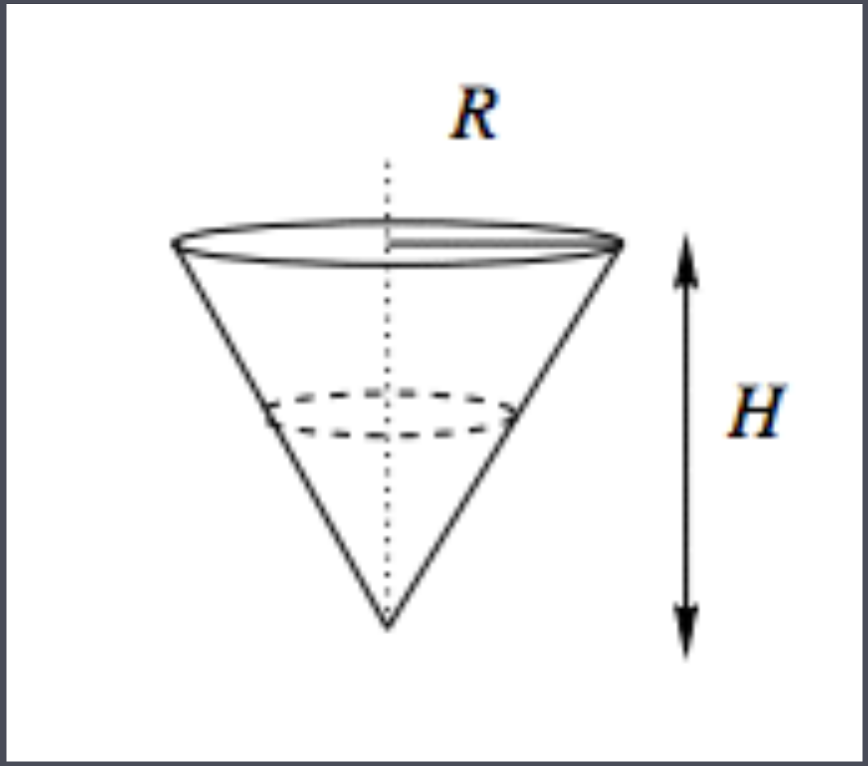

Plug $h=H$ into $h' = -k / (\pi (R^2/H^2) h^2)$

How many times do you think you should have to read a problem before finding the answer?

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Procedure

- Establish expectation(s) based on sketch or otherwise.
- Find equation relating Q_1 and Q_2.
- Take derivatives on both sides.
- Finally, plug in specific values.
- Reality check - compare answer against expectation.