Today...

- Assignment 1 - what does your score mean?
- Calculating the derivative from the definition.
- Limits and continuity examples.
- Reminders- OSH 2 Monday! WW2 Thurs 7AM!

Assignment 1 results and interpretation

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- If you didn't pass, think seriously about switching - talk to me if you need advice.
- If you didn't get 100% - fill in your gaps asap!

A WeBWorK limit example

Guess the value of the limit (if it exists) by evaluating the function at values close to where the limit is to be done. If it does not exist, enter DNE below.

$$\lim_{h \to 0} \frac{\sin(\frac{\pi}{4} + h) - \sin(\frac{\pi}{4})}{h}$$

Limit:

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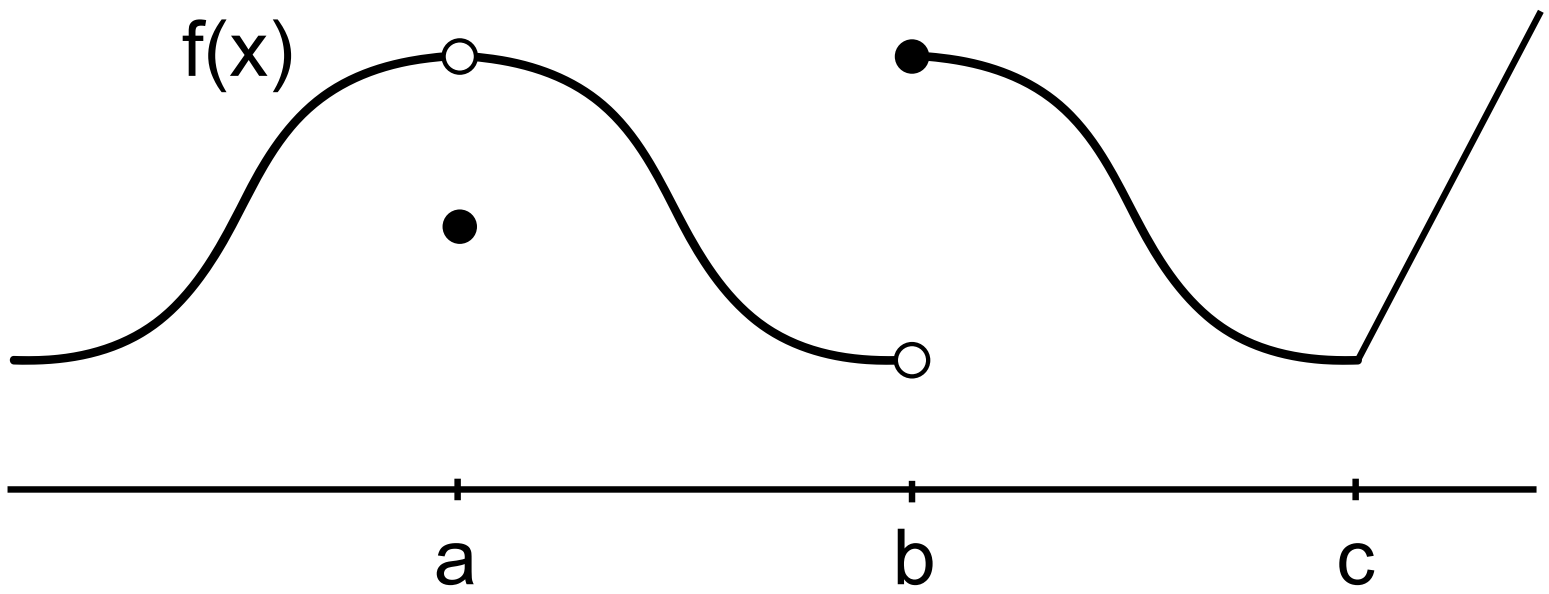
Limit:

Go over f'(2) where f(x) = 1/x on the board.

Limits



Which of the following are true?

1. $\lim_{x \to a} f(x) = f(a)$
2. $\lim_{x \to b} f(x) = f(b)$
3. $\lim_{x \to c} f(x)$ does not exist.
4. $\lim_{x \to a} f(x)$ exists.
5. $\lim_{x \to b} f(x)$ exists.

(A) 1, 4

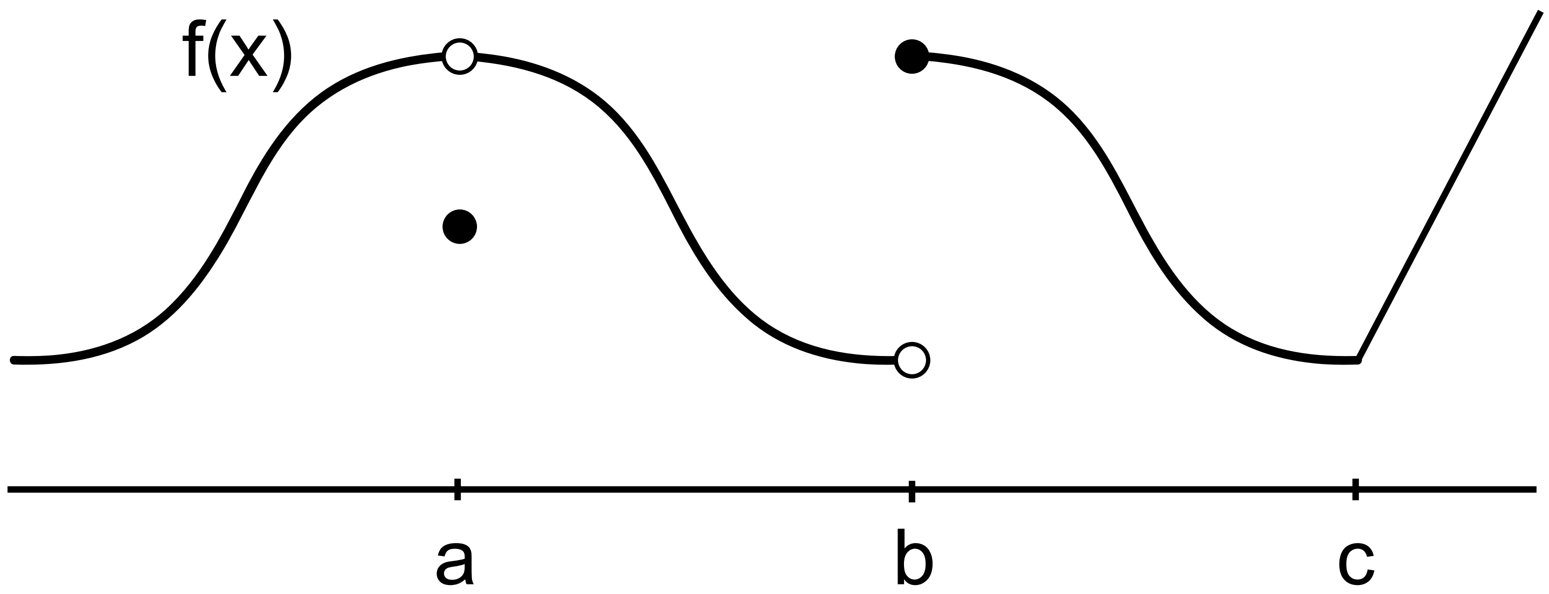
(B) 2, 5

(C) 3

(D) 4

(E) 5

Limits



(A) 1, 4

Which of the following are true?

(B) 2, 5

1. $\lim_{x \to a} f(x) = f(a)$

4. $\lim_{x \to a} f(x)$ exists.

(C) 3

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5. $\lim_{x \to b} f(x)$ exists.

(D) 4

3. $\lim_{x \to c} f(x)$ does not exist.

(E) 5

Left and right limits

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- The right limit at **a** - plug in x values approaching **a** from above (x>**a**):

$$\lim_{x \to a^+} f(x)$$

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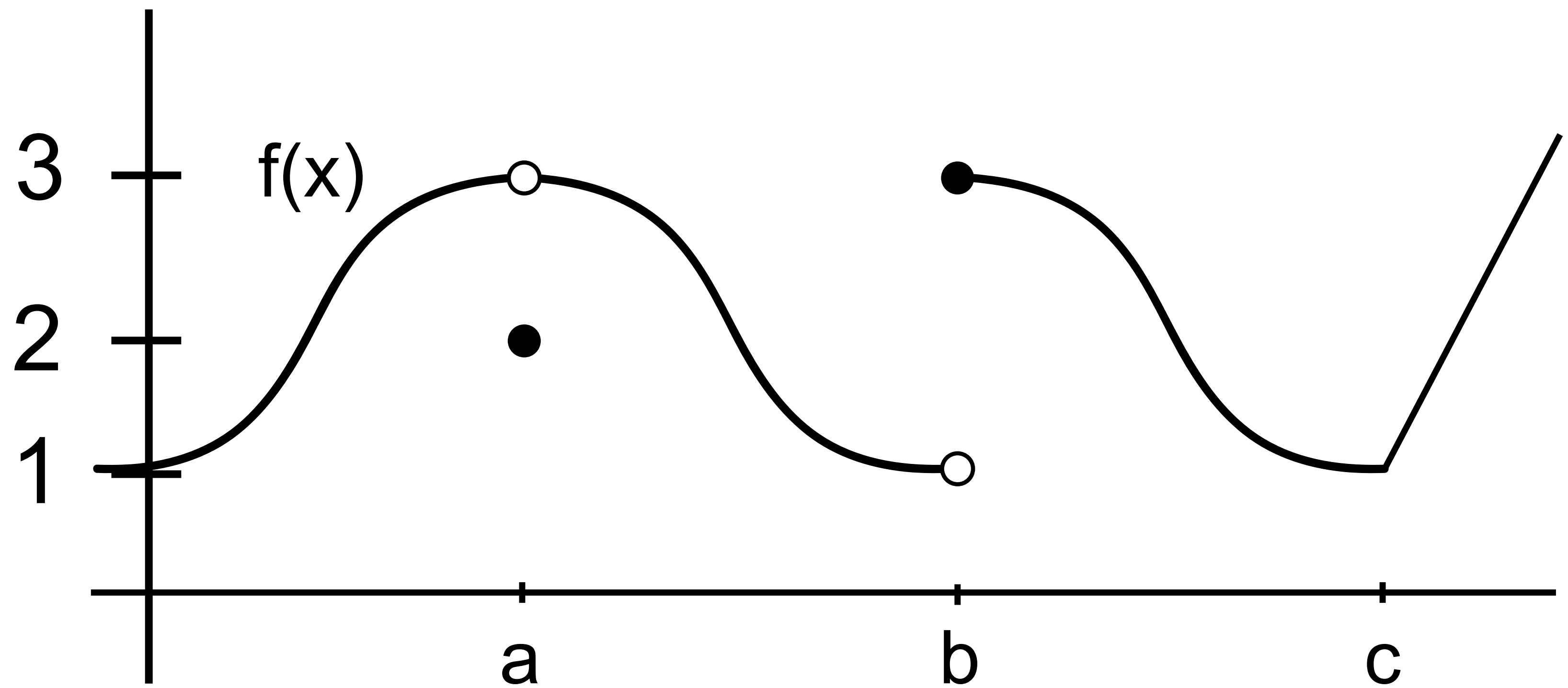
- The left limit at **a** - plug in x values approaching **a** from below (x<**a**):

$$\lim_{x \to a^-} f(x)$$

- When these exist and are equal, $\lim_{x \to a} f(x)$ exists

$$\lim_{x \to a} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x).$$

Limits



(A) $\lim_{x \to a} f(x) = 2$

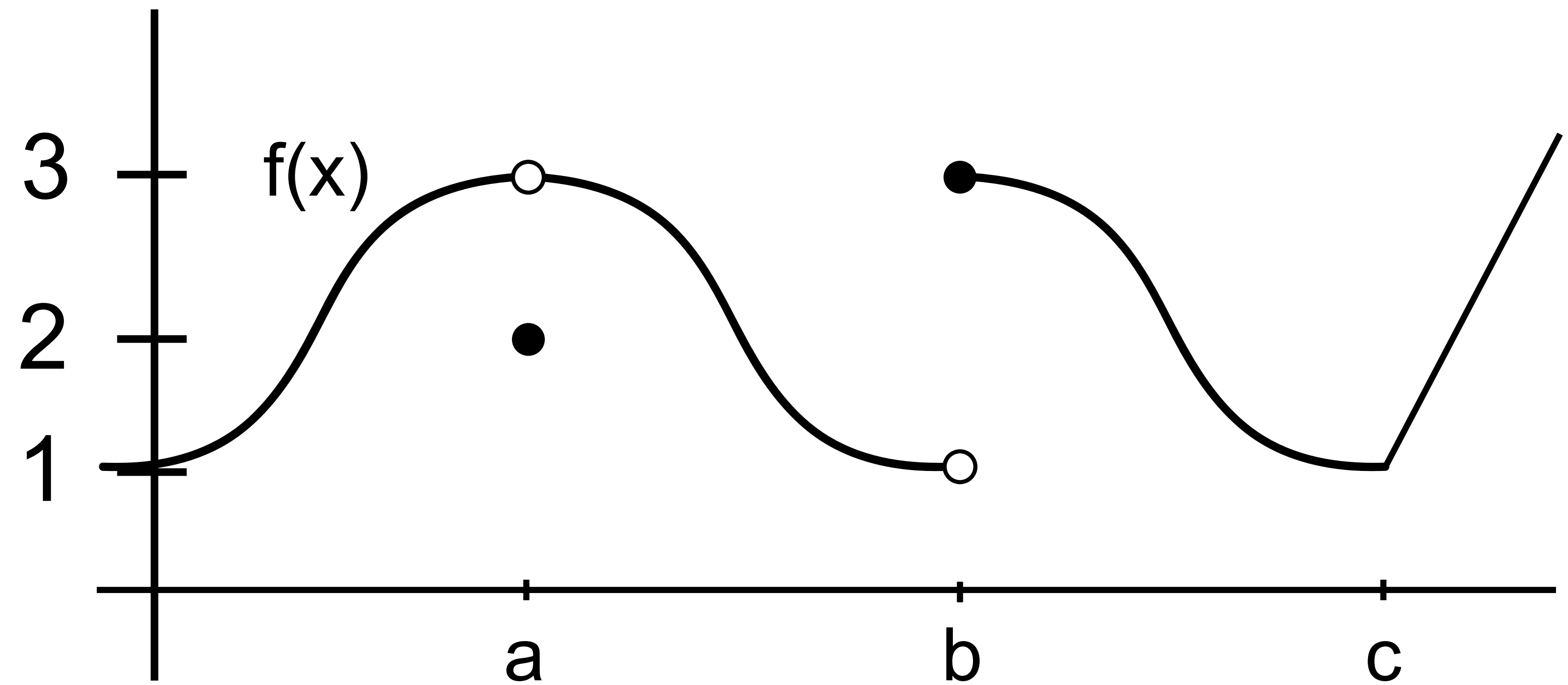
(B) $\lim_{x \to b^-} f(x) = 3$

(C) $\lim_{x \to a} f(x) = 3$

(D) $\lim_{x \to b} f(x) = 3$

(E) $\lim_{x \to b^+} f(x)$ does not exist

Limits



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Continuity

When $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$

we say that f(x) is continuous at x=a.

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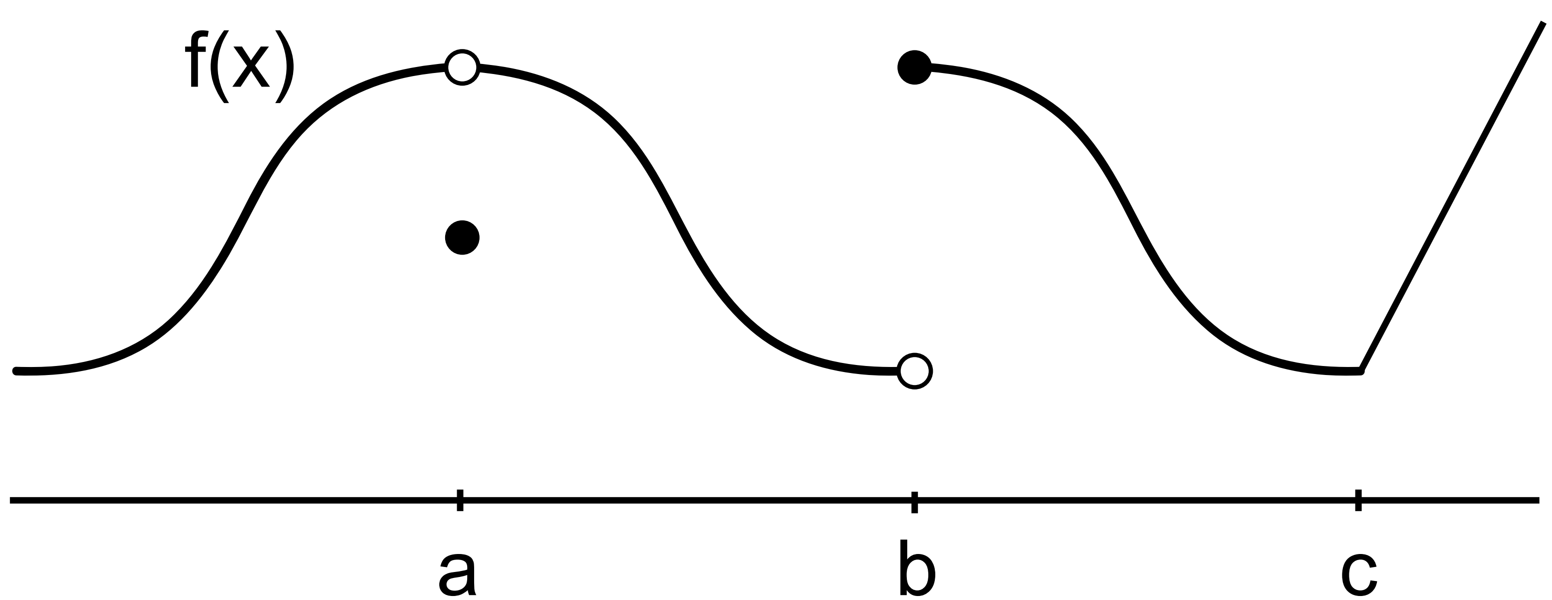

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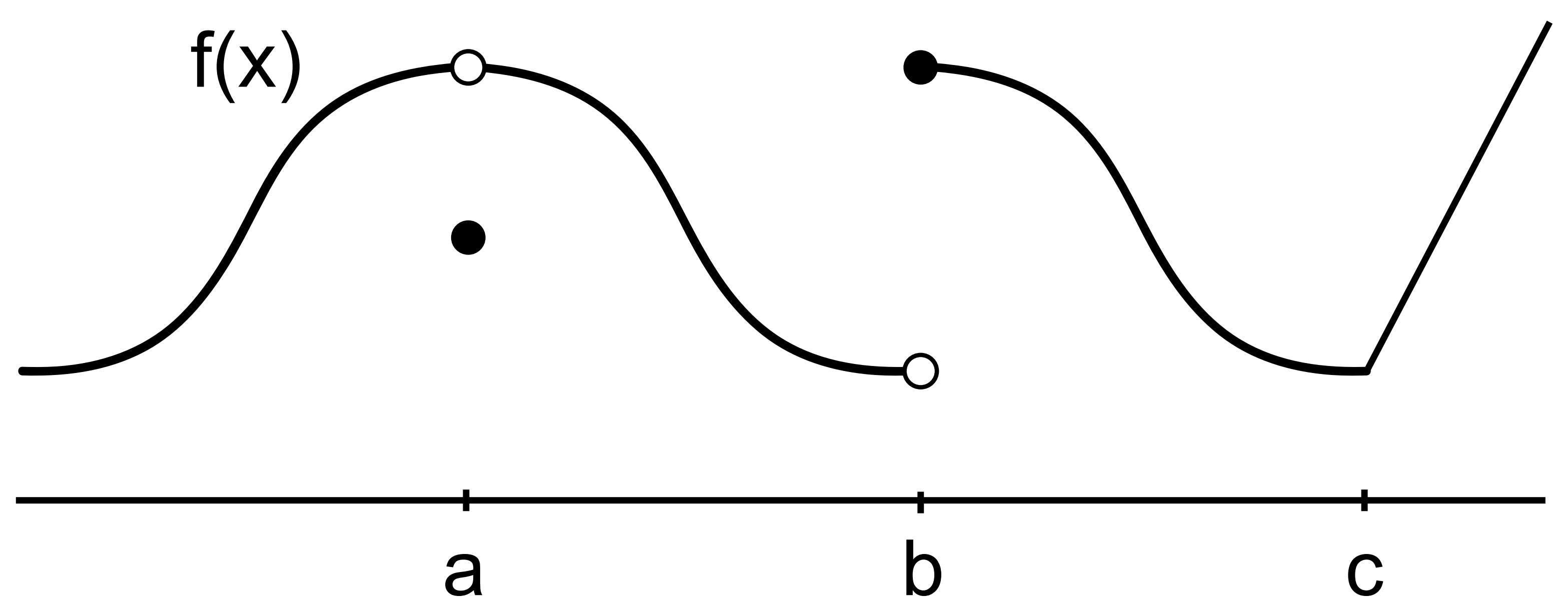

f(x) is continuous at all x except at x=a and x=b.

Continuous functions

- Examples of categories of continuous functions:
  - Polynomials
  - Exponentials
  - sin, cos
- These are all continuous for all real x.
- Desmos example...

Ensuring continuity

For what value of a is the following function continuous?

$$f(x) = \begin{cases} 4 - a^2 + 3x & x < 1 \\ x^2 + ax & x \geq 1 \end{cases}$$

(A) a = 3

(B) a = -3

(C) a = 0

(D) a = 1

(E) Don't know.

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Types of limits we'll talk about

- Points of continuity: $\lim_{x \to a} f(x) = f(a)$
- Hole-in-the-graph (like derivative limit)
- Limits at $\pm\infty$ (horizontal asymptotes)

What is $\lim_{x \to 1} \frac{x^2 - 4}{x - 2}$ **?**

(A) 3

(B) -3

(C) Not defined.

(D) ∞

(E) Don't know.

What is $\lim_{x \to 1} \frac{x^2 - 4}{x - 2}$?

(A) 3

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Limit at a point of continuity.

What is $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ **?**

(A) 0

(B) ∞

(C) 1

(D) 4

(E) Don’t know.

What is $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ **?**

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$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2}$$

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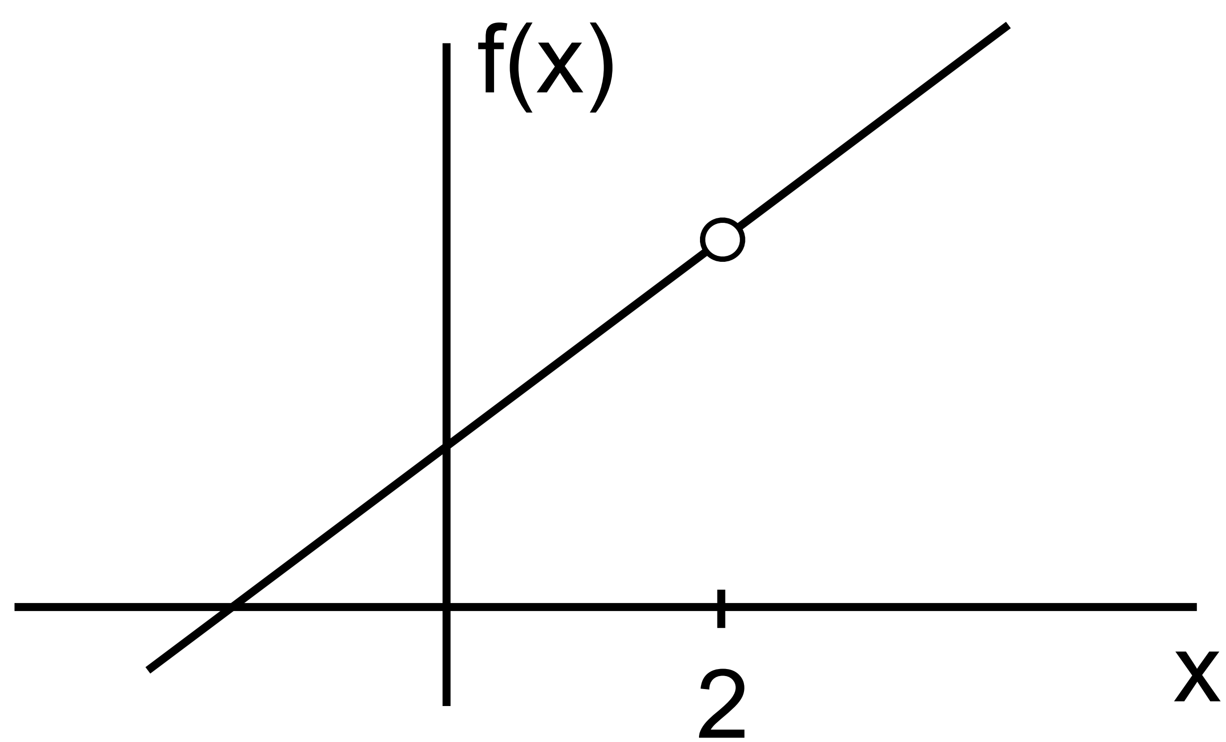

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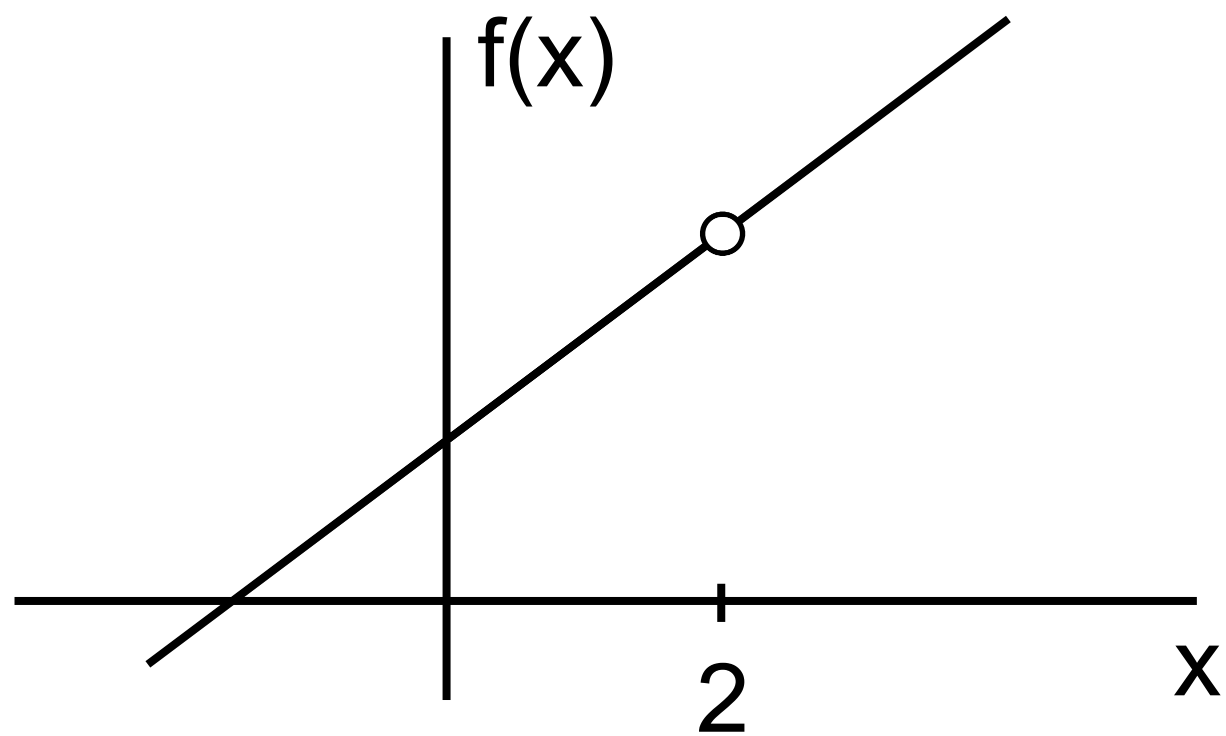

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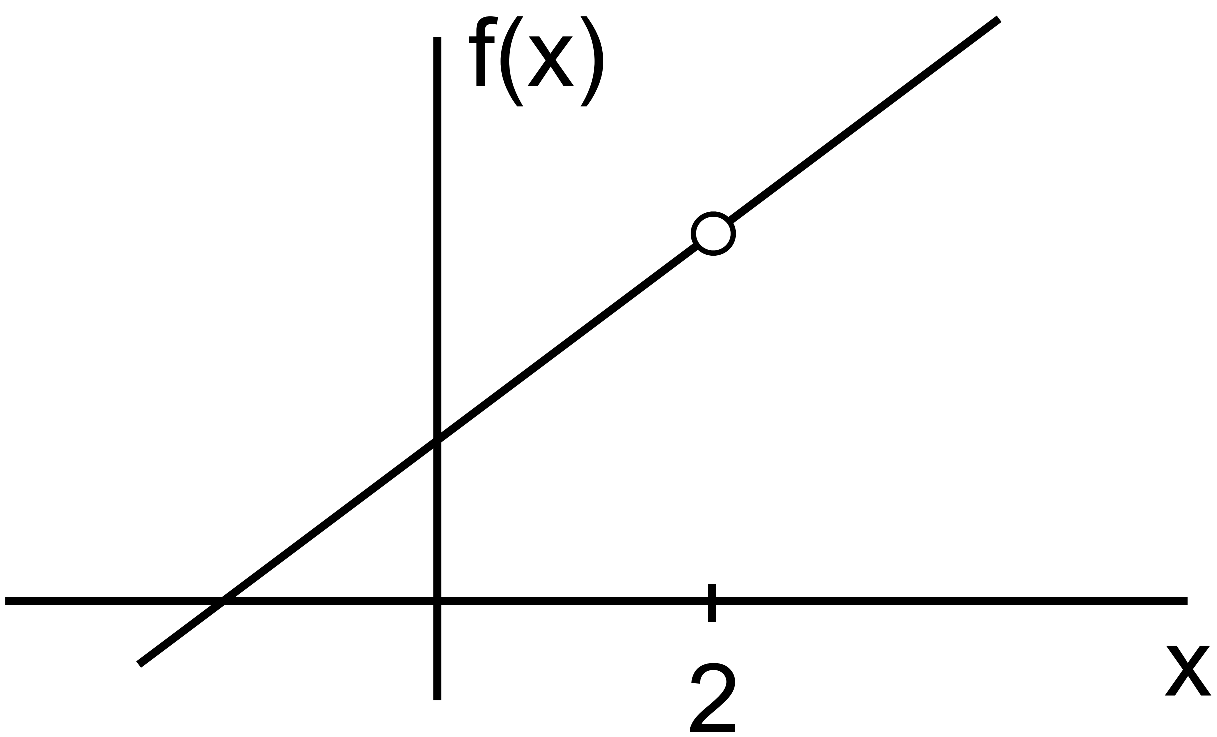

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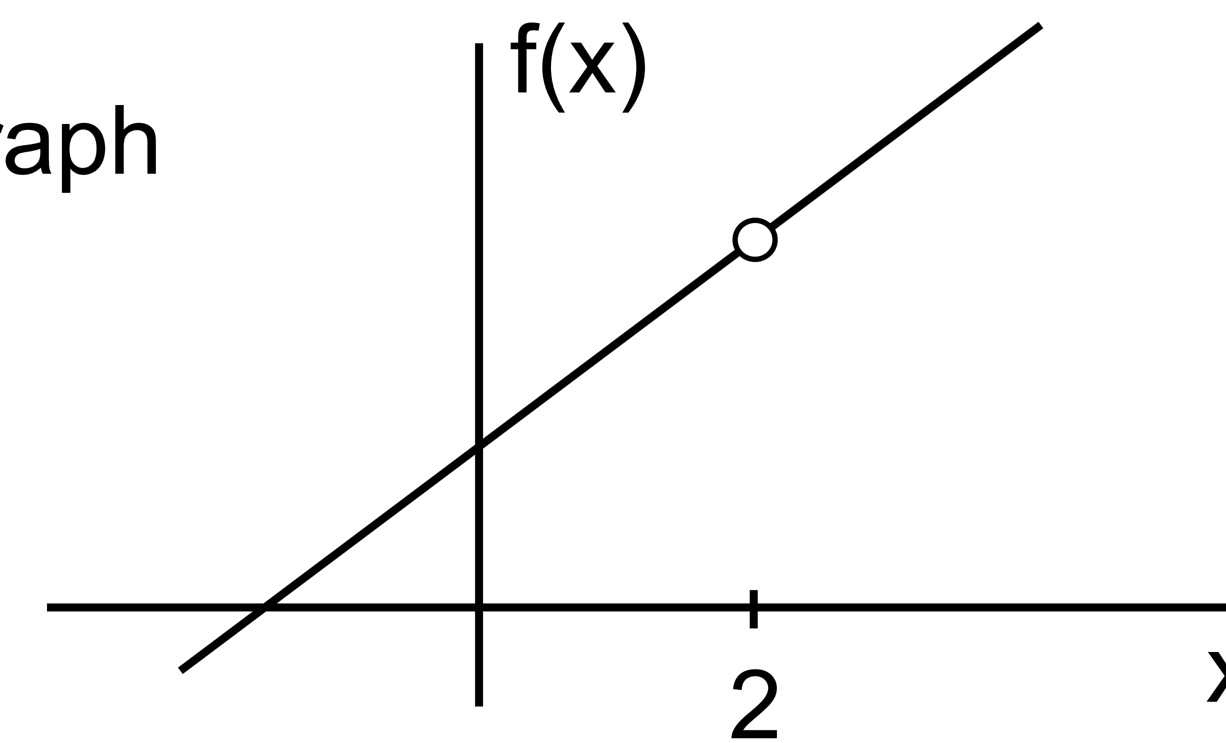
Hole-in-the-graph type limit



Is $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ actually the derivative of some function?

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(for you to think about)

What is $\lim_{x \to \infty} \frac{5x^3 + 3x^2 - x + 1}{2x^4 - x^2 + 2}$ **?**

(A) 0

(B) ∞

(C) 5/2

(D) 1/2

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Use asymptotics to simplify top and bottom.

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$$\frac{5x^3+3x^2-x+1}{2x^4-x^2+2} \approx \frac{5x^3}{2x^4} = \frac{5}{2x}$$

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