Today

- Power rule refresher
- Differentiation rules
- Tangent lines
- Reminders:
  - PL4.2 Wednesday 7am,
  - Assignment 3 Thursday 7am,
  - OSH 2 Friday 11:59 pm.
  - Sign up for midterm time/room should appear soon.

Power rule

- Binomial coefficients $(a+b)^n$
- $x^n + nx^{n-1}h + \ldots$terms that have h^2 or larger power

Rules for differentiation

- Addition rule
- Product rule
- Chain rule (for composition of functions)
- Quotient rule

Suppose $f(x) = g(x) + k(x)$ and that

$$g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$$

- What is f'(2)?

  (A) 4

  (B) 7

  (C) 10

  (D) 11

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Try g(x)=x and k(x)=x^2.

(if many choose B)

(B) 10

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Try g(x)=x and k(x)=x^2.

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If f'(x)=g'(x)k'(x) then

f(x) = (x) (x^2) so f'(x)= (1) (2x)=2x.

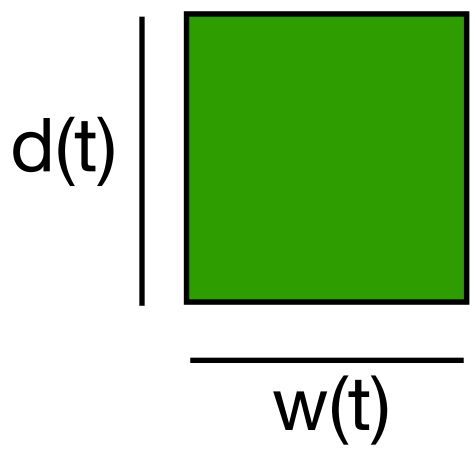
But f(x)=x^3 and power rule says

f'(x) = 3x^2.

So g'(x)k'(x) can't be right.

What is the correct derivative for f(x)=g(x)k(x)?

$$A(t) = d(t)w(t)$$



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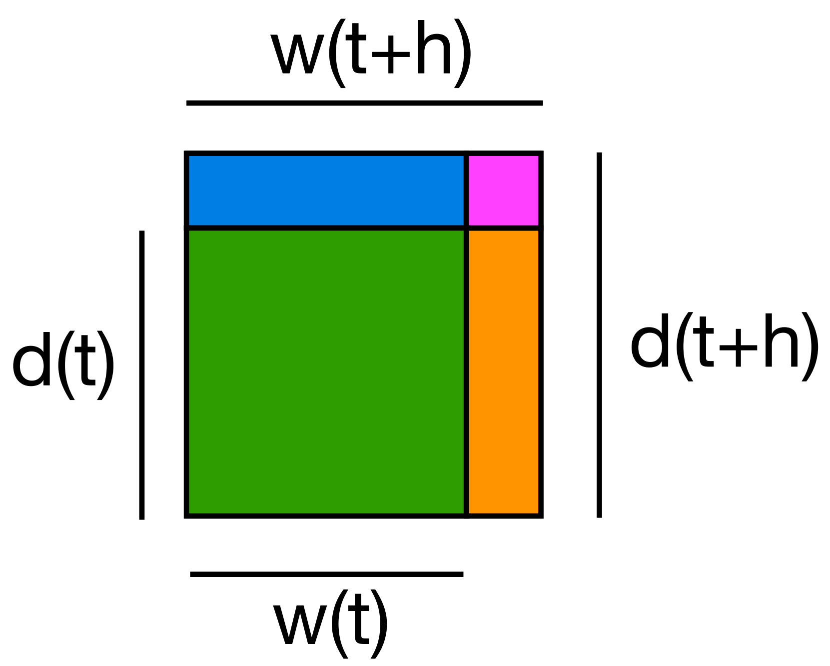

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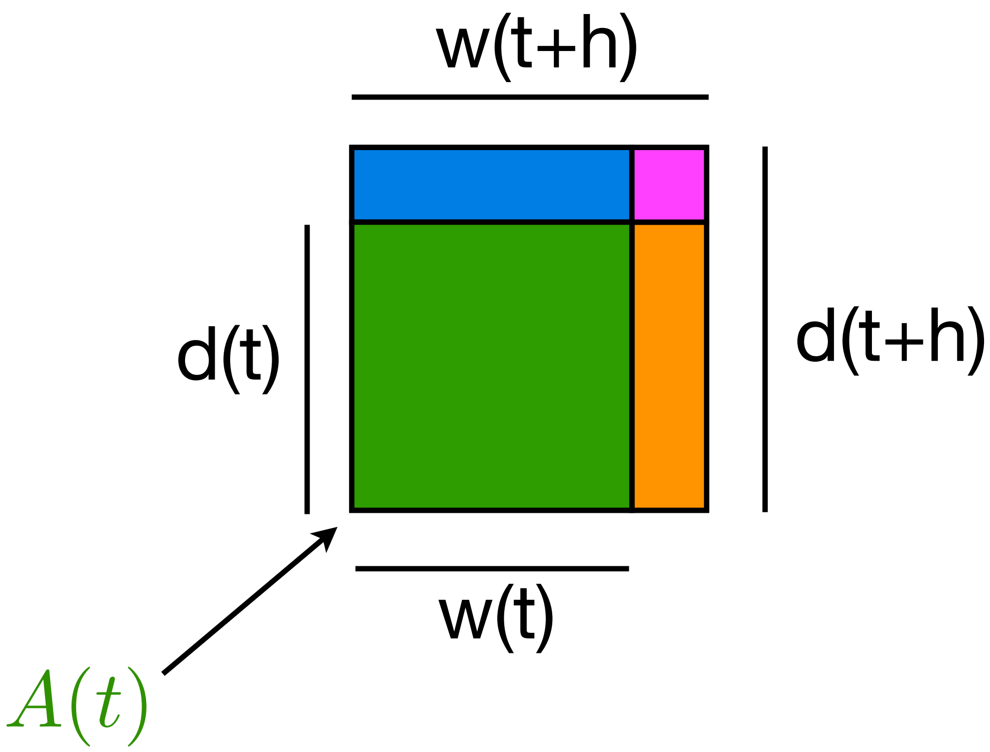

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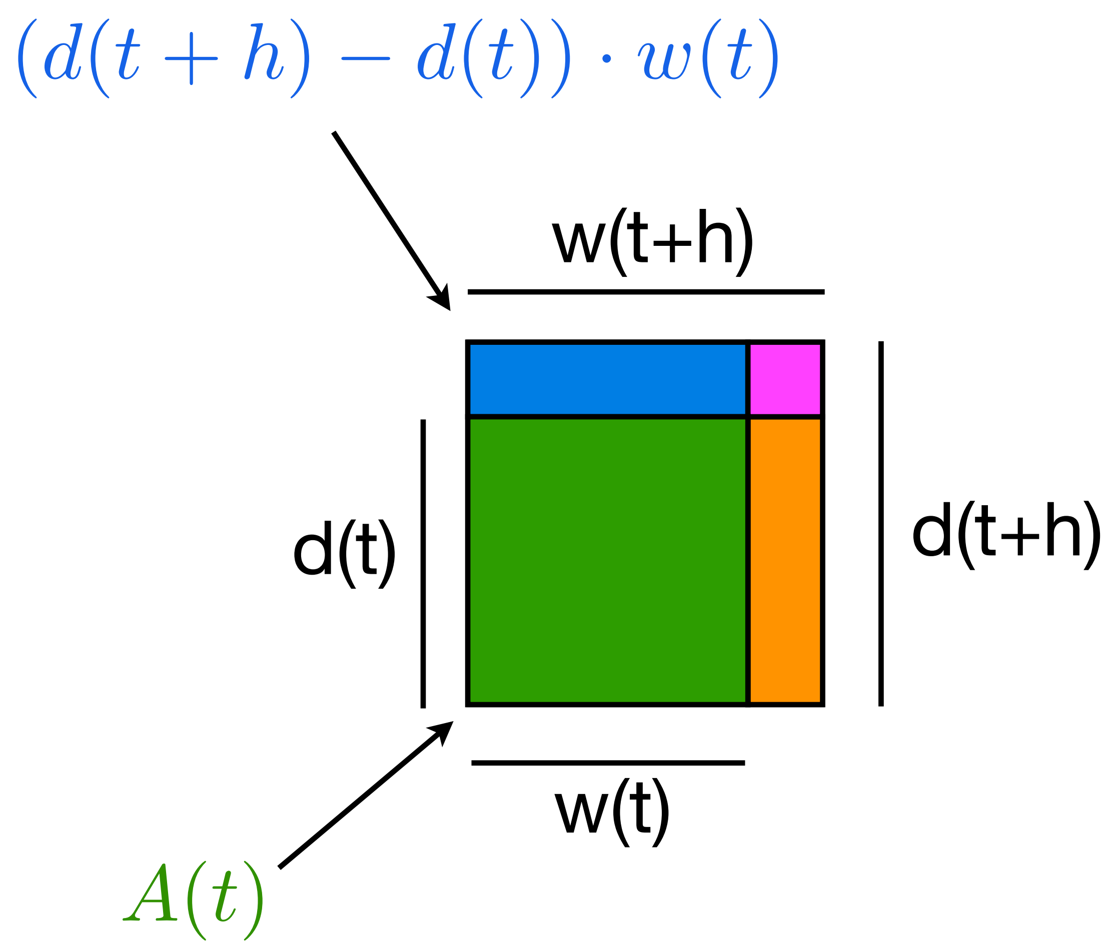

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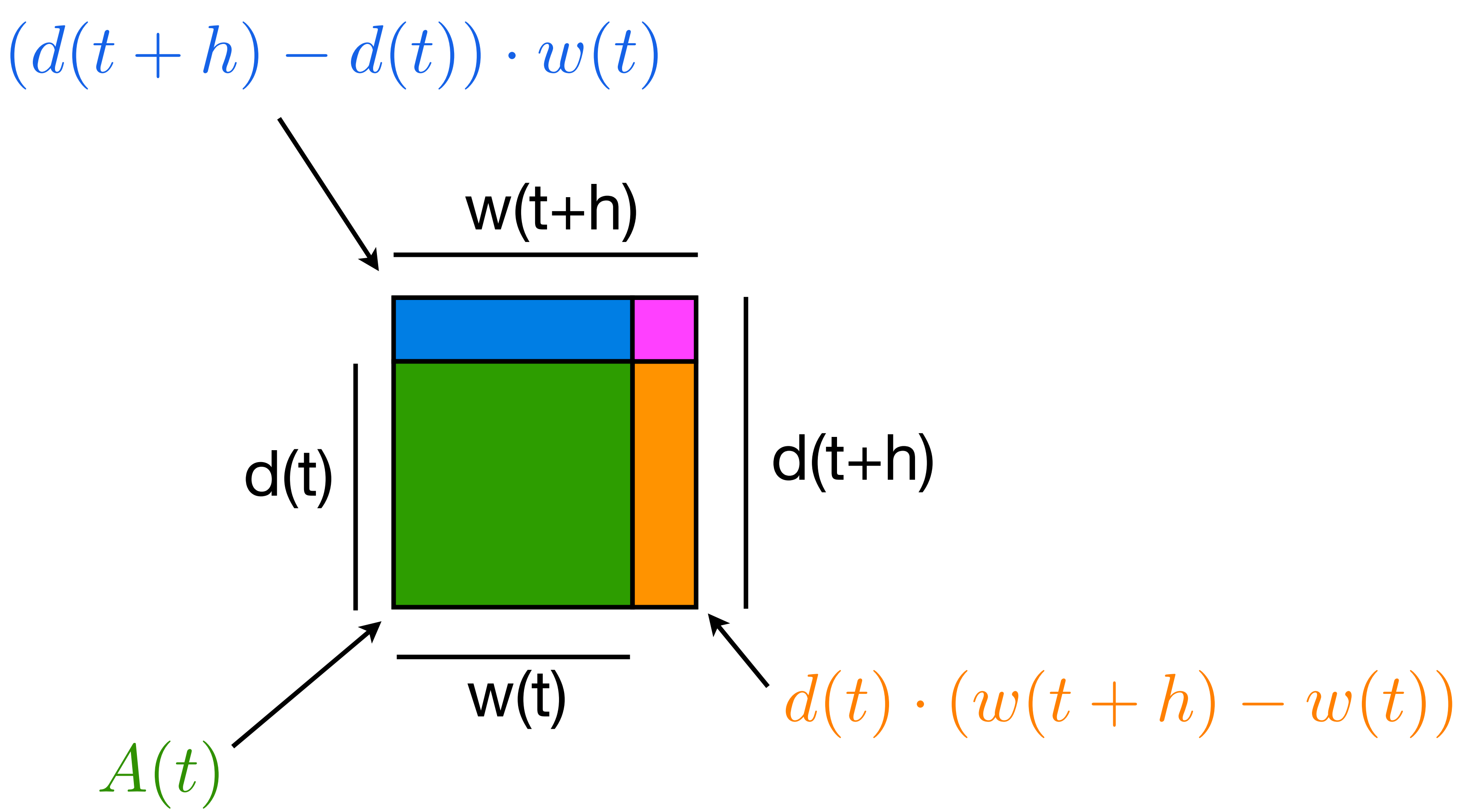

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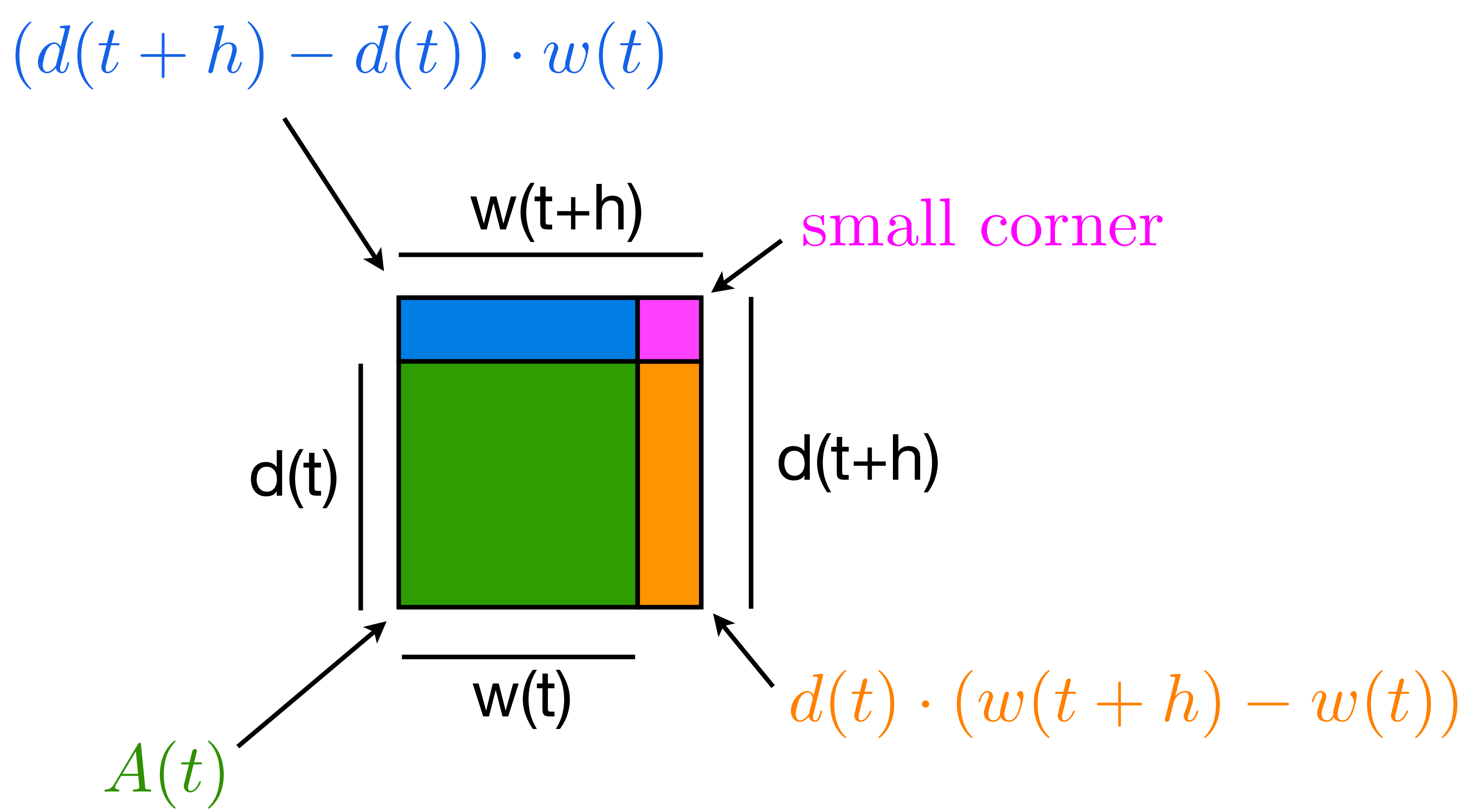

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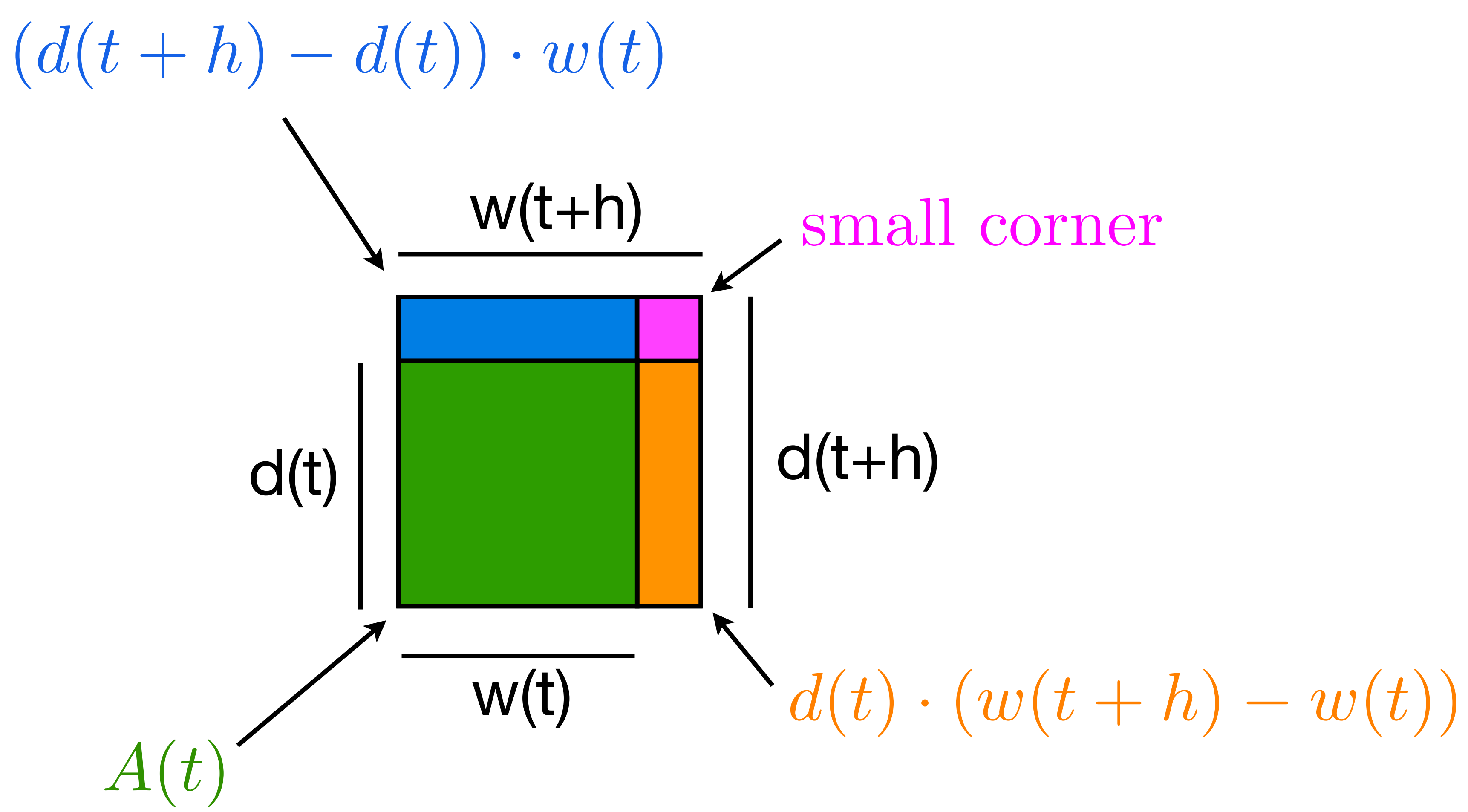
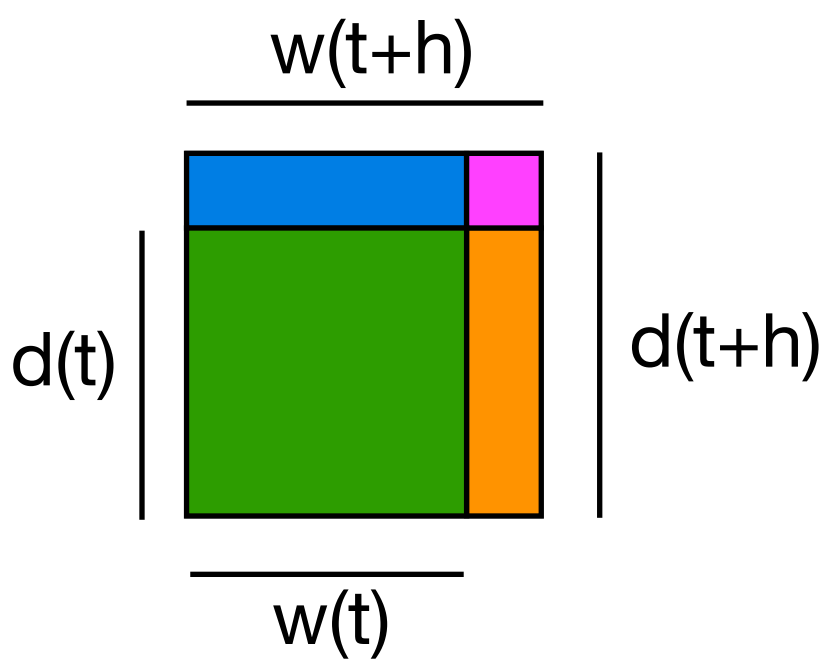


What is the correct derivative for f(x)=g(x)k(x)?

$$A(t) = d(t)w(t)$$

$$A(t+h) = A(t) + (d(t+h) - d(t)) \cdot w(t)$$
$$+ d(t) \cdot (w(t+h) - w(t)) + \text{small corner}$$



$$\frac{A(t+h) - A(t)}{h} \approx \frac{(d(t+h) - d(t)) \cdot w(t)}{h} + \frac{d(t) \cdot (w(t+h) - w(t))}{h}$$

$$A'(t) = d'(t)w(t) + d(t)w'(t)$$

Composition of functions

If $f(x) = 2x+3$ and $g(x) = -4x+2$,

A. $h(x) = f(\ g(x)\ ) = -8x+7$

B. $h(x) = f(\ g(x)\ ) = -8x-10$

C. $h(x) = f(\ g(x)\ ) = -8x^2-8x+6$

D. $h(x) = f(\ g(x)\ ) = -8x+5$

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If $h(x) = f(\ g(x)\ )$, then

A. $h'(x) = f'(x)\ g'(x)$

B. $h'(x) = f'(x)\ g(x) + f(x)\ g'(x)$

C. $h'(x) = f'(\ g'(x)\ )$

D. $h'(x) = f'(\ g(x)\ )\ g'(x)$

Composition of functions

If $h(x) = f(\,g(x)\,)$, then

A. $h'(x) = f'(x)\,g'(x)$

B. $h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

C. $h'(x) = f'(\,g'(x)\,)$

D. $h'(x) = f'(\,g(x)\,)\,g'(x)$ <---- Chain Rule

Composition of functions

If $h(x) = (x^3-2x+1)^6$, then $h'(x)=?$

A. $6\,(x^3-2x+1)^5$

B. $(x^3-2x+1)^6\,(3x^2-2)$

C. $6\,(x^3-2x+1)^5\,(3x^2-2)$

D. $6\,(x^3-2x+1)^5\,(x^3-2x+1)$

E. Are you kidding? It will take me weeks to multiply those out.

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$f(g) = g^6$

$g(x) = x^3-2x+1$

$h(x) = f(g(x))$

(not shown in class)

Geometry of function composition

(not shown in class)

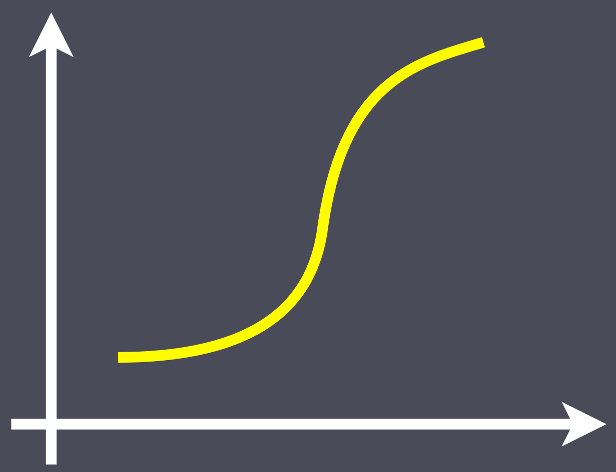
Geometry of function composition

- A function f(x) takes x values to y values.

(not shown in class)

Geometry of function composition

- A function f(x) takes x values to y values.
- When the slope is steep(shallow) at a point, the distance between nearby points on the x axis get stretched out (squished) on the y axis.



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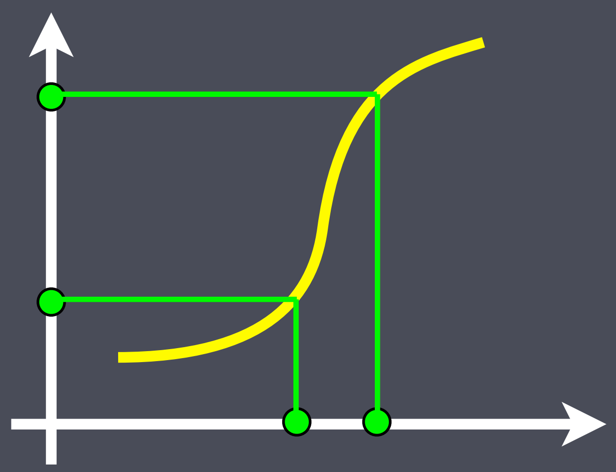

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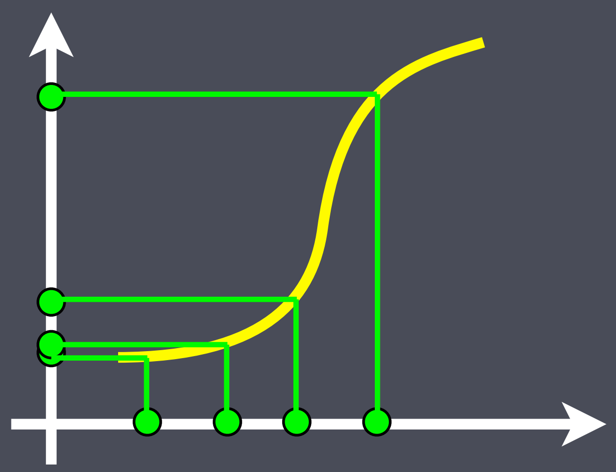

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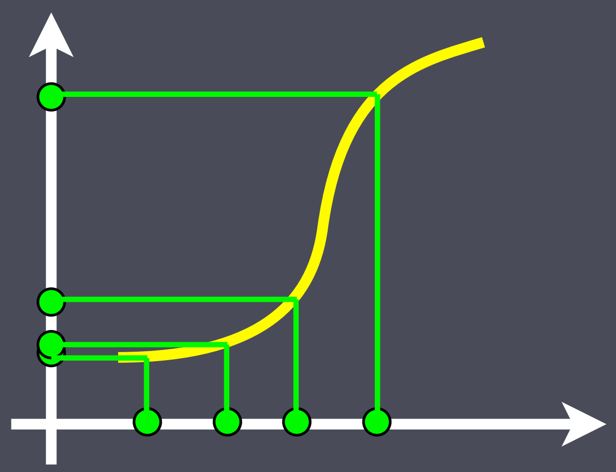
Geometry of function composition

- A function f(x) takes x values to y values.
- When the slope is steep(shallow) at a point, the distance between nearby points on the x axis get stretched out (squished) on the y axis.
- So $f'(x)$ is the stretch factor near x.



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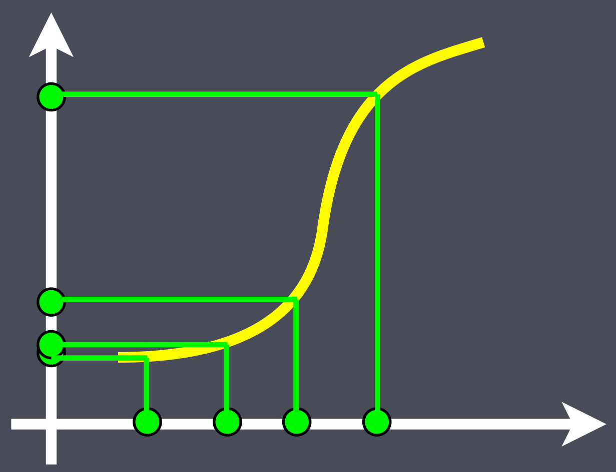
Geometry of function composition

- A function f(x) takes x values to y values.
- When the slope is steep(shallow) at a point, the distance between nearby points on the x axis get stretched out (squished) on the y axis.
- So f′(x) is the stretch factor near x.
- Where you are matters!



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Geometry of function composition

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Geometry of function composition

- When you compose functions, $h(x)=f(g(x))$, you first stretch/squish near x according to g and then stretch/squish near $g(x)$ according to f.

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- When you do one and then the other, you multiply their effects.

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- When you do one and then the other, you multiply their effects.
- Where you are on the function matters - multiply the stretch factor of g near x: $g'(x)$, by the stretch factor of f near $g(x)$: $f'(x)$.

Rules for differentiation - summary

- Addition rule:
  - $f(x) = g(x) + h(x)$ ----> $f'(x) = g'(x) + h'(x)$
- Product rule:
  - $f(x) = g(x)\, h(x)$ ----> $f'(x) = g'(x)\, h(x) + g(x)\, h'(x)$
- Chain rule:
  - $f(x) = g(\, h(x)\,)$ ----> $f'(x) = g'(\, h(x)\,)\, h'(x)$
- Quotient rule:
  - $f(x) = g(x) / h(x) = g(x)\, (h(x))^{-1}$ <---- apply product and chain rules or

Suppose $f(x) = g(x)/k(x)$ and that

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- What is f’(2)?

(A) -13

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