Math 102, Section 103, Quiz 2
Friday, Oct 6

Clear communication is an important skill to practice, so simplify and justify all answers unless otherwise directed, show your work, and use proper notation.

1. The acceleration of an object at time t is given by $\boxed{3}$

$$a(t) = 20t^3 - 6t$$

(a) Give a general equation for the position of the object.

(b) At time $t = \dfrac{1}{5}$, is its velocity increasing or decreasing, or is there not enough information to know?

Solution: (a) Since acceleration is the second derivative of position, to find the equation of the position, we antidifferentiate twice.

$$\begin{aligned} a(t) &= 20t^3 - 6t \\ v(t) &= 5t^4 - 3t^2 + C \\ s(t) &= \boxed{t^5 - t^3 + Ct + D} \end{aligned}$$

where C and D are some constants.

(b) At time $t = \frac{1}{5}$, the acceleration is $\frac{20}{5^3} - \frac{6}{5} = \frac{4}{25} - \frac{30}{25} < 0$. Since acceleration is the derivative of velocity, and acceleration is negative, the velocity is $\boxed{\text{decreasing}}$.

2. Use two iterations of Newton's Method, starting with $x_0 = 0$, to approximate a root of $f(x) = x^3 + x + 1$. $\boxed{2}$

Solution: Recall Newton's method works with the iteration $x_{k+1} = \dfrac{f(x_k)}{f'(x_k)}$.

$$f'(x) = 3x^2 + 1$$
$$x_1 = 0 - \frac{f(0)}{f'(0)} = -\frac{1}{1} = -1$$
$$x_2 = -1 - \frac{f(-1)}{f'(-1)} = -1 - \frac{-1}{4} = \boxed{-\frac{3}{4}}$$

3. Use a linear approximation to find a reasonable rational number (i.e. a fraction of integers) approximating $\sqrt[5]{30}$. $\boxed{3}$

Solution: We use the equation $f(x) = x^{1/5}$, and the point $(32, 2)$, to find our tangent line approximation.

- $f'(x) = \frac{1}{5}x^{-4/5}$
- $f'(32) = \frac{1}{5 \cdot 2^4} = \frac{1}{5 \cdot 16} = \frac{1}{80}$
- So, the tangent line to the curve $y = f(x)$ when $x = 32$ has equation $y - 2 = \frac{1}{80}(x - 32)$
- Then we approximate $f(30) \approx 2 + \frac{1}{80}(30 - 32) = 2 - \frac{1}{40} = \frac{79}{40}$

4. The function $f(x) = \text{arcsec(x)}$ is differentiable, with $f'(x) = \dfrac{1}{\sqrt{x^4 - x^2}}$. $\boxed{2}$

Using this knowledge, find the derivative of $g(x) = \text{arcsec}\big(x^2 + x\big)$.
You do not need to simplify your answer.

Note: you're not crazy–we ***have not*** *talked about arcsecant in class. All you need to know about arcsecant to solve this problem is its derivative.*

Solution: Using the chain rule,

$$g'(x) = \frac{1}{\sqrt{(x^2+x)^4 - (x^2+x)^2}} \cdot (2x+1)$$

5. Which of the previous questions did you get **completely right**? Fill in the appropriate box(es). 2 (bonus)

#1 #2 #3 #4

☐ I didn't get any of the questions completely right.