Name: __Solution__   Quiz Score:____/20

Student Number:_____

| Answer questions in the space provided. Show your work. |

1. For functions $p(x), q(x)$ and $r(x)$, function and derivative values at $x = \pi$ are given by:

$$p(\pi) = 1, q(\pi) = 2, r(\pi) = 3,$$
$$p'(\pi) = 3, q'(\pi) = 2, r'(\pi) = 1.$$

(a) (2 points) For $f(x) = p(x)q(x)$, determine $f'(\pi)$.

$$f'(x) = p'(x)q(x) + p(x)q'(x) \implies f'(\pi) = p'(\pi)q(\pi) + p(\pi)q'(\pi)$$
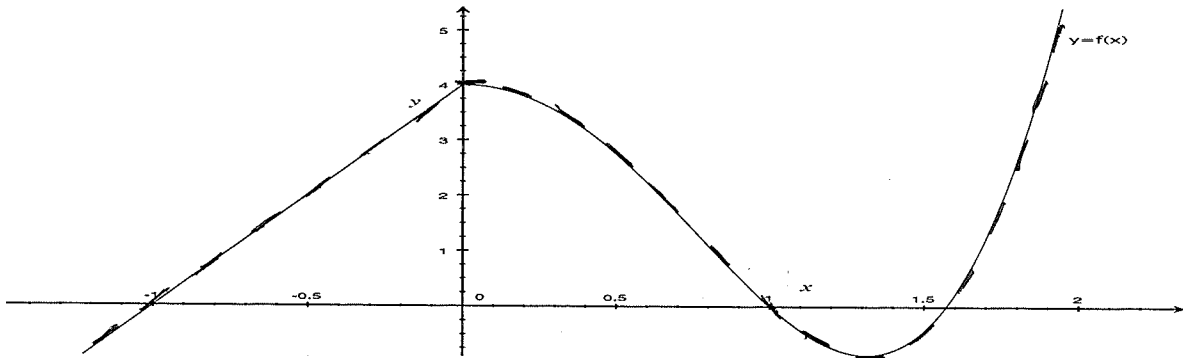
$$= (3)(2) + (1)(2) = 8$$

(b) (3 points) For $g(x) = \frac{p(x)q(x)}{r(x)}$, determine $g'(\pi)$.

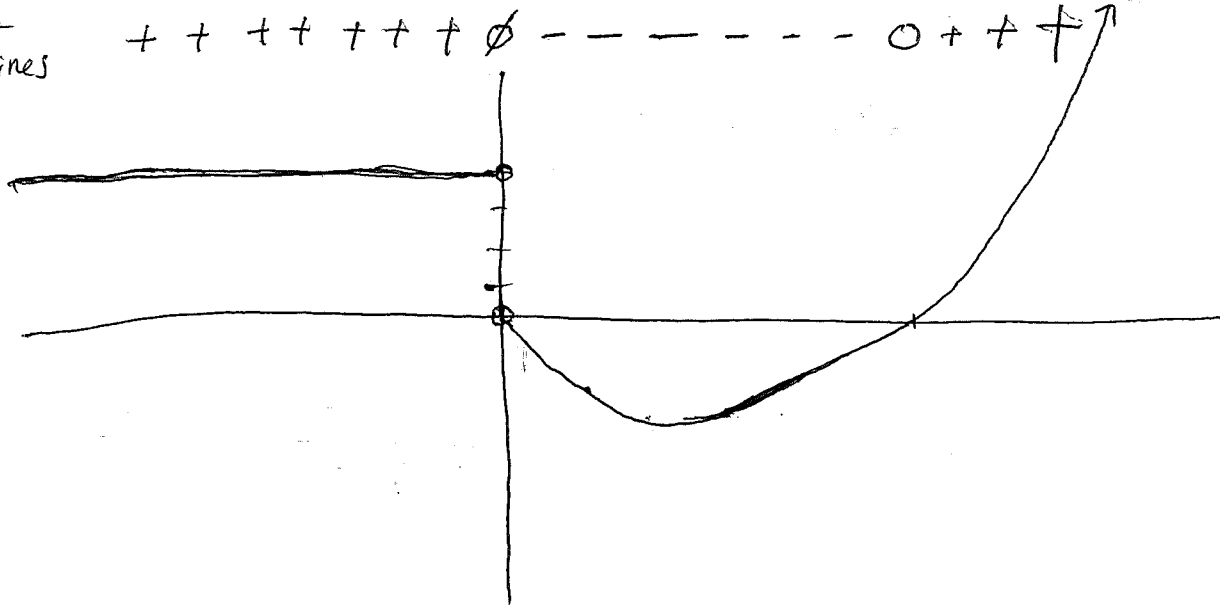$$g(x) = \frac{f(x)}{r(x)} \implies g'(x) = \frac{r(x)f'(x) - r'(x)f(x)}{(r(x))^2}$$

$$\implies g'(\pi) = \frac{r(\pi)f'(\pi) - r'(\pi)f(\pi)}{r(\pi)^2}$$

$$= \frac{(3)(8) - (1)[(1)(2)]}{3^2} = \frac{24-2}{9} = \frac{22}{9}$$

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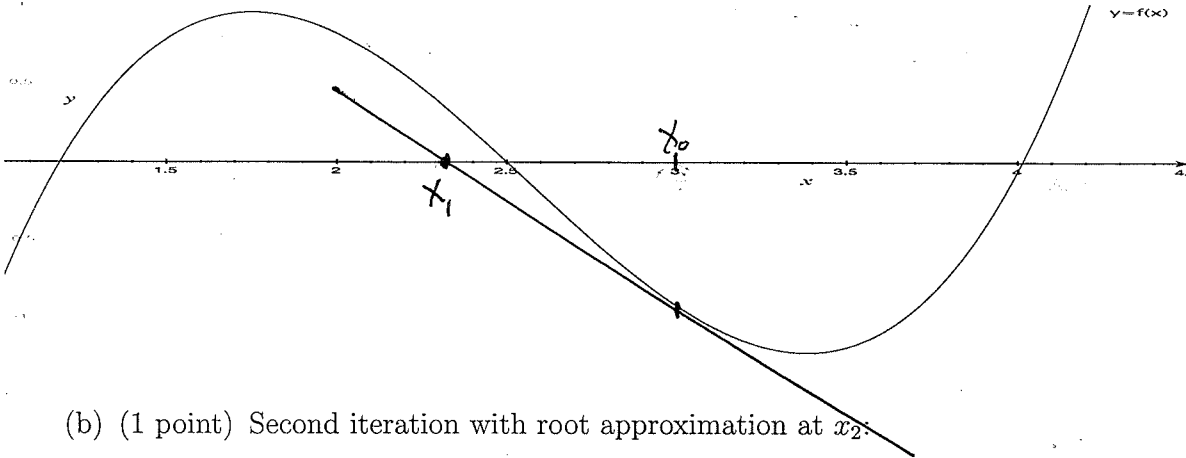2. (4 points) Below the graph of the function $f(x)$, sketch $f'(x)$.

Slopes of tangent lines

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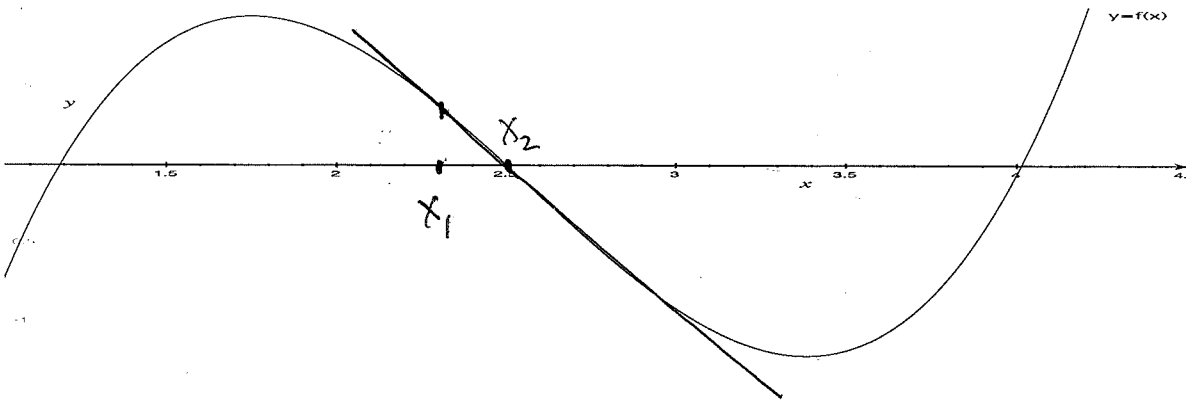
3. From (and on) the graph(s) of the function provided, sketch two iterations of Newton's method, starting with the initial estimate to a zero of the function at $x_0 = 3$.

   (a) (2 points) First iteration with root approximation at x_1:

   (b) (1 point) Second iteration with root approximation at x_2:

   (c) (2 points) Estimate values of x_1 and x_2 from your sketches.

$$x_1 \approx 2.3$$

$$x_2 \approx 2.5$$

4. $f(x)$ is a function such that $f(2) = 1$ and $f'(2) = 3$.

(a) (2 points) Determine the equation of the tangent line to $f(x)$ at $x = 2$.

$$\frac{y - f(2)}{x - 2} = f'(2) \quad \Rightarrow \quad \frac{y - 1}{x - 2} = 3$$

$$\Rightarrow \quad y - 1 = 3(x - 2) \quad \Leftrightarrow \quad y = 1 + 3(x - 2)$$

(b) (2 points) Using linear approximation, approximate $f(2.5)$.

$$f(2.5) \approx 1 + 3(2.5 - 2)$$
$$= 1 + 3(0.5)$$
$$= 1 + 3/2$$
$$= 5/2$$

(c) (2 points) If $x = 2$ is an estimate to a zero of $f(x)$, use one iteration of Newton's method to find a new estimate to a zero of f(x).

$$y = 1 + 3(x - 2) = 0$$
$$\Leftrightarrow \quad 3(x - 2) = -1$$
$$\Leftrightarrow \quad x - 2 = -1/3$$
$$\Leftrightarrow \quad x = 2 - 1/3$$
$$\Leftrightarrow \quad x = 5/3$$