Today

- Related rates with trig
- Zebra Danio

Reminders:
- Friday is the last day of classes.
- Exam: Dec 12 @ 8:30 am – SRC B
Trig-related rates

These usually come down to a triangle that changes in time. For example...
If the height of an isosceles triangle with base 2m changes at a rate \( h' = 3 \) m/s, how quickly is the angle opposite the base changing when \( h = \sqrt{3} \) m?

Relate the two changing quantities (\( h \) and \( \theta \)):

(A) \( \sin(\theta) = \frac{2}{h} \)

(B) \( \sin(\frac{\theta}{2}) = \frac{1}{h} \)

(C) \( \sin(\frac{\theta}{2}) = \frac{1}{\sqrt{1+h^2}} \)

(D) \( \tan(\theta) = \frac{2}{h} \)

(E) \( \tan(\frac{\theta}{2}) = \frac{1}{h} \)
If the height of an isosceles triangle with base 2m changes at a rate \( h' = 3 \, \text{m/s} \), how quickly is the angle opposite the base changing when \( h = \sqrt{3} \, \text{m} \)?

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(D) \( \tan(\theta) = 2/h \)

(E) \( \tan(\theta/2) = 1/h \)

\( \theta \) \hspace{2cm} h \hspace{2cm} 2

This will get messy.
If the height of an isosceles triangle with base 2m changes at a rate \( h' = 3 \) m/s, how quickly is the angle opposite the base changing when \( h = \sqrt{3} \) m?

Take derivatives to relate their rates of change (\( h' \) and \( \theta' \)):

\begin{align*}
\tan(\theta/2) &= 1/h \\
\sec^2(\theta/2) \frac{\theta'}{2} &= -h'/h^2 \\
\theta' &= -2 \frac{h'}{(h^2 \sec^2(\theta/2))} = -2 \frac{h'}{h^2} \cos^2(\theta/2)
\end{align*}

\[= -2 \cos^2(\theta/2) = -3/2 \text{ radians/s} \]

\[\theta = \ldots (A) \pi/6 \quad (B) \pi/4 \quad (C) \pi/3 \quad (D) 2\pi/3 \quad (E) \pi.\]
Zebra Danio escape response

Zebra Danio escape response

ZD tries to escape when $\alpha'$ is above a threshold value.

\[
\frac{d\alpha}{dt} = -\frac{dx}{dt} \cos^2 \left( \frac{\alpha}{2} \right) \frac{S}{x^2}
\]
What is $\cos^2(a)$ when $\tan(a)=\frac{p}{q}$?

(A) $\frac{p^2+q^2}{q^2}$

(B) $\frac{p^2+q^2}{p^2}$

(C) $\frac{p^2}{p^2+q^2}$

(D) $\frac{q^2}{p^2+q^2}$

(E) $\frac{p^2}{q^2}$
What is $\cos^2(a)$ when $\tan(a) = p/q$?

(A) $(p^2 + q^2) / q^2$
(B) $(p^2 + q^2) / p^2$
(C) $p^2 / (p^2 + q^2)$
(D) $q^2 / (p^2 + q^2)$
(E) $p^2 / q^2$

\[
\frac{d\alpha}{dt} = -\frac{dx}{dt} \cos^2 \left( \frac{\alpha}{2} \right) \frac{S}{x^2} \\
= -\frac{dx}{dt} \frac{x^2}{x^2 + \frac{S^2}{4}} \frac{S}{x^2} \\
= -\frac{dx}{dt} \frac{S}{x^2 + \frac{S^2}{4}} = v \frac{S}{x^2 + \frac{S^2}{4}}
\]
Assuming the Zebra Danio reacts to a rapidly changing optical angle $\alpha$, it will try to escape from...

(A) ...a very large predator (large $S$).

(B) ...a very small predator (small $S$).

(C) ...a predator that is far away (large $x$).

(D) ...a slow-moving predator (small $v$).

(E) ...a fast-moving predator (large $v$).

\[
\frac{d\alpha}{dt} = v \frac{S}{x^2 + \frac{S^2}{4}}
\]
Assuming the Zebra Danio reacts to a rapidly changing optical angle $\alpha$, it will try to escape from...

(A) ...a very large predator (large $S$).
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$$\frac{d\alpha}{dt} = v \frac{S}{x^2 + \frac{S^2}{4}}$$
If the ZD reacts when \( \alpha' > \omega_{\text{crit}} \) then...

Hold predator distance \( x \) constant, plot \( \alpha' = \frac{v S}{x^2 + S^2/4} \) as function of \( S \).

- (A) ...a very large predator.
- (B) ...a very small predator.
- (C) ...a predator that is far away.
- (D) ...a slow-moving predator.
- (E) ...a fast-moving predator.
If the ZD reacts when \( \alpha' > \omega_{\text{crit}} \) then...

Hold predator distance \( x \) constant, plot \( \alpha' = \frac{v S}{(x^2+S^2/4)} \) as function of \( S \).

(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.
If the ZD reacts when $\alpha' > \omega_{\text{crit}}$ then...

Hold predator size $S$ constant, plot $\alpha' = v S/(x^2 + S^2/4)$ as function of $x$. 

(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.
Triangle with two sides of fixed length, angle between them changes.

Relate the two changing quantities:

(A) \( a^2 = b^2 + c^2 \)

(B) \( a^2 = b^2 + c^2 - 2bc \cos(\theta) \)

(C) \( a/\sin(A) = b/\sin(B) \)

(D) \( \sin(\theta) = a/b \)