Today

- Composition and chain rule, quotient rule
- Antiderivatives of power functions and polynomials
- Tangent lines
- Reminders:
  - Assignment3 Thursday 7am,
  - OSH 2 Friday 11:59 pm.
  - Sign up for midterm time/room.
Composition of functions

If \( f(x) = 2x+3 \) and \( g(x) = -4x+2 \),

A. \( h(x) = f( g(x) ) = -8x+7 \)

B. \( h(x) = f( g(x) ) = -8x-10 \)

C. \( h(x) = f( g(x) ) = -8x^2-8x+6 \)

D. \( h(x) = f( g(x) ) = -8x+5 \)
Composition of functions

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Composition of functions

If \( h(x) = f(g(x)) \), then

A. \( h'(x) = f'(x) \cdot g'(x) \)

B. \( h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \)

C. \( h'(x) = f'(g'(x)) \)

D. \( h'(x) = f'(g(x)) \cdot g'(x) \)
Composition of functions

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C. \( h'(x) = f'(g'(x)) \)

D. \( h'(x) = f'(g(x)) \cdot g'(x) \quad \text{----- Chain Rule} \)
Composition of functions

If \( h(x) = (x^3-2x+1)^6 \), then \( h'(x) = ? \)

A. \( 6 (x^3-2x+1)^5 \)

B. \( (x^3-2x+1)^6 (3x^2-2) \)

C. \( 6 (x^3-2x+1)^5 (3x^2-2) \)

D. \( 6 (x^3-2x+1)^5 (x^3-2x+1) \)

E. Are you kidding? It will take me weeks to multiply those out.
Composition of functions

If \( h(x) = (x^3 - 2x + 1)^6 \), then \( h'(x) = \) ?

A. \( 6 \ (x^3 - 2x + 1)^5 \)

B. \( (x^3 - 2x + 1)^6 \ (3x^2 - 2) \)

C. \( 6 \ (x^3 - 2x + 1)^5 \ (3x^2 - 2) \)

D. \( 6 \ (x^3 - 2x + 1)^5 \ (x^3 - 2x + 1) \)

E. Are you kidding? It will take me weeks to multiply those out.
Rules for differentiation - summary

• Addition rule:
  \[ f(x) = g(x) + h(x) \quad ----> \quad f'(x) = g'(x) + h'(x) \]

• Product rule:
  \[ f(x) = g(x) h(x) \quad ----> \quad f'(x) = g'(x) h(x) + g(x) h'(x) \]

• Chain rule:
  \[ f(x) = g(h(x)) \quad ----> \quad f'(x) = g'(h(x)) h'(x) \]

• Quotient rule:
  \[ f(x) = \frac{g(x)}{h(x)} = g(x) (h(x))^{-1} \quad <---- \text{apply product and chain rules} \]
or
  \[ f(x) = g(x) / h(x) = g(x) (h(x))^{-1} \quad <---- \text{apply product and chain rules} \]
Suppose \( f(x) = g(x)/k(x) \) and that
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

• What is \( f'(2) \)?

   (A) -13

   (B) -13/25

   (C) -13/9

   (D) 17/25
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• What is \( f'(2) \)?

(A) -13
(B) -13/25
(C) -13/9
(D) 17/25
Antiderivatives – going backward

If \( f'(x) = 6x^2 + 4x - 1 \), then

(A) \( f(x) = 12x + 4 \)

(B) \( f(x) = 2x^3 + 2x^2 - x \)

(C) \( f(x) = 2x^3 + 2x^2 - x + 2 \)

(D) \( f(x) = 2x^3 + 2x^2 - x + C \)
Antiderivatives – going backward

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(A) \( f(x) = 12x + 4 \)

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(D) \( f(x) = 2x^3 + 2x^2 - x + C \)

Slopes at each \( x \) don't change with vertical shift.
This is $f'(x)$. Draw $f(x)$.

Only determined up to a vertical shift.
Position-Velocity-Acceleration

If $x(t)$ is position as a function of time,

- velocity $v(t) = x'(t)$,
- acceleration $a(t) = v'(t) = x''(t)$.

Constant acceleration $a$:

- $v(t) = at + C = at + v_0$ so that $v(0) = v_0$.
- $x(t) = a/2 \ t^2 + v_0t + D = a/2 \ t^2 + v_0t + x_0$

Classic “projectile motion” (ball falling)
Tangent lines – simple ex

• Let \( f(x) = x^3 + 2x^2 - x + 2 \).

• Find tangent line at \( x=3 \).

• Need equation of line
  
  • slope is \( m = f'(3) \), point on line is \( (3,f(3)) \)
  
  • Either \( y = mx + b \) or \( y = m(x-a) + f(a) \)...

(A) \( y = 3x + 44 \)  
(B) \( y = 38x + 44 \)  
(C) \( y = 38(x-3) + 44 \)  
(D) \( y = 44 \)
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(D) \( y = 44 \)