Today

- Critical points
- First and second derivative tests
- Sketching

Critical points

- A critical point of f(x) is a point **a** at which **f'(a)=0** or **f'(a) is not defined** even though f(a) is defined.
- Use of critical points:
  - Critical points of f(x) might be minima or maxima of f(x). Not always though.
  - Critical points of f'(x) might be minima or maxima of f'(x) and hence inflection points of f(x). Not always though.

First derivative test

First Derivative Test

Suppose that $x = c$ is a critical point of $f(x)$ then,

1. If $f'(x) > 0$ to the left of $x = c$ and $f'(x) < 0$ to the right of $x = c$ then $x = c$ is a relative maximum.
2. If $f'(x) < 0$ to the left of $x = c$ and $f'(x) > 0$ to the right of $x = c$ then $x = c$ is a relative minimum.
3. If $f'(x)$ is the same sign on both sides of $x = c$ then $x = c$ is neither a relative maximum nor a relative minimum.

Second derivative test

Second Derivative Test

Suppose that $x = c$ is a critical point of $f'(c)$ such that $f'(c) = 0$ and that $f''(x)$ is continuous in a region around $x = c$. Then,

1. If $f''(c) < 0$ then $x = c$ is a relative maximum.
2. If $f''(c) > 0$ then $x = c$ is a relative minimum.
3. If $f''(c) = 0$ then $x = c$ can be a relative maximum, relative minimum or neither.

Second derivative test

Second Derivative Test

Suppose that $x = c$ is a critical point of $f'(c)$ such that $f'(c) = 0$ and that $f''(x)$ is continuous in a region around $x = c$. Then,

1. If $f''(c) < 0$ then $x = c$ is a relative maximum.
2. If $f''(c) > 0$ then $x = c$ is a relative minimum.
3. If $f''(c) = 0$ then $x = c$ can be a relative maximum, relative minimum or neither.

These tests both tell you (almost) the same thing!

$g(x) = 12x^3 - 12x^2$ has...

(A) a maximum at $x=0$ and a minimum at $x=1/3$.

(B) a minimum at $x=0$ and a maximum at $x=1/3$.

(C) a maximum at $x=0$ and an inflection pt at $x=1/3$.

(D) an inflection pt at $x=0$ and a minimum at $x=1/3$.

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(D) an inflection pt at $x=0$ and a minimum at $x=1/3$.

$f(x) = 3x^4 - 4x^3$ has...

(A) a maximum at $x=0$ and a minimum at $x=1$.

(B) a minimum at $x=0$ and a maximum at $x=1$.

(C) a maximum at $x=0$ and an inflection pt at $x=1$.

(D) an inflection pt at $x=0$ and a minimum at $x=1$.

$f(x) = 3x^4 - 4x^3$ has...

(A) a maximum at $x=0$ and a minimum at $x=1$.

(B) a minimum at $x=0$ and a maximum at $x=1$.

(C) a maximum at $x=0$ and an inflection pt at $x=1$.

(D) an inflection pt at $x=0$ and a minimum at $x=1$.