Math 102 Midterm Examination
Thursday, October 26

Email address:

(This may be used to help match your exam to your name in our records.)

These files will be scanned, and your name will be read by a computer, so please write clearly and darkly.

Time: 90 minutes

Content: 7 multiple choice, 5 full answer
55 points total

Clear communication is an important skill to practice, so simplify and justify all answers unless otherwise directed, show your work (except for multiple choice), and use proper notation.

Multiple Choice: *7 questions*

1. Suppose the velocity of an object at time t is $v(t)$. Which of the following gives the acceleration of the object? $\boxed{3}$

   A. The derivative of $v(t)$.

   B. The second derivative of $v(t)$.

   C. The antiderivative of $v(t)$.

   D. The antiderivative of $v(t)$, plus a constant C.

   E. None of the above.

2. Which of the following describes the point $(1, 1)$ in the function $f(x) = x^4 - 4x^3 + 4x^2$? $\boxed{3}$

   A. local minimum

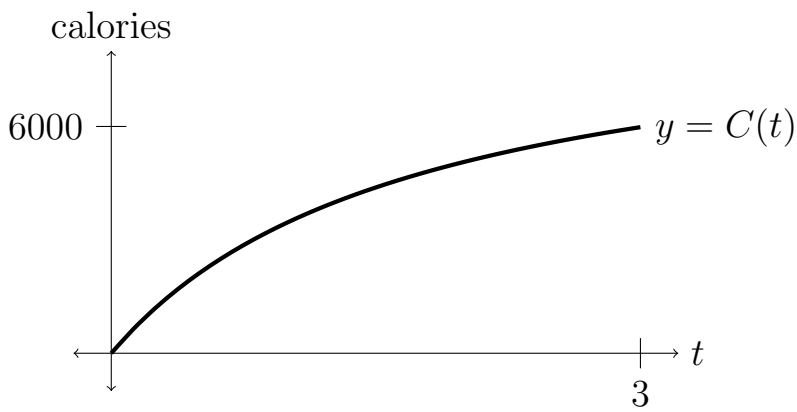
   B. local maximum

   C. inflection point

   D. root

   E. none of the above

3. The graph below represents the number of calories $C(t)$ consumed by a bear after t hours in a berry patch. $\boxed{3}$



Over the course of 3 hours, the bear consumed 6000 calories. That's 2000 calories per hour. The quantity 2000 represents the:

A. value of $C(t)$ at $t = 3$.

B. difference $C(3) - C(0)$.

C. average rate of change of $C(t)$ from $t = 0$ to $t = 3$.

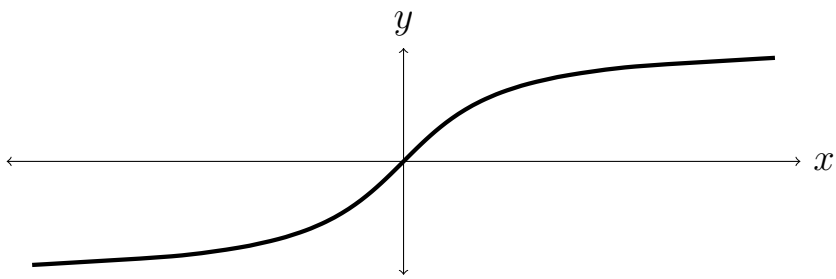
D. instantaneous rate of change of $C(t)$ from $t = 0$ to $t = 3$.

E. slope of the tangent line of $C(t)$ at $t = 3$.

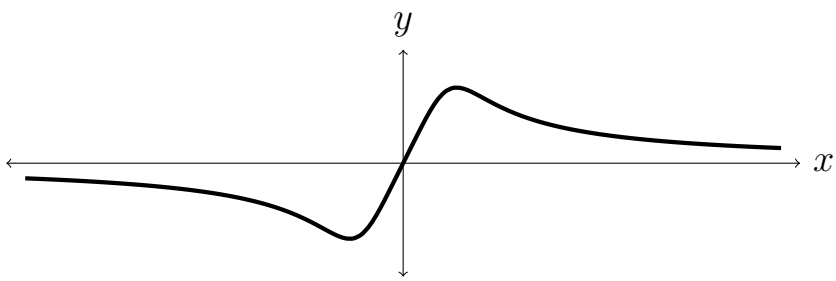
4. Which of the following looks **most** like the graph of $y = \dfrac{x}{\sqrt{x^4+4}}$? $\boxed{3}$

In the graphs below, the x-axis and y-axis might have different scales.

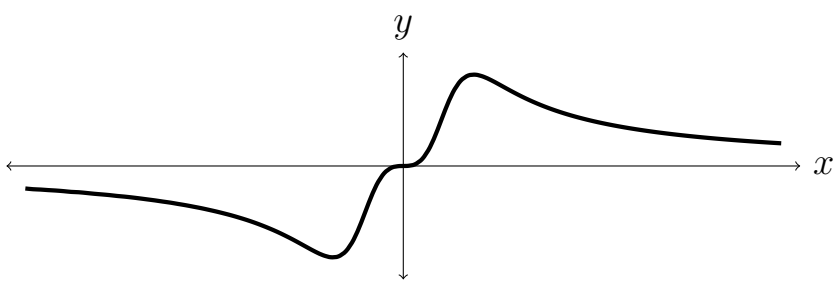
A.



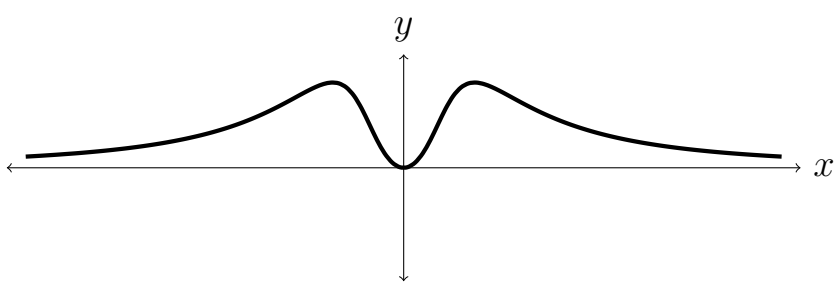
B.



C.



D.



5. The spreadsheet below will be used to approximate a root of $f(x) = x^5 + 3x^3 - 7$ using Newton's Method with $x_0 = 1$. An arrow indicates the content of a cell will be copied down its column. What should go in cells A1, B1, and C1? $\boxed{3}$

Circle the numeral next to the correct entry for each.

	A	**B**	**C**
1			
2	`=A1-B1/C1`	↓	↓
3	↓	↓	↓
4	↓	↓	↓

Entry for A1:

(i.) 0

(ii.) 1

(iii.) `=A1^5+3*A1^3-7`

(iv.) `=B1^5+3*B1^3-7`

(v.) `=5*A1^4+9*A1^2`

(vi.) `=5*B1^4+9*B1^2`

Entry for B1:

(i.) 0

(ii.) 1

(iii.) `=A1^5+3*A1^3-7`

(iv.) `=B1^5+3*B1^3-7`

(v.) `=5*A1^4+9*A1^2`

(vi.) `=5*B1^4+9*B1^2`

Entry for C1:

(i.) 0

(ii.) 1

(iii.) `=A1^5+3*A1^3-7`

(iv.) `=B1^5+3*B1^3-7`

(v.) `=5*A1^4+9*A1^2`

(vi.) `=5*B1^4+9*B1^2`

6. Which of the following statements is true for **any** differentiable function f? If more than one statement is true, choose (E). $\boxed{3}$

A. If the graph of $f'(x)$ looks like the line with equation $y = 3x$ when zooming in at $x = 0$, then $f(x)$ has a local minimum at 0.

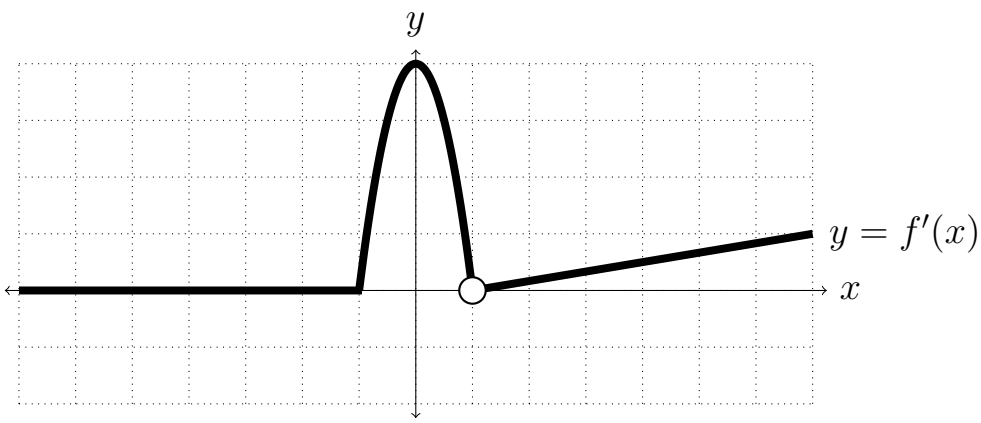
B. If $f''(\pi) = 0$, then the concavity of $f(x)$ changes at π.

C. If $f'(x)$ is continuous, then $f''(x)$ exists.

D. If the function $f(x)$ considered on the interval $2 \leq x \leq 3$ has an absolute minimum over this interval, then $f'(x)$ has a zero.

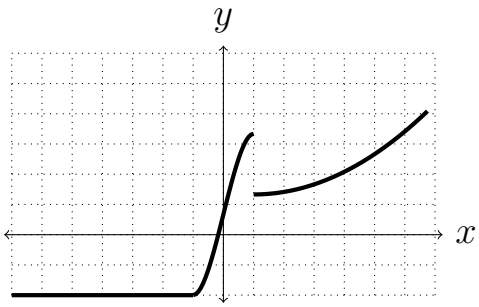
E. More than one of the above statements is true.

7. Suppose $f(x)$ is a function whose derivative $f'(x)$ is shown below. $\boxed{3}$

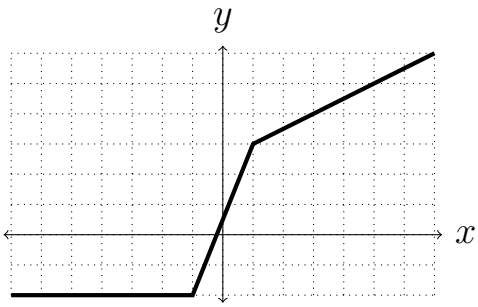


Which of the following graphs could be $f(x)$?

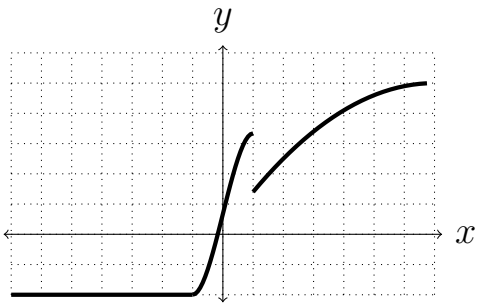
A.



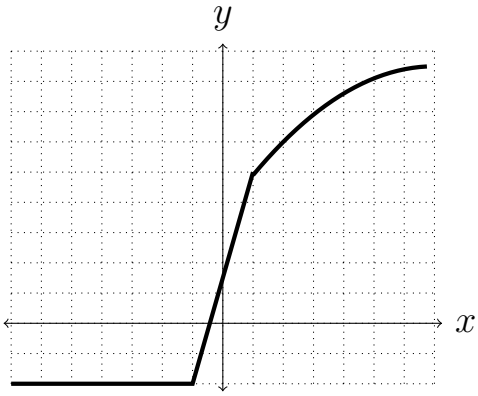
B.



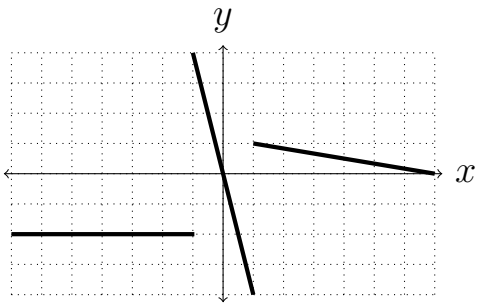
C.



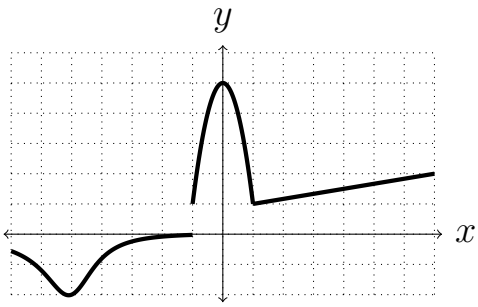
D.



E.



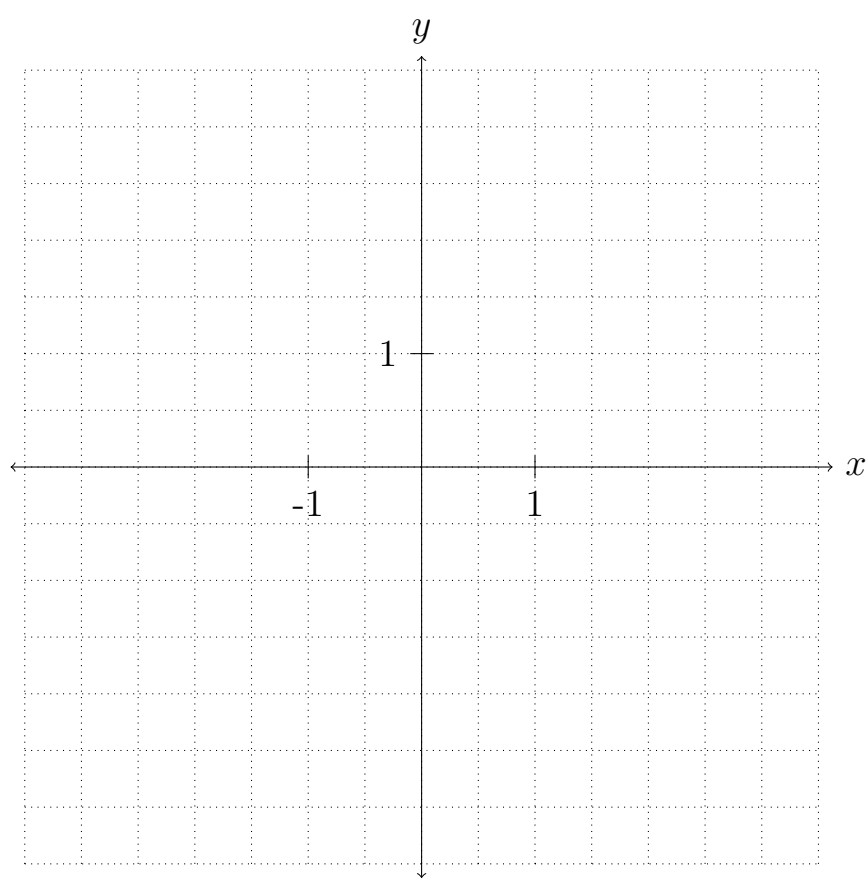
F.



Full Answer: *5 questions*

8. Sketch the graph $y = f(x) = \frac{1}{2}x^2\left(1 - \frac{1}{3}x\right)$. $\boxed{7}$
Your graph should include all roots, local extrema, and inflection points, at the appropriate x- and y-values.
You may use the following without proving them:

- $f'(x) = x\left(1 - \frac{1}{2}x\right)$
- $f''(x) = 1 - x$



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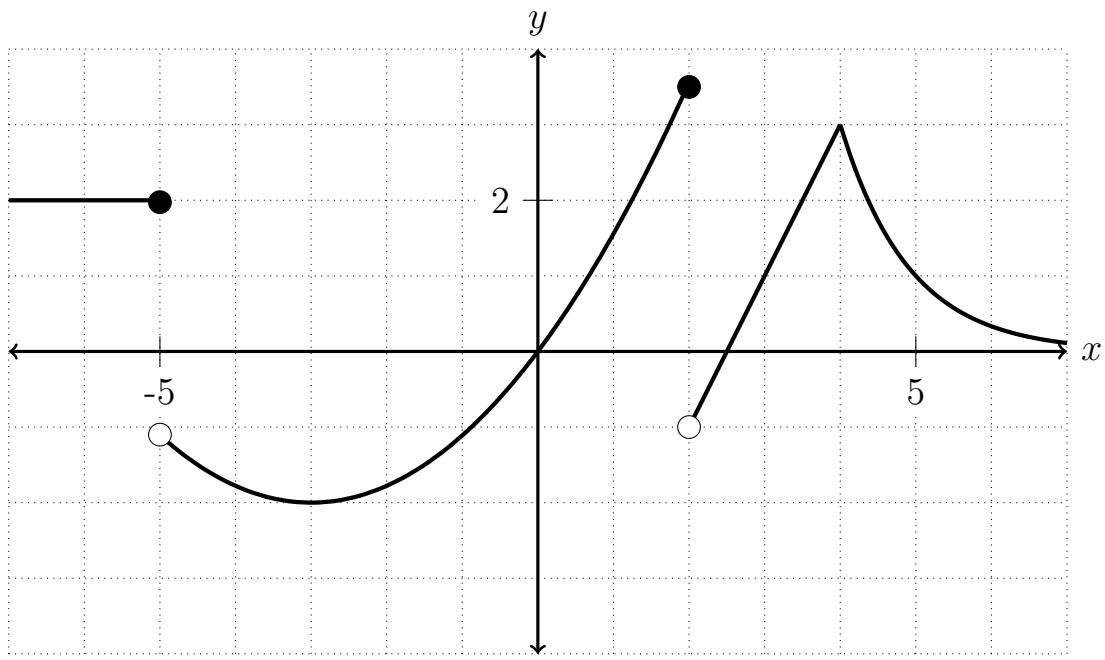
9. Shown below is the graph $y = f(x)$. Compute the five limits below, or write "DNE" if they don't exist. $\boxed{5}$

You do not need to justify your answers for this problem.



$$\lim_{x \to 0} f(x) = \boxed{}$$

$$\lim_{x \to -5^+} f(x) = \boxed{}$$

Note x goes to -5 *from the right*

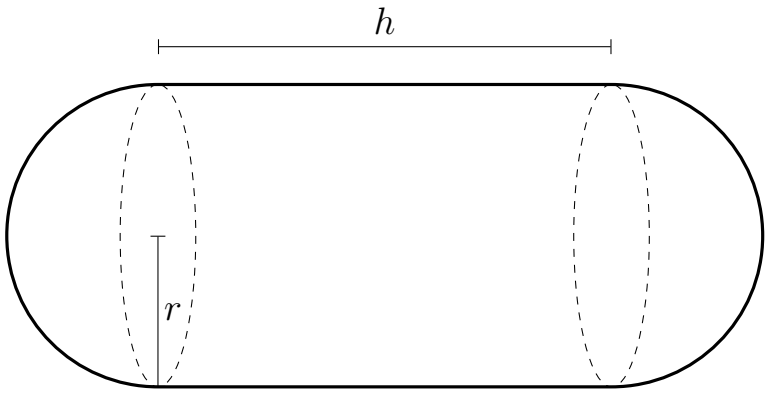
$$\lim_{x \to 2} f(x) = \boxed{}$$

$$\lim_{h \to 0^-} \frac{f(4+h) - f(4)}{h} = \boxed{}$$

Note h goes to 0 *from the left*

$$\lim_{h \to 0} \frac{f(h-3) - f(-3)}{h} = \boxed{}$$

10. A cell of the bacterium E. coli has the shape of a cylinder with two hemispherical caps, as shown below. Consider this shape, with h the height of the cylinder, and r the radius of the cylinder and hemispheres. $\boxed{8}$



(a) Find the value of h that leads to the largest volume for a fixed constant surface area, S= constant. (6 points)

(b) Describe or sketch the shape you found in (a). (1 point)

(c) A typical E. coli cell has $h = 1\mu$m and $r = 0.5\mu$m. Based on your results in (a) and (b), would you agree that E. coli has a shape that maximizes its volume for a fixed surface area? Explain your answer in one sentence. (1 point)

Note: 1 μm = 10^{-6} m.

The cylindrical part of the shape in the figure has volume $V = \pi r^2 h$ and surface area $S = 2\pi r h$. A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.

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11. A fox and a hare are running. At time $t = 0$, they are at the same position. $\boxed{8}$

The fox's velocity is a constant v, where $0 < v < 4$.

The hare's position at time t is given by

$$H(t) = \frac{12t}{3+t}$$

(a) What is the position of the fox at time t, $F(t)$? (Note it starts at the same place as the hare.) (2 points)

(b) Just after $t = 0$, the hare is ahead of the fox. When do the fox and hare meet again next? (2 points)

(c) When is the distance between the two animals increasing, and when is it decreasing, between $t = 0$ and their meeting from part (b)? (4 points)

This page left blank for your work on Question 11.

12. (a) Use a linear approximation to find a reasonable rational number approximating $\sqrt{98}$. Simplify your answer. (4 points) $\boxed{6}$

(b) Let $c > \sqrt{2}$. Use a linear approximation to find a reasonable number approximating $\sqrt{c^2 - 2}$, in terms of c. Simplify your answer. (2 points)

13. Predict your score: fill in the appropriate box. 2 (bonus)

43-55 points	29-42 points	15-28 points	0-14 points

If your score (excluding the bonus points for this question!) falls within the selected range, you'll get two bonus points. The purpose of this question is to encourage you to think critically about your own understanding. It is possible to earn 100% on this exam without answering this question.