Today

- Solving linear DEs
- Murder on 13 ave.
- Reminder: midterm Tuesday 6 pm!

A drug delivered by IV accumulates at a constant rate k_{IV}. The body metabolizes the drug proportional to the amount of the drug.

(A) $d'(t) = k_{IV} - k_m d(t)$

(B) $d'(t) = (k_{IV} - k_m) d(t)$

(C) $d'(t) = k_{IV} d(t) - k_m$

(D) $d'(t) = -k_{IV} + k_m d(t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m\, d(t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m d(t)$
- Take derivative of this $c(t)$:

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m\, d(t)$
- Take derivative of this $c(t)$: $c'(t) = -k_m\, d'(t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m d(t)$
- Take derivative of this c(t): $c'(t) = -k_m d'(t)$
- New equation for c(t):

(A) $c'(t) = -k_m c(t)$

(B) $c'(t) = -k_{IV} c(t)$

(C) $c'(t) = k_m c(t)$

(D) $c'(t) = -k_m (k_{IV} - k_m d(t))$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m\, d(t)$
- Take derivative of this $c(t)$: $c'(t) = -k_m\, d'(t)$
- New equation for $c(t)$:

(A) $c'(t) = -k_m\, c(t)$

(B) $c'(t) = -k_{IV}\, c(t)$

(C) $c'(t) = k_m\, c(t)$

(D) $c'(t) = -k_m\, (k_{IV} - k_m\, d(t))$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m d(t)$
- Take derivative of this $c(t)$: $c'(t) = -k_m d'(t)$
- New equation for $c(t)$:

(A) $c'(t) = -k_m c(t)$

(B) $c'(t) = -k_{IV} c(t)$

(C) $c'(t) = k_m c(t)$

(D) $c'(t) = -k_m (k_{IV} - k_m d(t))$

What about the initial condition, $c(0) = ?$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m d(t)$
- Take derivative of this $c(t)$: $c'(t) = -k_m d'(t)$
- New equation: $c'(t) = -k_m c(t)$, $c(0)=k_{IV}$.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m d(t)$
- Take derivative of this c(t): $c'(t) = -k_m d'(t)$
- New equation: $c'(t) = -k_m c(t)$, $c(0)=k_{IV}$.
- This means the solution to the d(t) eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(C) $d(t) = k_{IV}/k_m (1-\exp(-k_m t))$

(B) $d(t) = k_{IV} \exp(k_m t)$

(D) $d(t) = k_{IV}/k_m \exp(-k_m t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- Replace RHS by: $c(t) = k_{IV} - k_m\, d(t)$
- Take derivative of this c(t): $c'(t) = -k_m\, d'(t)$
- New equation: $c'(t) = -k_m\, c(t)$, $c(0) = k_{IV}$.
- This means the solution to the d(t) eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(B) $d(t) = k_{IV} \exp(k_m t)$

(C) $d(t) = k_{IV}/k_m\, (1-\exp(-k_m t))$

(D) $d(t) = k_{IV}/k_m\, \exp(-k_m t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.

What happens to $d(t)$ as $t \to \infty$?

- New equation: $c'(t) = -k_m\, c(t)$, $c(0) = k_{IV}$.
- This means the solution to the $d(t)$ eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(B) $d(t) = k_{IV} \exp(k_m t)$

(C) $d(t) = k_{IV}/k_m\, (1 - \exp(-k_m t))$

(D) $d(t) = k_{IV}/k_m\, \exp(-k_m t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.

What happens to $d(t)$ as $t \to \infty$?

$d(t) \to k_{IV}/k_m$

- New equation: $c'(t) = -k_m\, c(t)$, $c(0) = k_{IV}$.
- This means the solution to the $d(t)$ eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(B) $d(t) = k_{IV} \exp(k_m t)$

(C) $d(t) = k_{IV}/k_m\, (1 - \exp(-k_m t))$

(D) $d(t) = k_{IV}/k_m\, \exp(-k_m t)$

General case

- Any problem of the form $y' = a - by$ with IC $y(0)=y_0$ has solution

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- Check:
  - LHS: $y'(t)$ = (on the blackboard)
  - RHS: $a - by$ = (on the blackboard)
  - $y(0) = a/b + (y_0 - a/b)\, e^0 = y_0$

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- If b>0 then as t--> ∞, y(t) --> a/b.

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- If b>0 then as t--> ∞, y(t) --> a/b.
- When b>0, the characteristic time is 1/b.

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- If b>0 then as t--> ∞, y(t) --> a/b.
- When b>0, the characteristic time is 1/b.
- Notice that if $y_0 = a/b$ then y(t) = a/b.

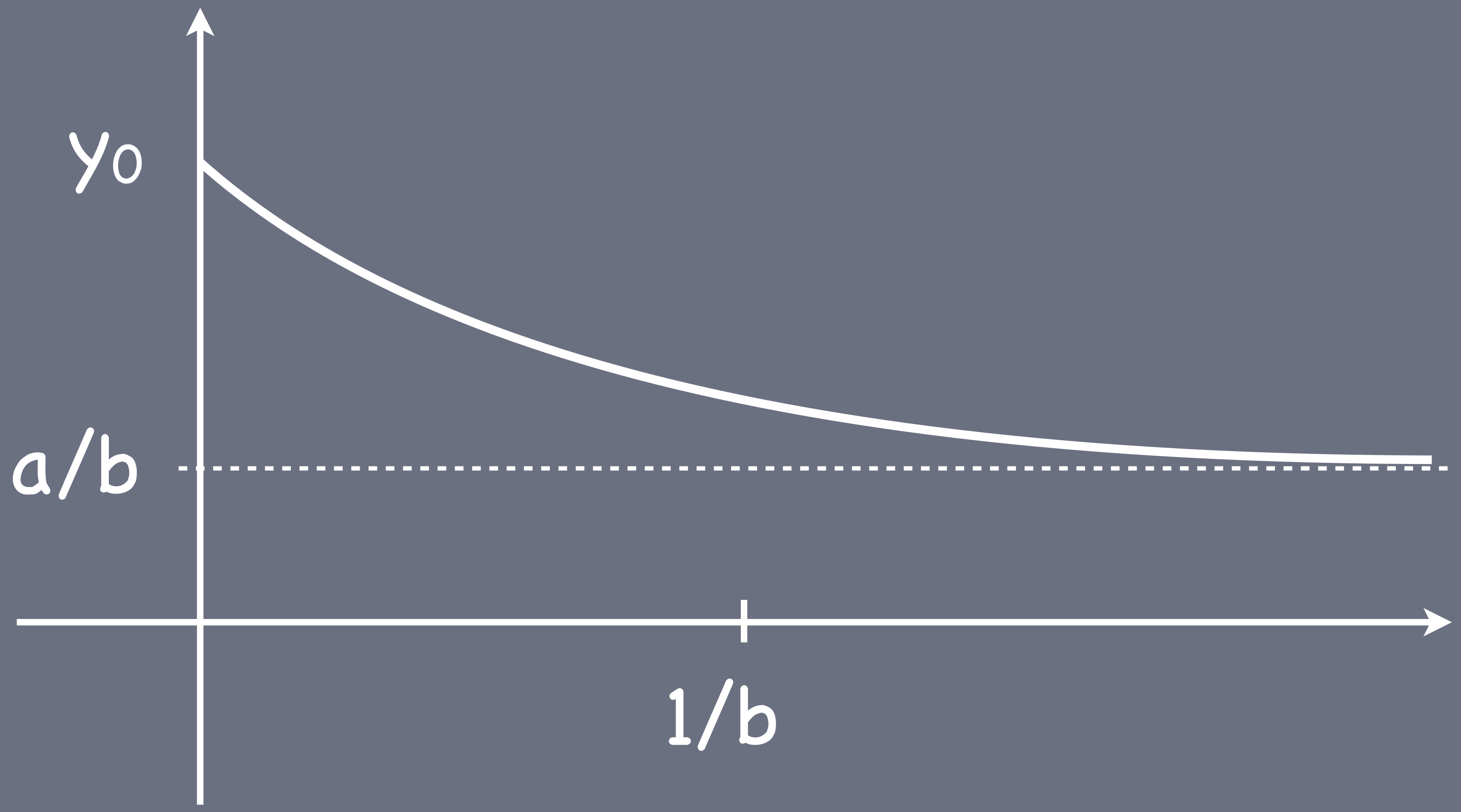
General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

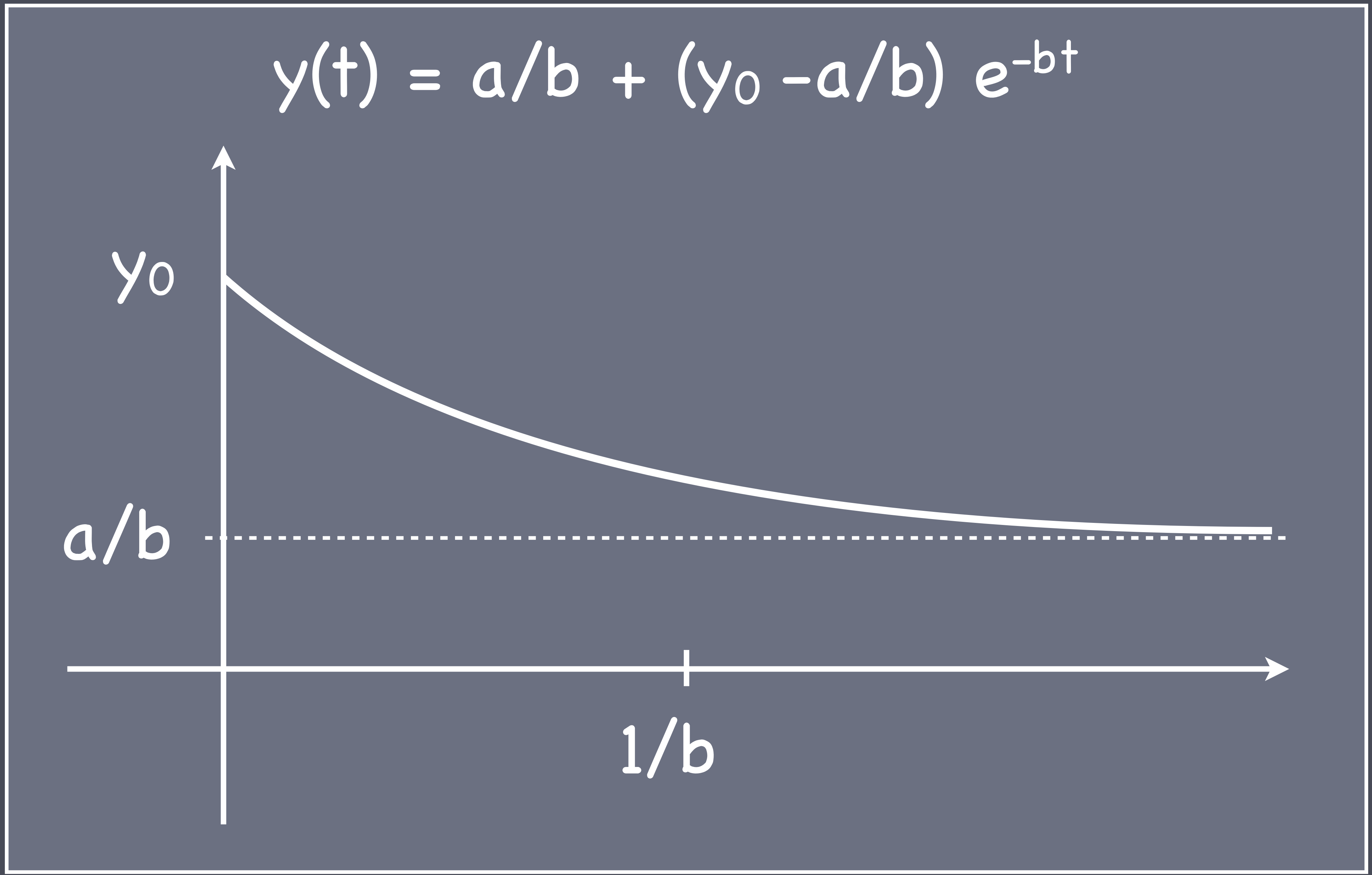
- If $b>0$ then as $t \to \infty$, $y(t) \to a/b$.
- When $b>0$, the characteristic time is $1/b$.
- Notice that if $y_0=a/b$ then $y(t) = a/b$.
- Constant solutions like this are called steady states.

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

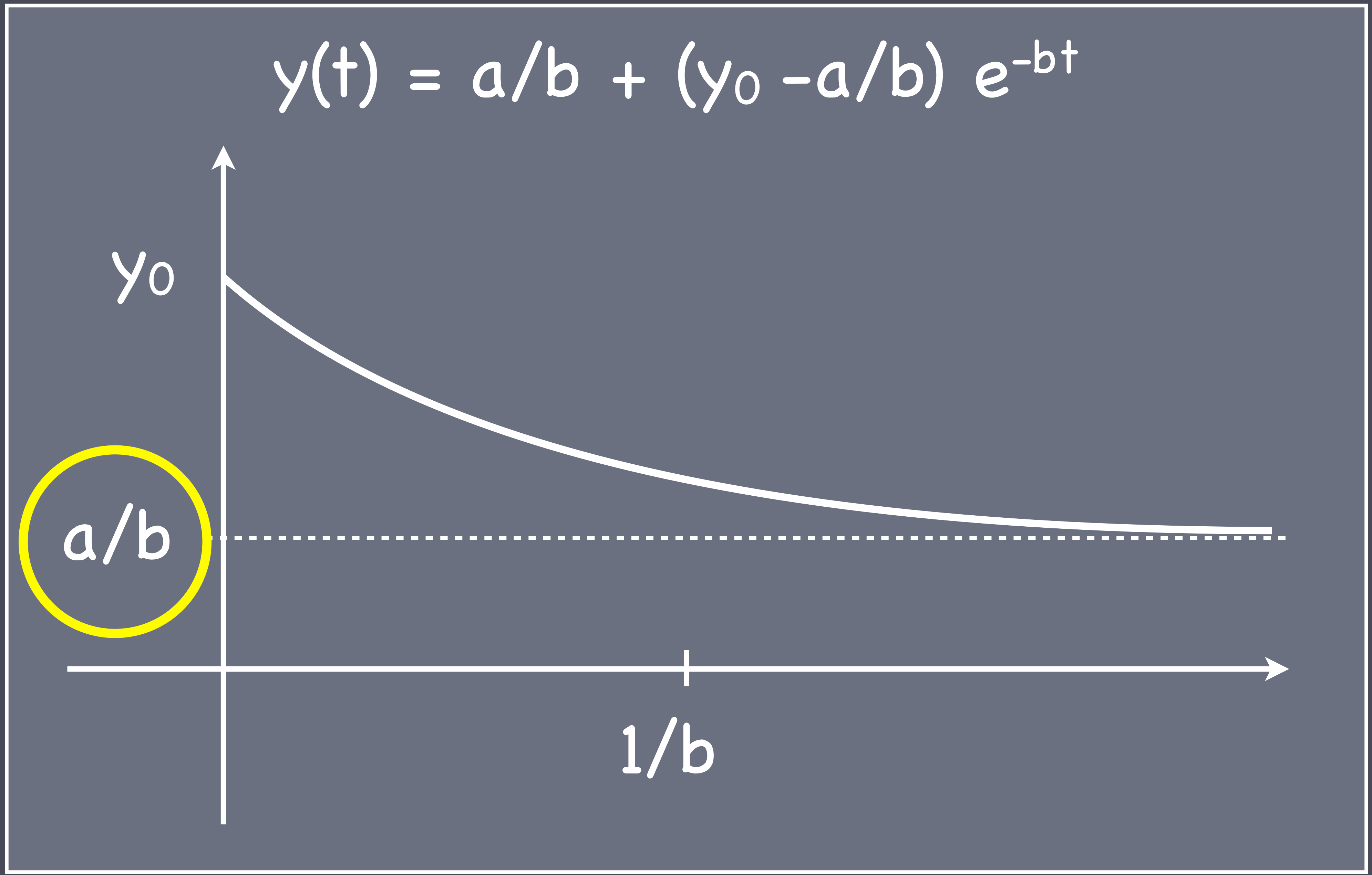


General case



Where is y(t) going?

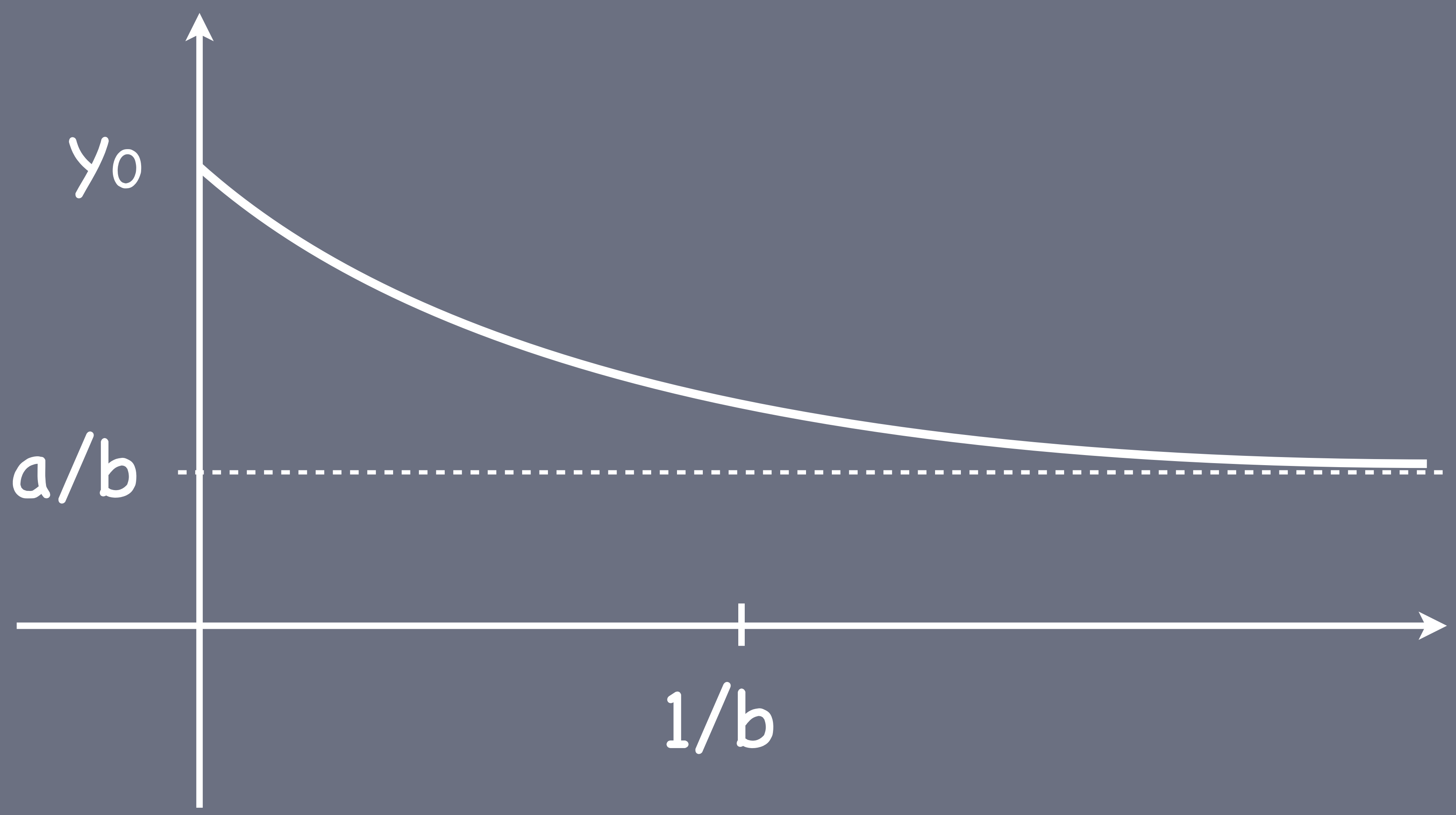
General case



Where is y(t) going? To the steady state a/b.

General case

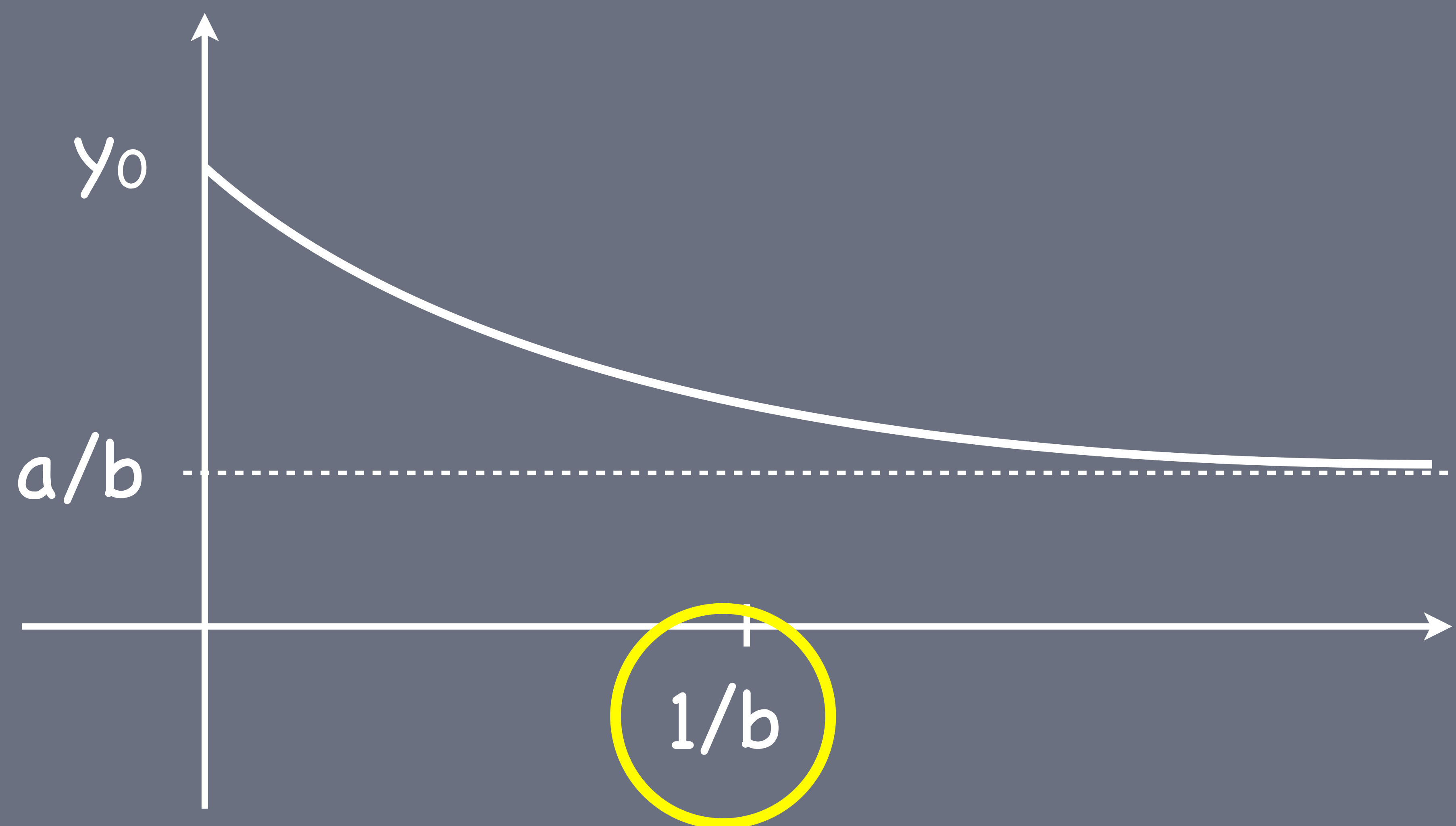
$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$



When will it get there?

General case

$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$

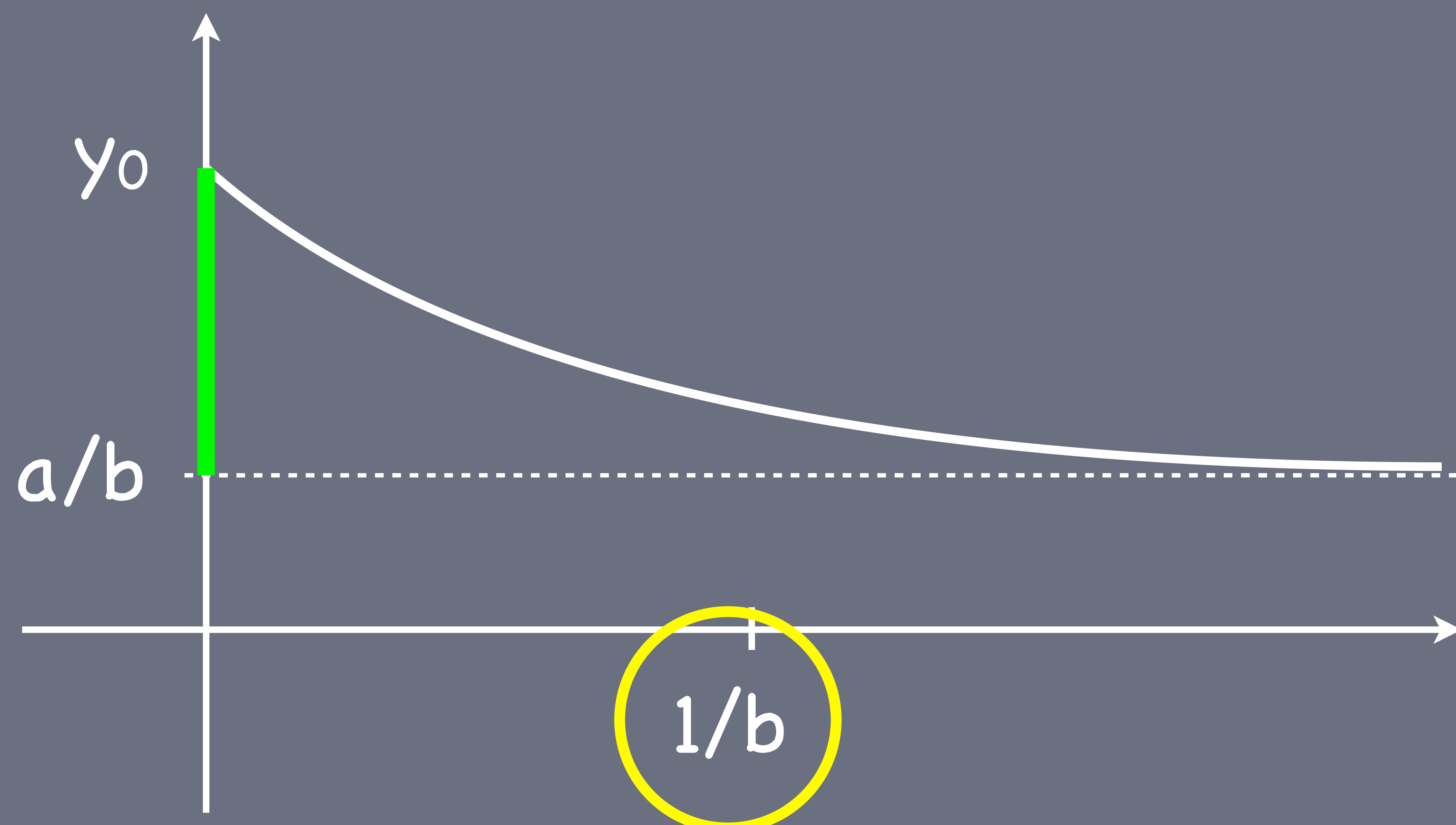


When will it get there?

Never but at t=1/b it will be 1/e of the way.

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

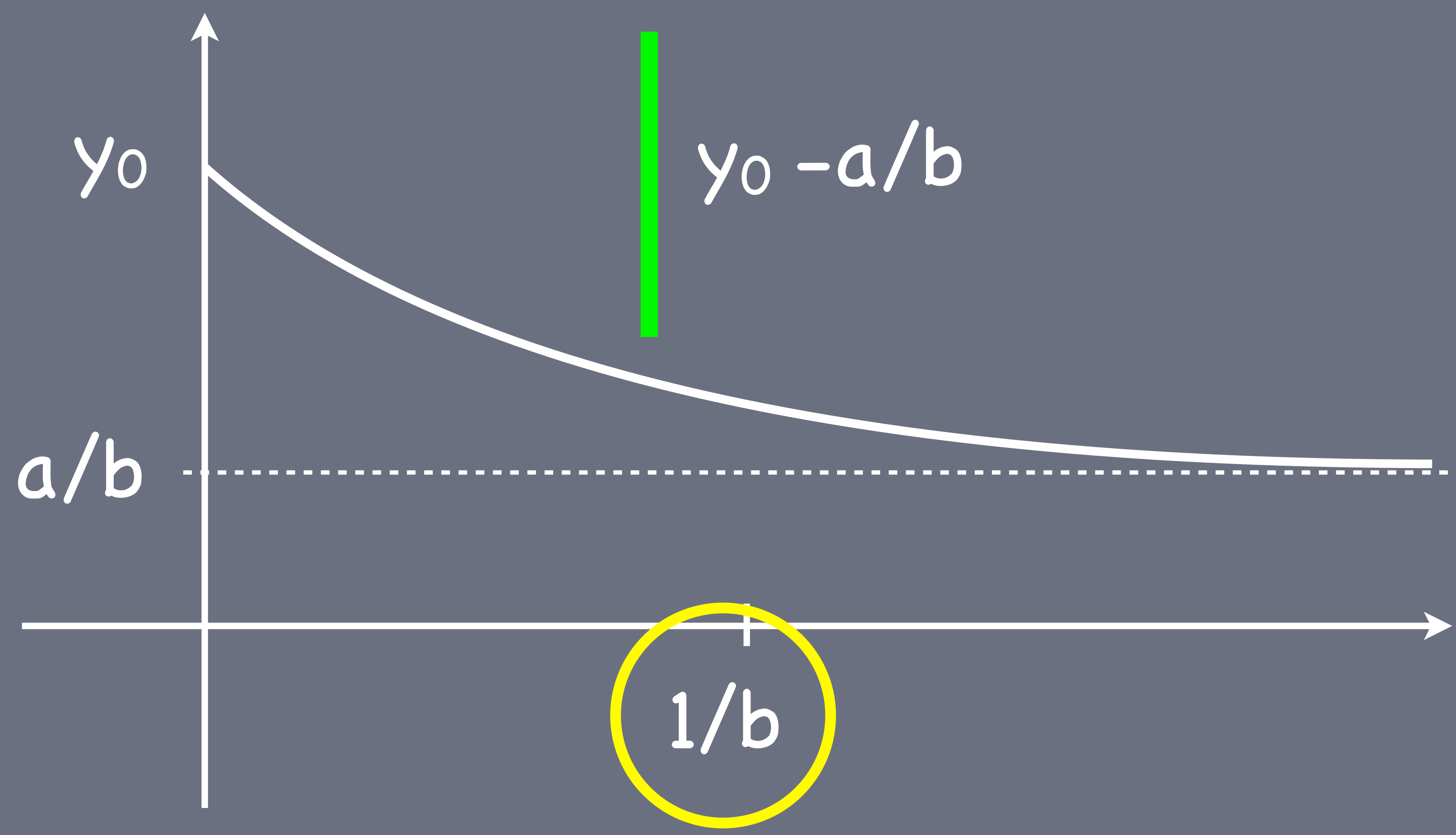


When will it get there?

Never but at t=1/b it will be 1/e of the way.

General case

$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$

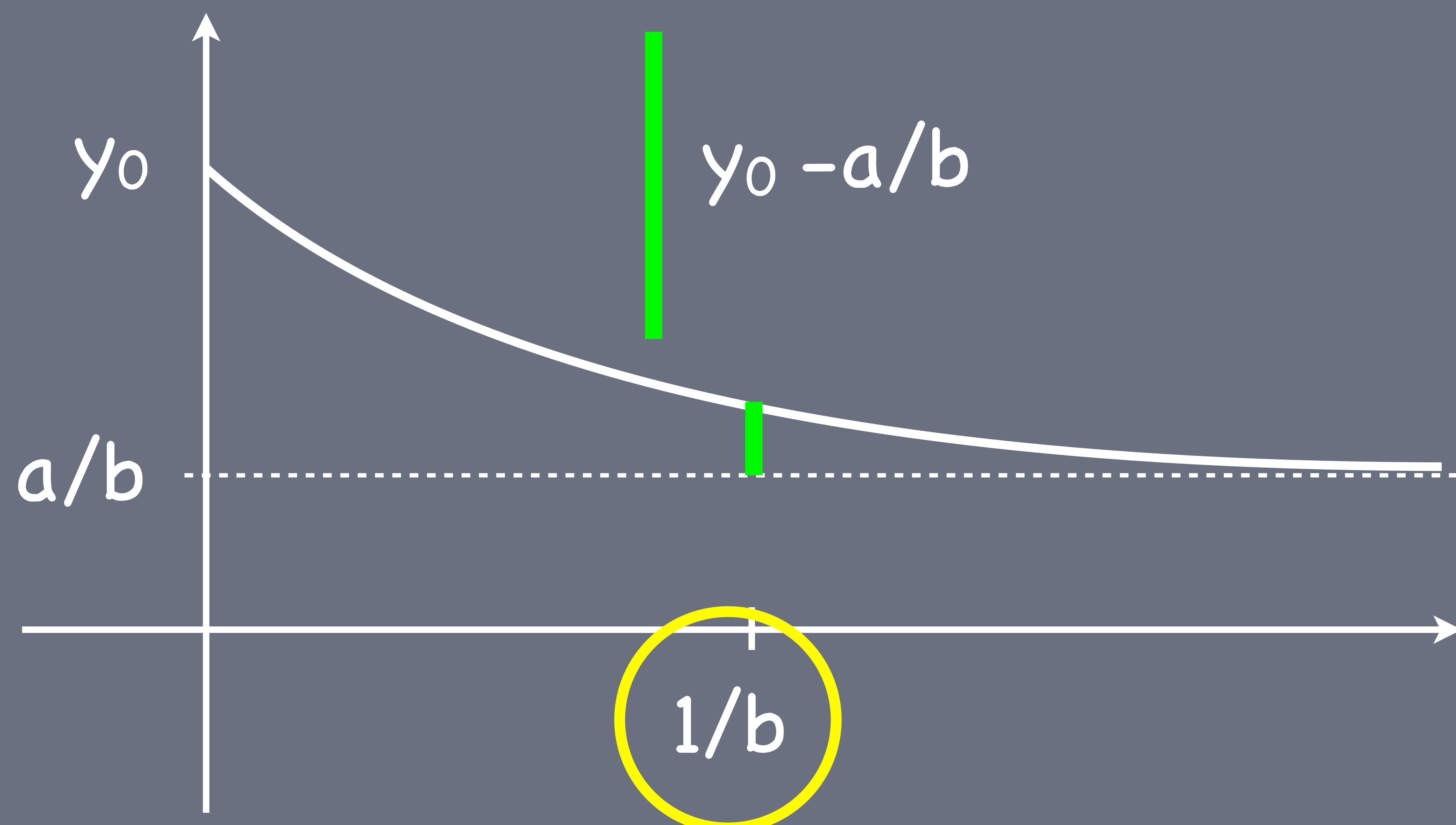


When will it get there?

Never but at $t=1/b$ it will be $1/e$ of the way.

General case

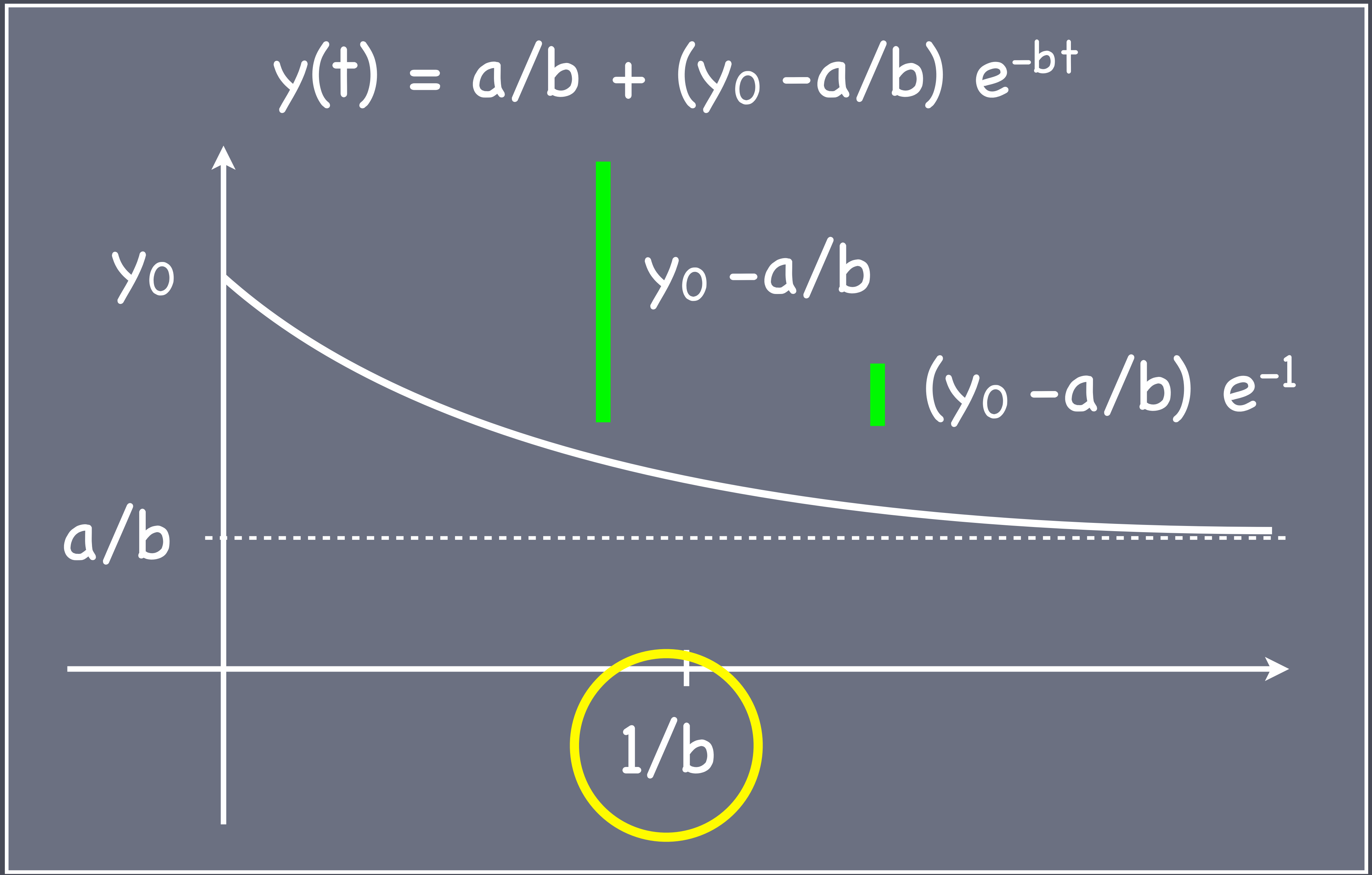
$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$



When will it get there?

Never but at t=1/b it will be 1/e of the way.

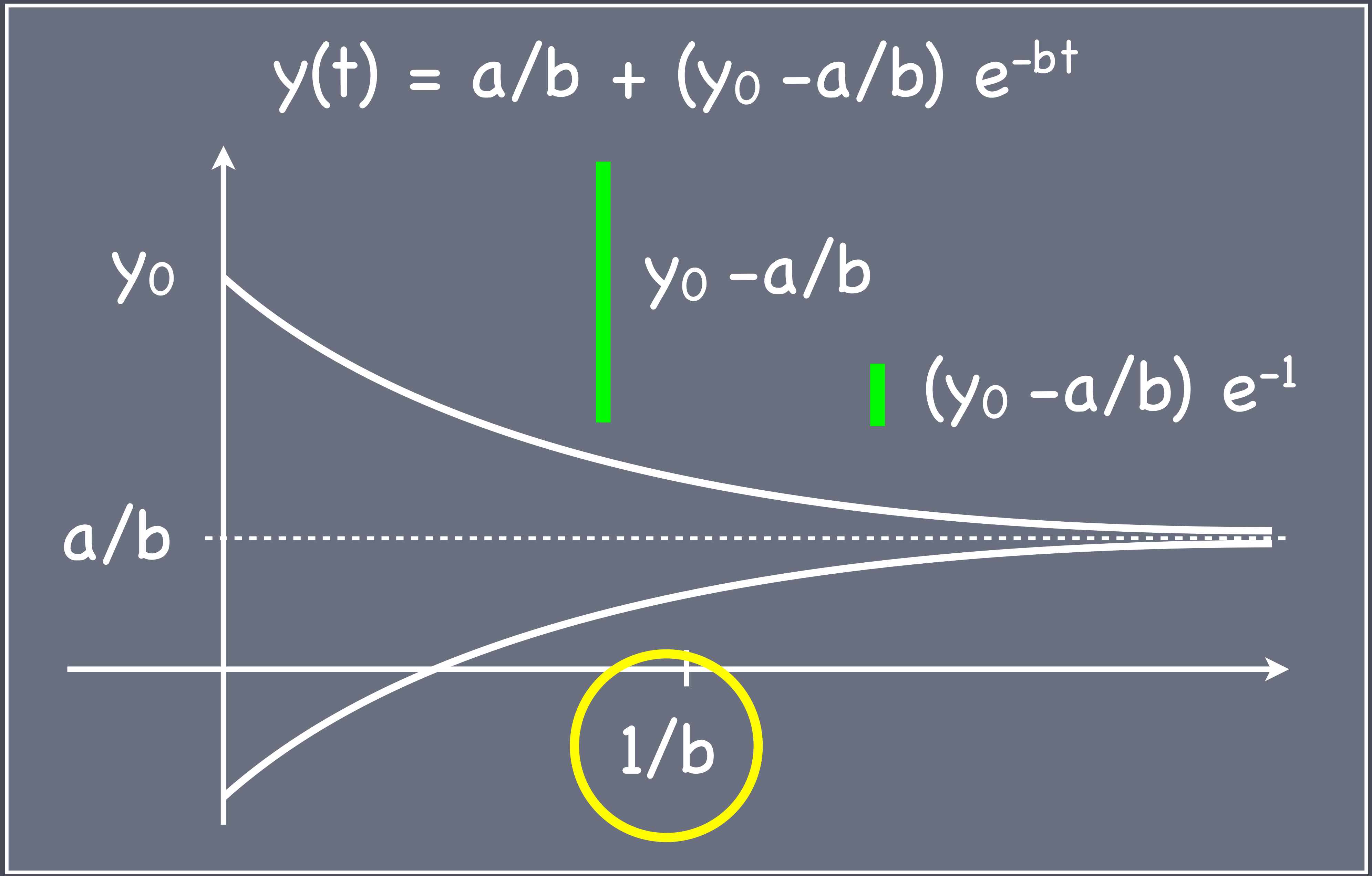
General case



When will it get there?

Never but at $t=1/b$ it will be $1/e$ of the way.

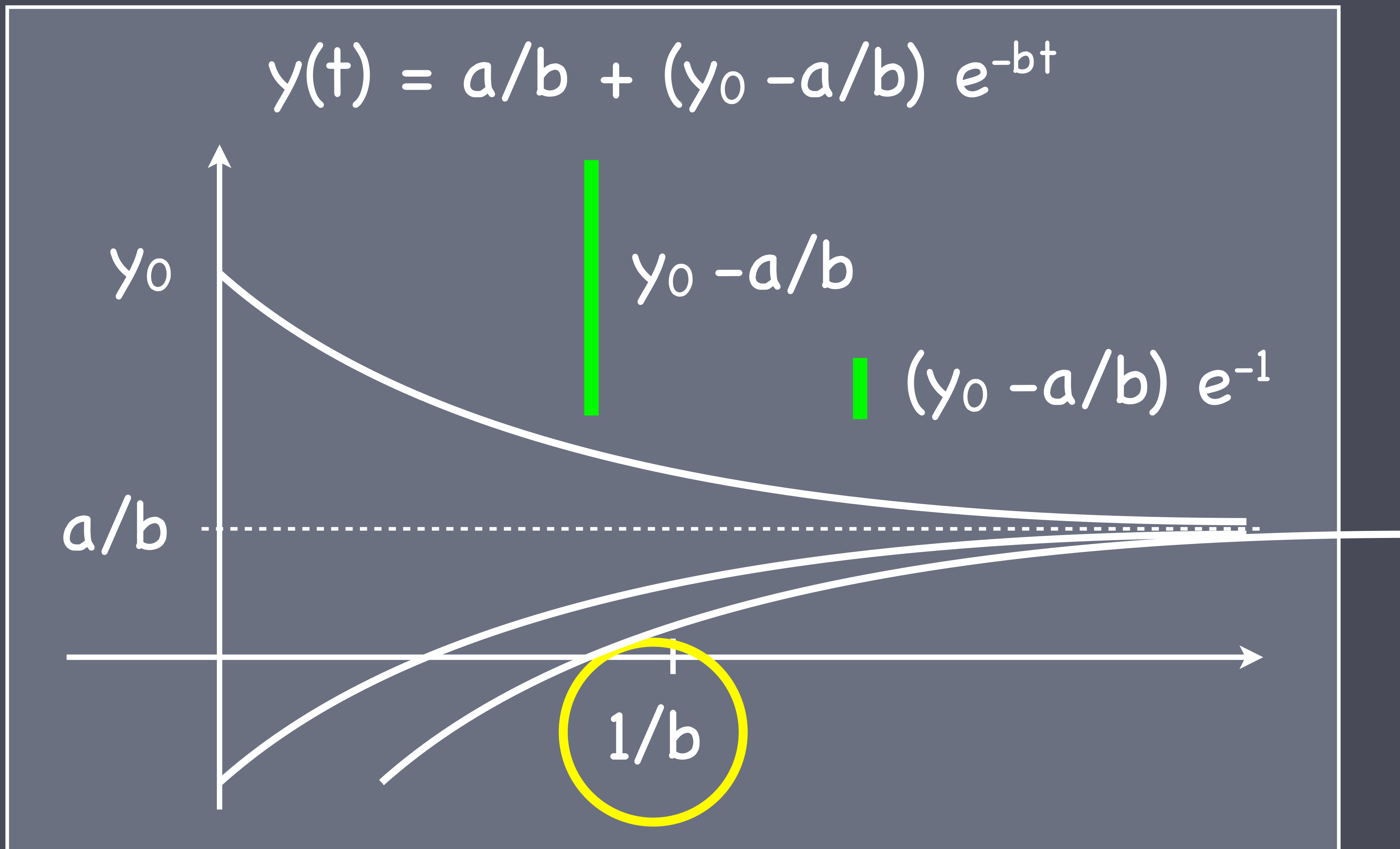
General case



When will it get there?

Never but at $t=1/b$ it will be $1/e$ of the way.

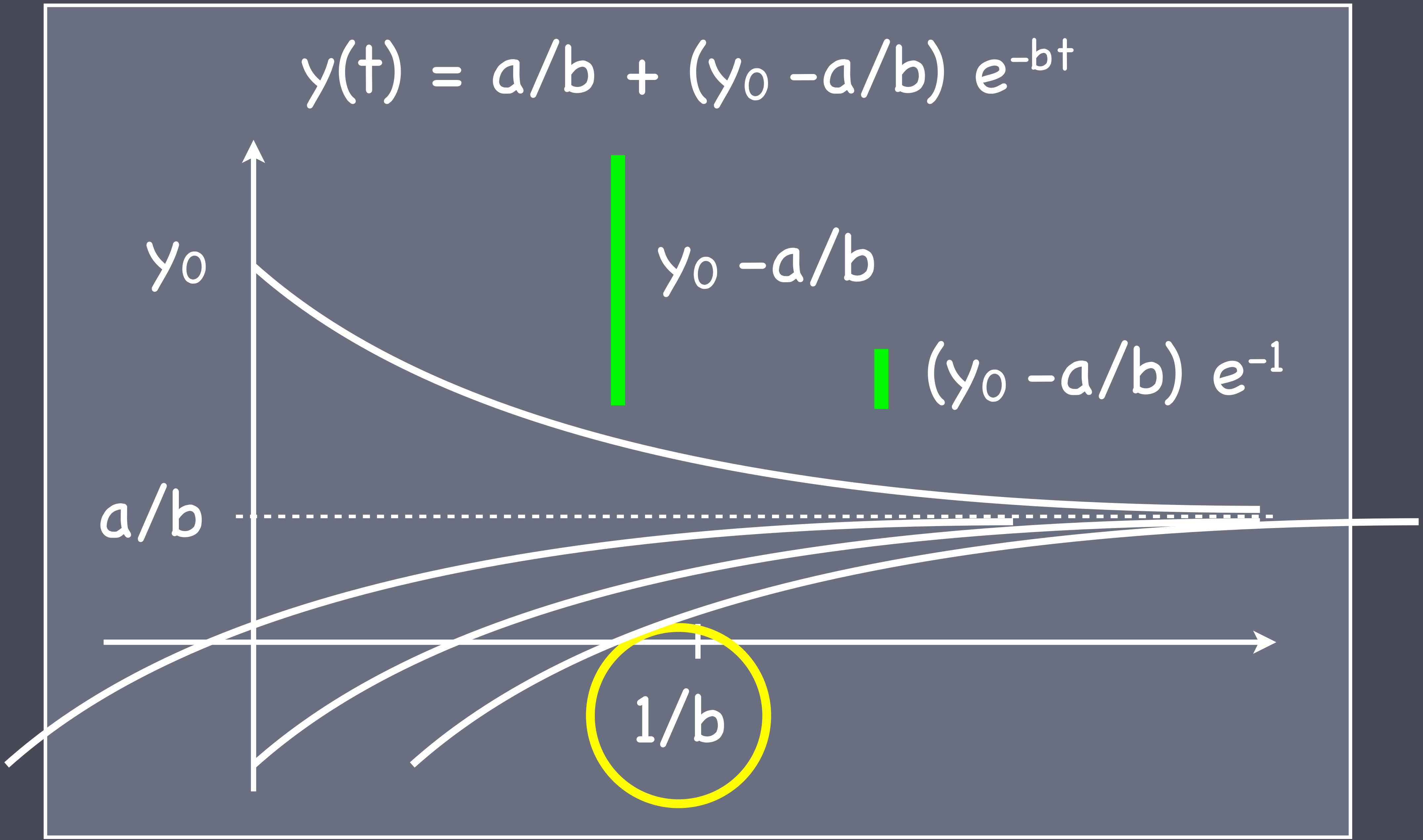
General case



When will it get there?

Never but at $t=1/b$ it will be $1/e$ of the way.

General case



When will it get there?

Never but at $t=1/b$ it will be $1/e$ of the way.

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E - T(t))$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$

  $a = kE$, $b = k$.

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$

  $a = kE$, $b = k$.

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$

  $a = kE,\ b = k.$

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

  $a = k_{IV},\ b = k_m$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$

  $a = kE$, $b = k$.

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

  $a = k_{IV}$, $b = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$

  $a = kE$, $b = k$.

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

  $a = k_{IV}$, $b = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$

  $a = g$, $b = \delta$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E - T(t))$

  $a = kE,\ b = k.$

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

  $a = k_{IV},\ b = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$

  $a = g,\ b = \delta$

- General form, factored: $y'(t) = b\,(a/b - y)$.

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E - T(t))$

  $a = kE$, $b = k$.

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

  $a = k_{IV}$, $b = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$

  $a = g$, $b = \delta$

- General form, factored: $y'(t) = b\,(a/b - y)$.

  steady state (a/b)

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$

  $a = kE$, $b = k$.

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

  $a = k_{IV}$, $b = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$

  $a = g$, $b = \delta$

- General form, factored: $y'(t) = b\,(a/b - y)$.

1 / characteristic time

steady state

What do you need to know?

- Given a word description, write down an equation for the quantity q(t) described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...
  - Ex. Blah changes proportional to the difference between blah and fixed #.
- Substitute as in the drug problem to get $y'=ky$ and state that $y(t)=Ce^{kt}$ solves it.
- Substitute back to find q(t).
- Determine C using the initial condition.
- Answer questions about the resulting exponential q(t).

Newton's Law of Cooling (NLC)

- When an object cools by convection, it can be modeled by Newton's Law of Cooling:

  Heat is lost proportional to the difference between the object's temperature T(t) and the surrounding's temperature E.

$$T'(t) = k\,(\,E - T(t)\,)$$

Note: heat can be lost in other ways (e.g. radiation). Newton's Law of Cooling is a model that is sometimes appropriate and sometimes not.

What do you expect $\lim_{t \to \infty} T(t)$ to be?

(A) E

(B) kE

(C) 0

(D) E–T(0)

(E) T(0)

What do you expect

$\lim_{t \to \infty} T(t)$ to be?

(A) E

(B) kE

(C) 0

(D) E–T(0)

(E) T(0)

$$T'(t) = k\,(E - T(t))$$

$$T(0)=T_0$$

$$T'(t) = k\,(E - T(t)) = kE - kT(t)$$

$$T(0)=T_0$$

$$T'(t) = k\,(E - T(t)) = kE - kT(t)$$

$$T(0)=T_0$$

$$a=kE,\ b=k$$

$$T'(t) = k\,(E - T(t)) = kE - kT(t)$$

$$T(0)=T_0$$

$$a=kE,\ b=k$$

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

$$T'(t) = k\,(E - T(t)) = kE - kT(t)$$

$$T(0)=T_0$$

$$a=kE,\ b=k$$

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

$$T(t) = E + (T_0 - E)\, e^{-kt}$$

All Hallow's Eve

- On Oct 31, 2014, a string of trick-or-treaters were seen walking along the 400 block of East 13th ave.
  - 8:38 pm - Tina
  - 8:55 pm - Jinsong
  - 9:05 pm - Maria
  - 9:12 pm - Ali-reza
  - 9:27 pm - Chadni

All Hallow's Eve

- At 8:15 am, the woman who lived at 444 East 13 ave was found dead in the front yard of her home.
  - The VPD has asked you to figure out who did it.
  - You arrive on the scene, and tell the police you will have an answer for them soon.
  - What is your next move? Discuss.