Today

- No class on Nov 29 - clicker poll for alternate review date.
- Linear approximation
- Newton's method

No class on Nov 29

- I prefer to have a review session on

(A) Dec 4

(B) Dec 5

(C) Dec 6

(D) Dec 9

(E) Dec 10

No class on Nov 29

- I prefer to have a review session on

(A) Dec 4

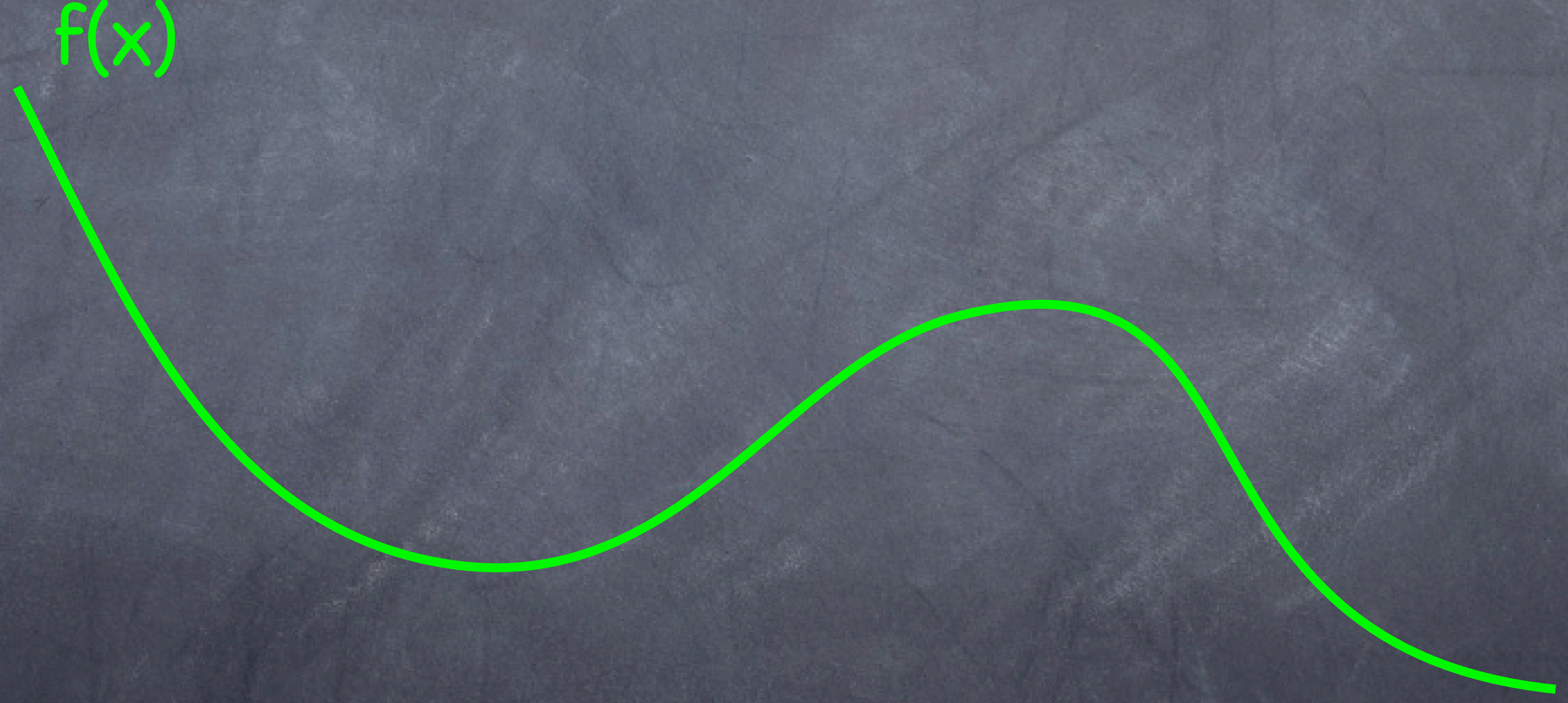
(B) Dec 5

(C) Dec 6 @ 1 pm

(D) Dec 9 @ 10 am

(E) Dec 10

Linear approximation



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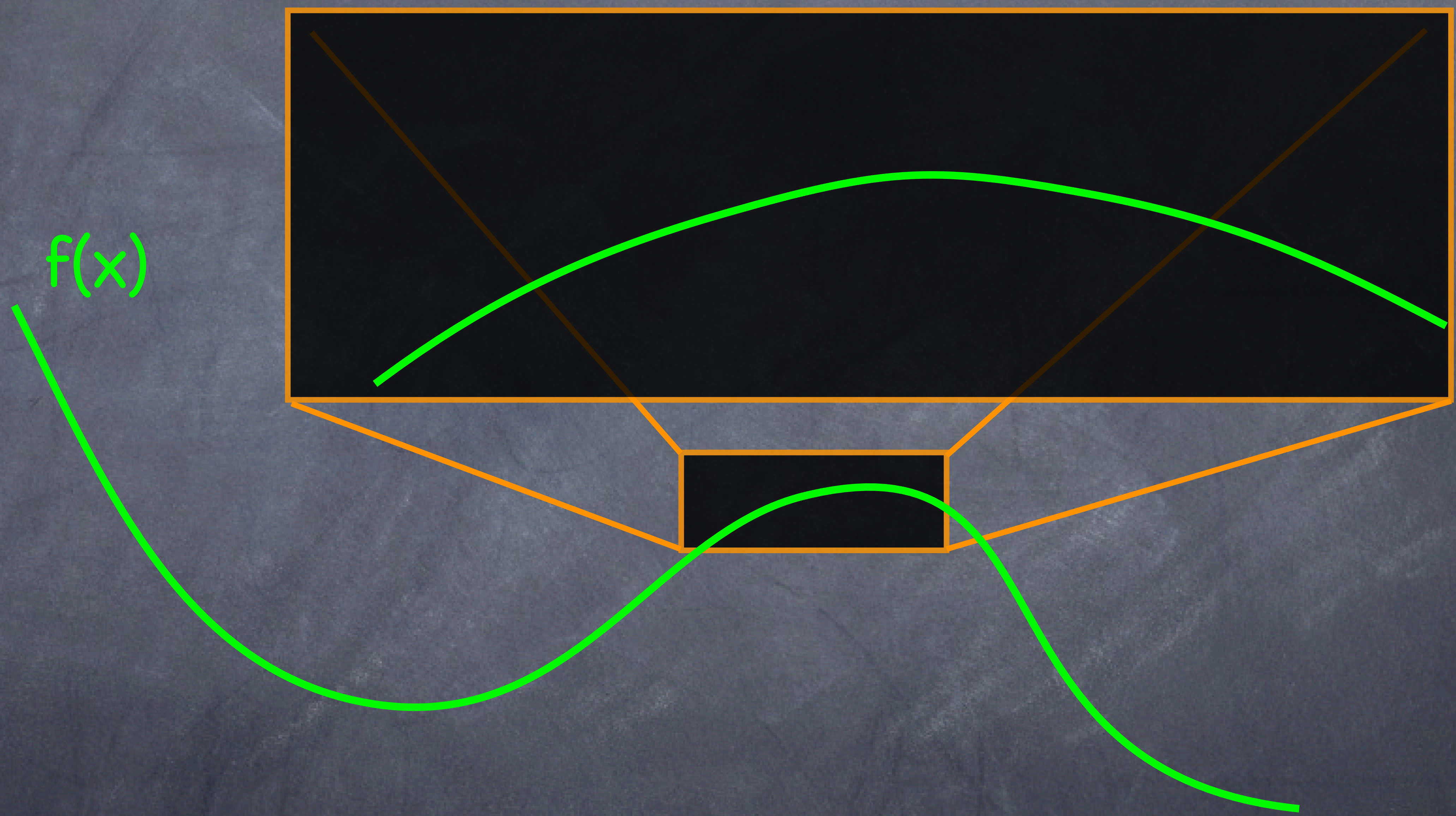
If a is near b and both f(a) and f'(a) are easy to calculate, use tangent line to approximate f(b).

Linear approximation



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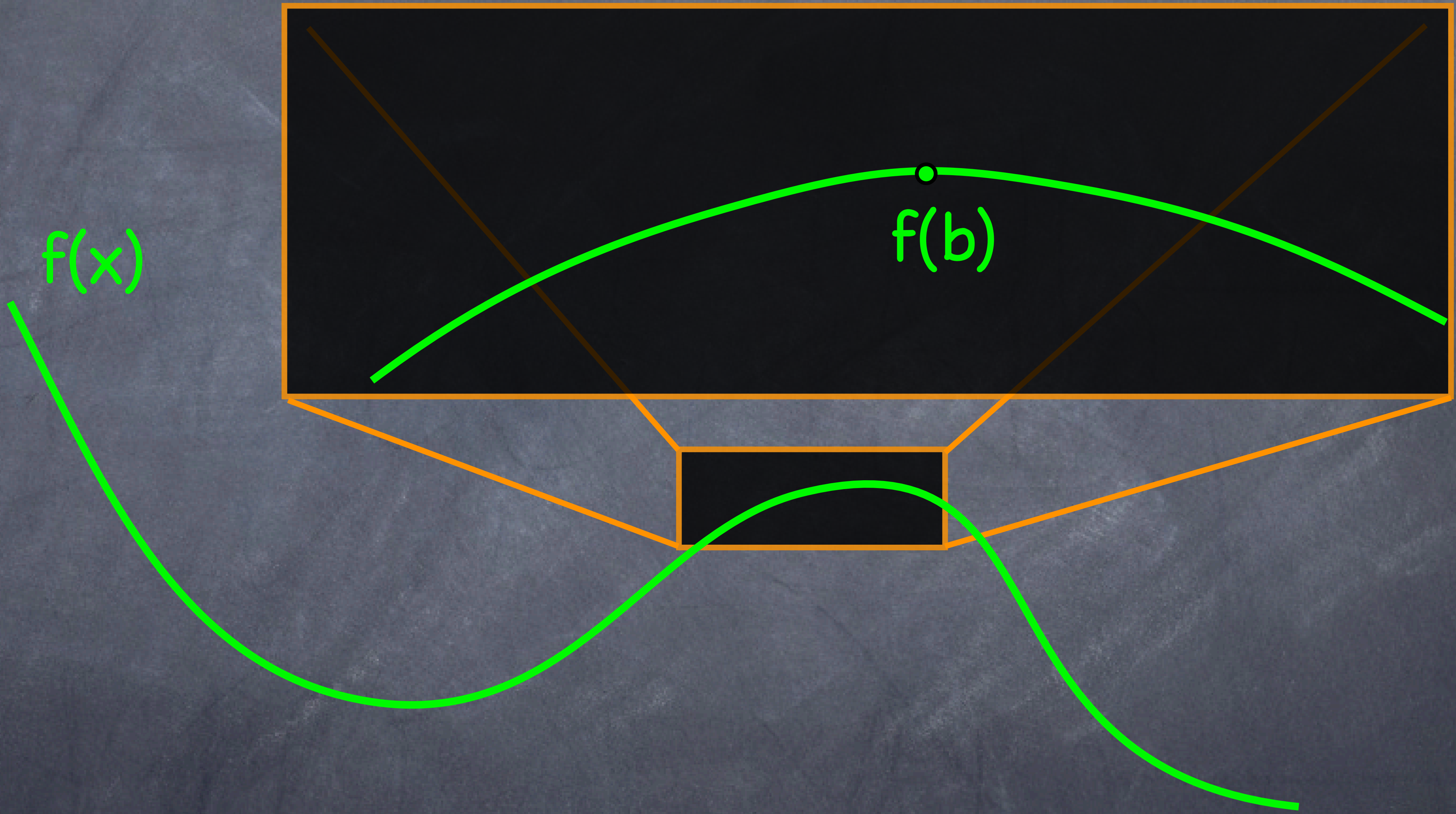

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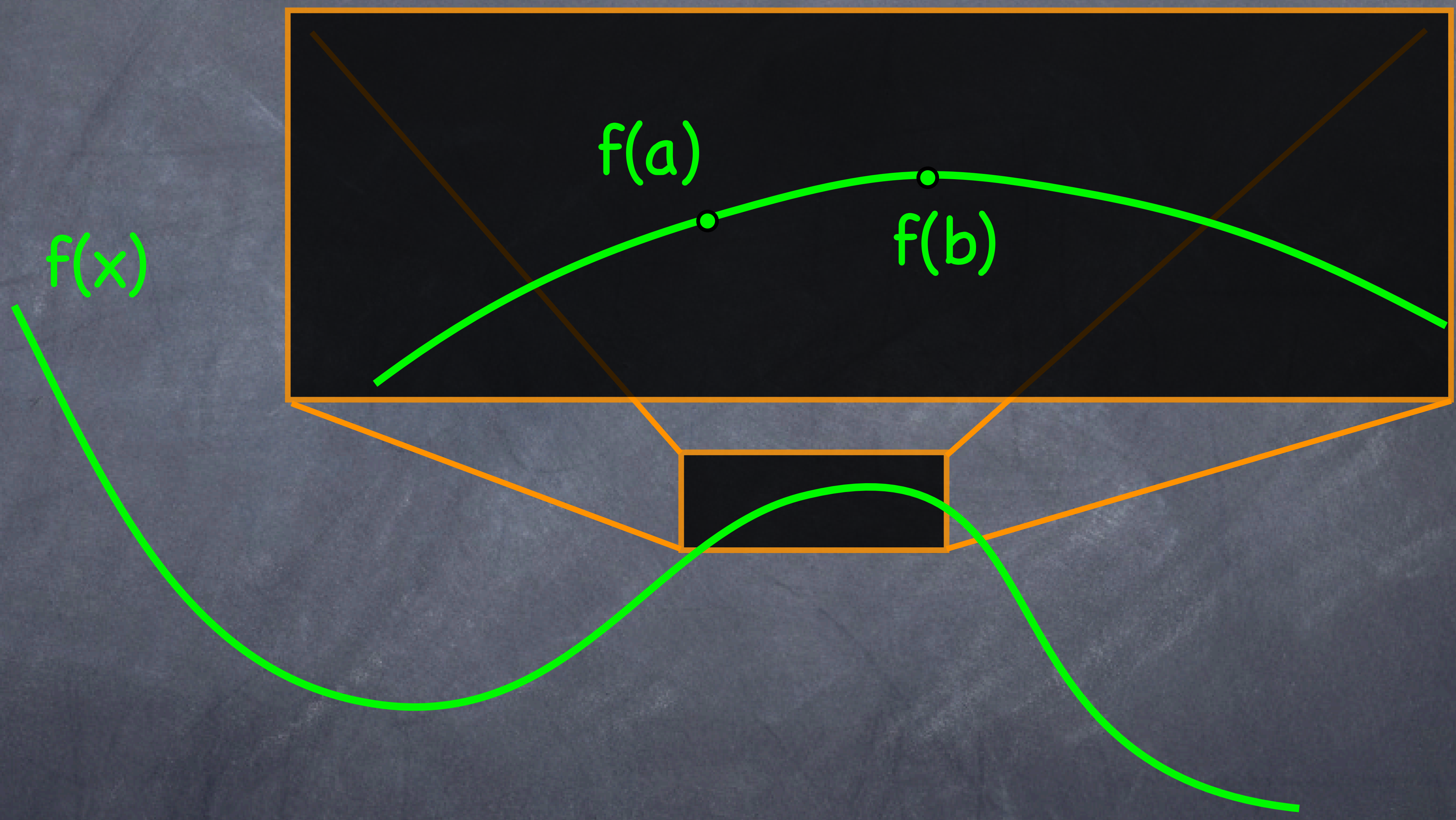

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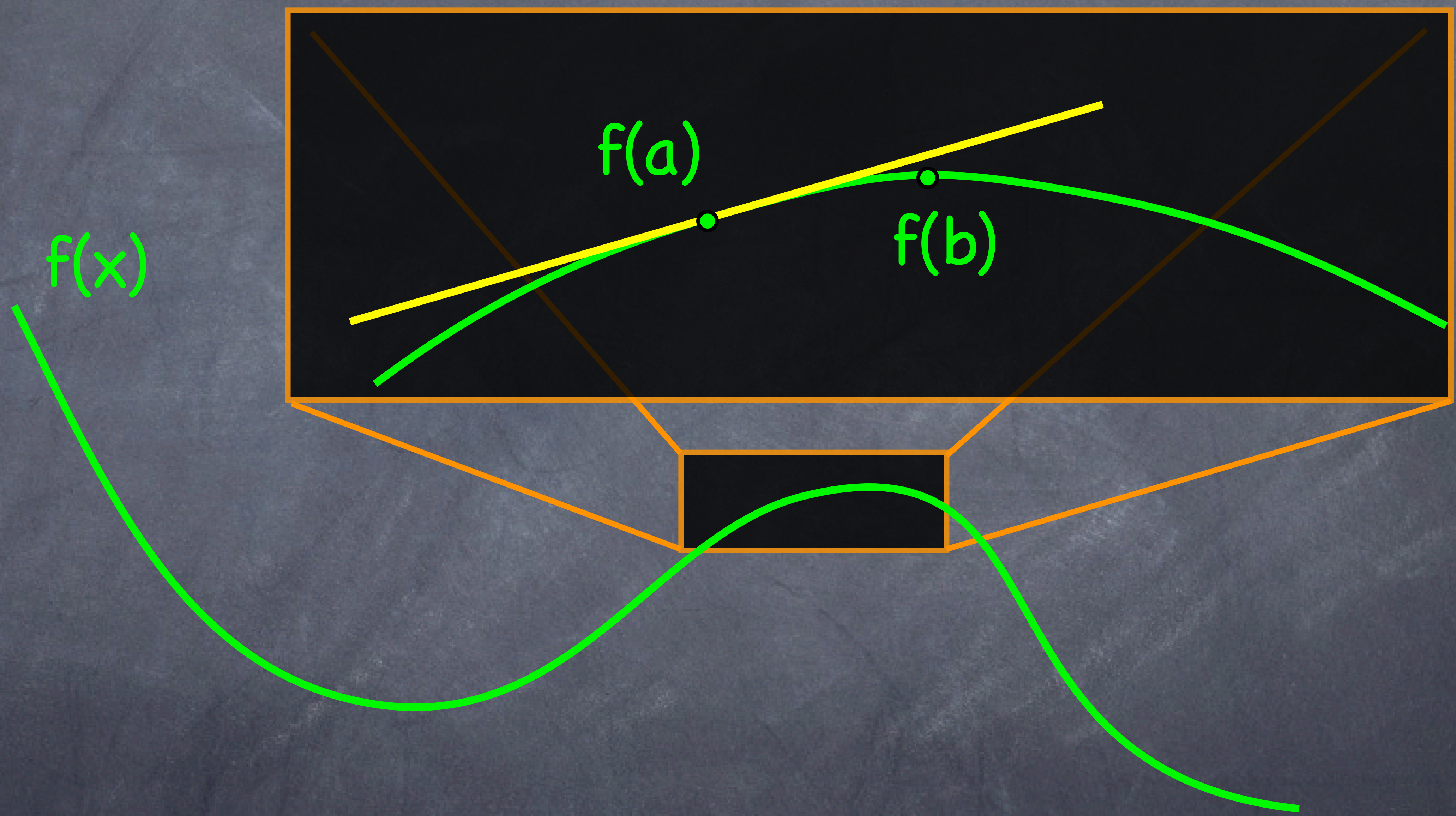

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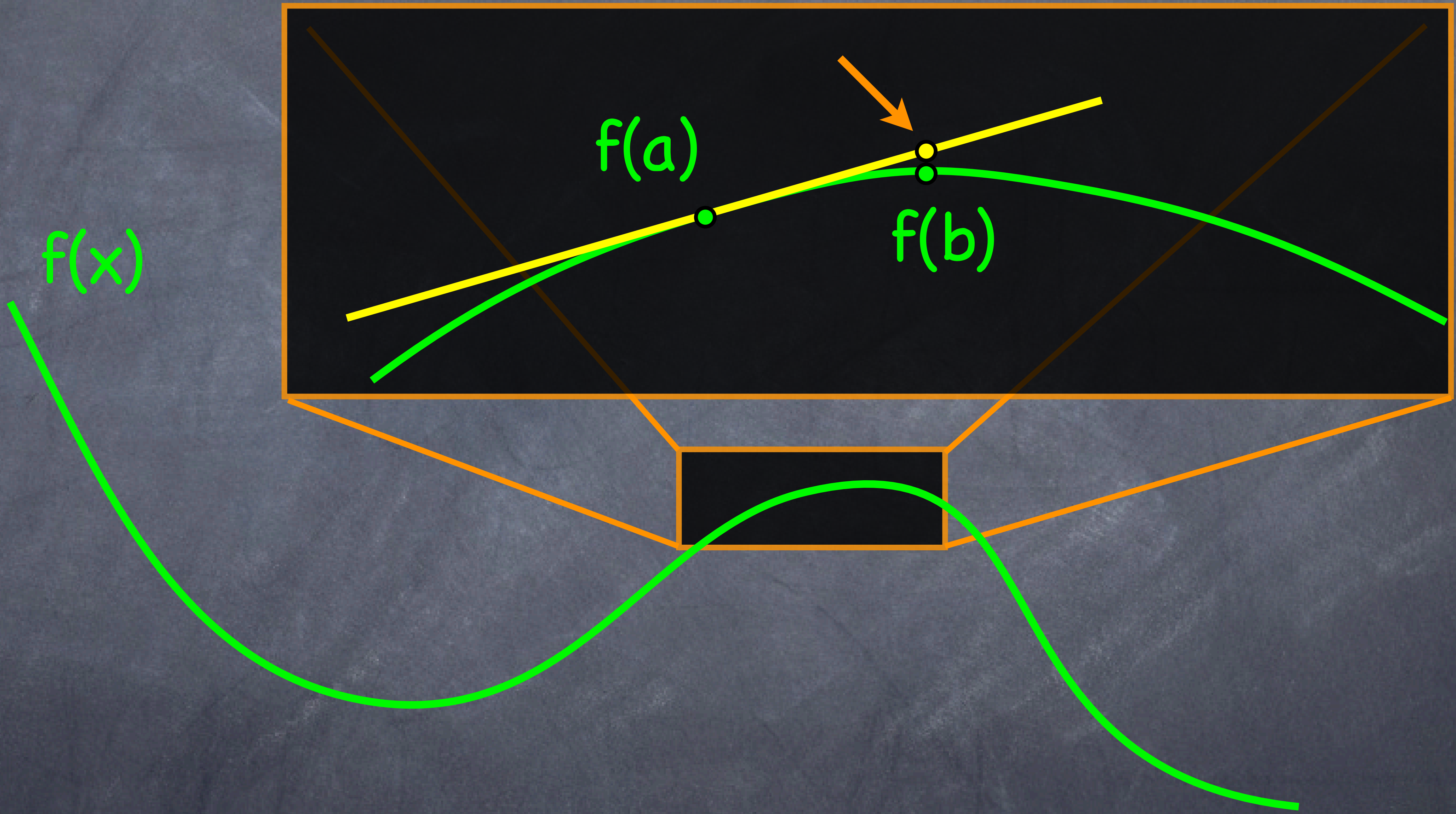

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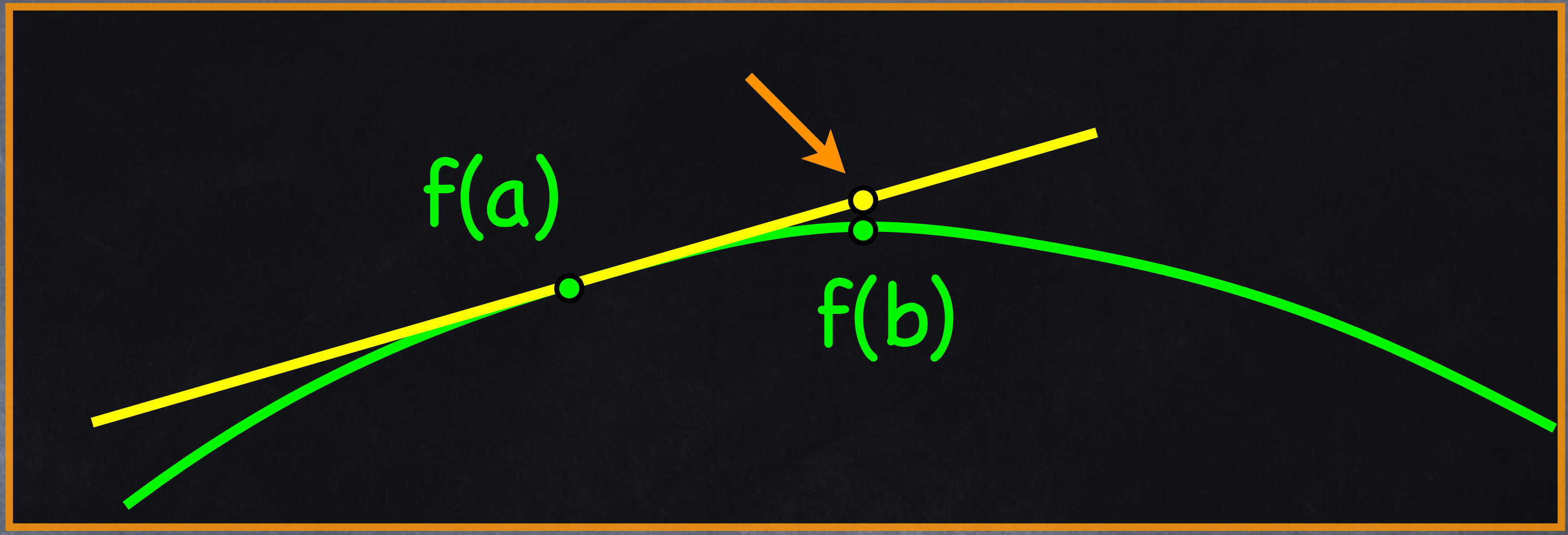

(A) $f(b) \approx f(b)+f'(b)(x-b)$

(B) $f(b) \approx f(a)+f'(a)(x-a)$

(C) $f(b) \approx f(a)+f'(a)(b-a)$

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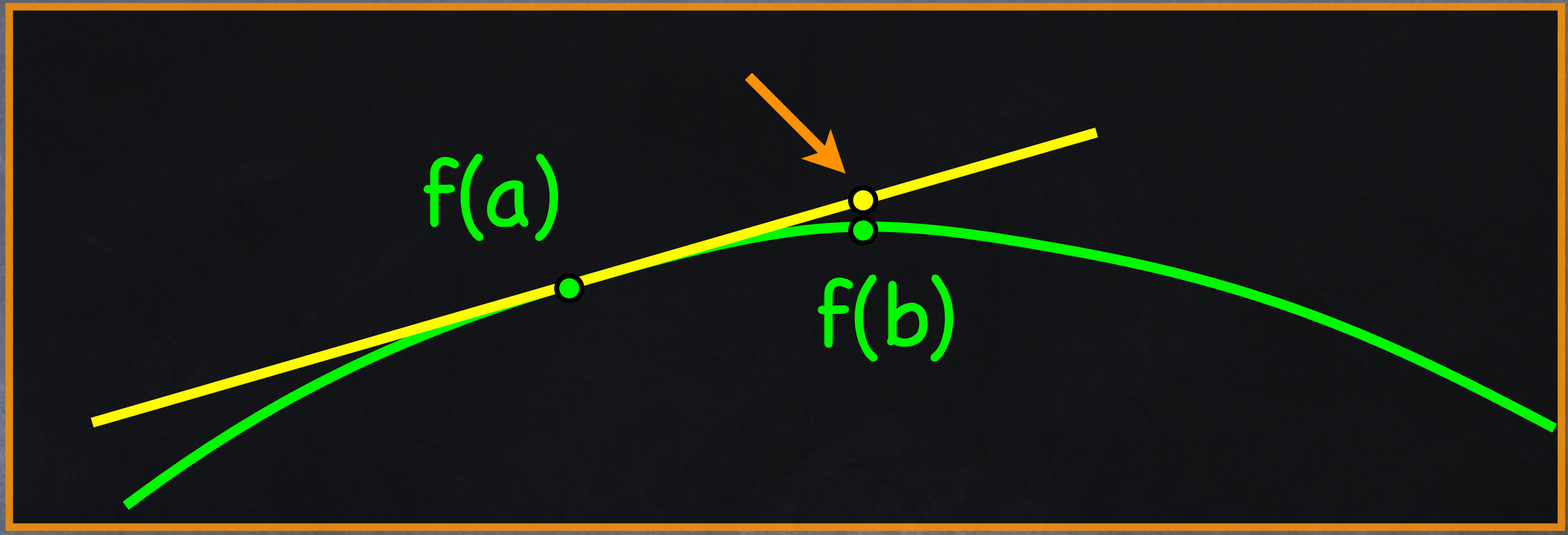
(E) $f(a) \approx f(b)+f'(b)(x-b)$

Linear approximation



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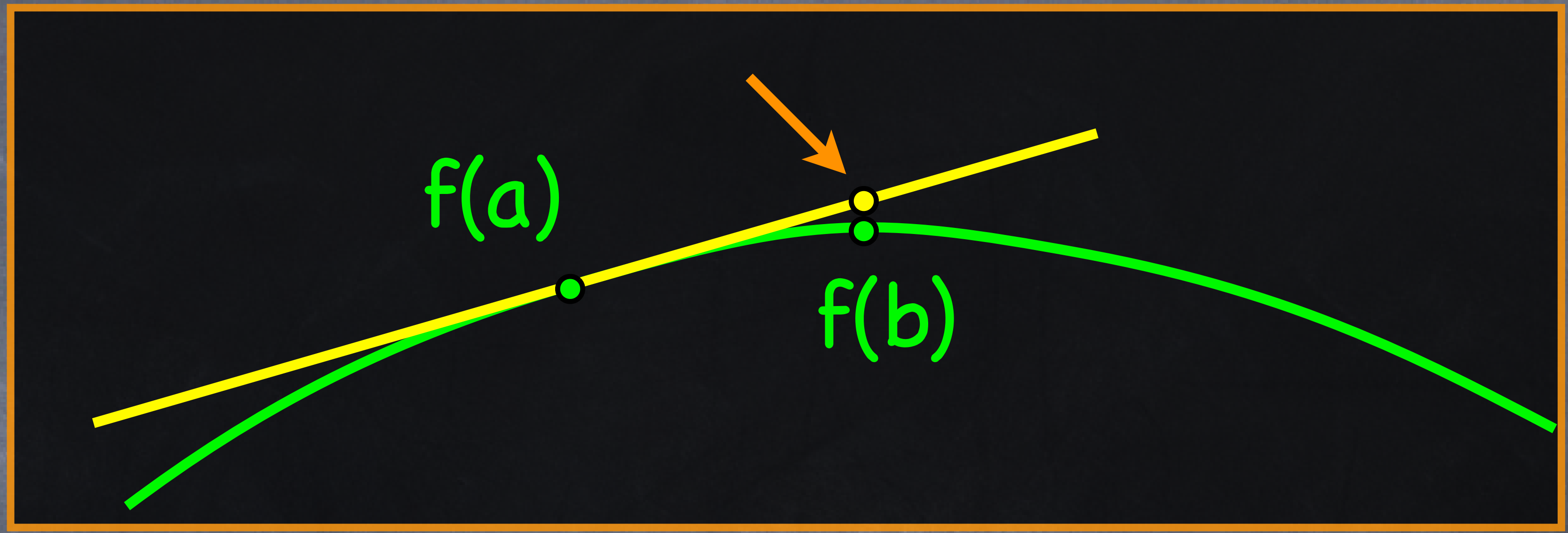

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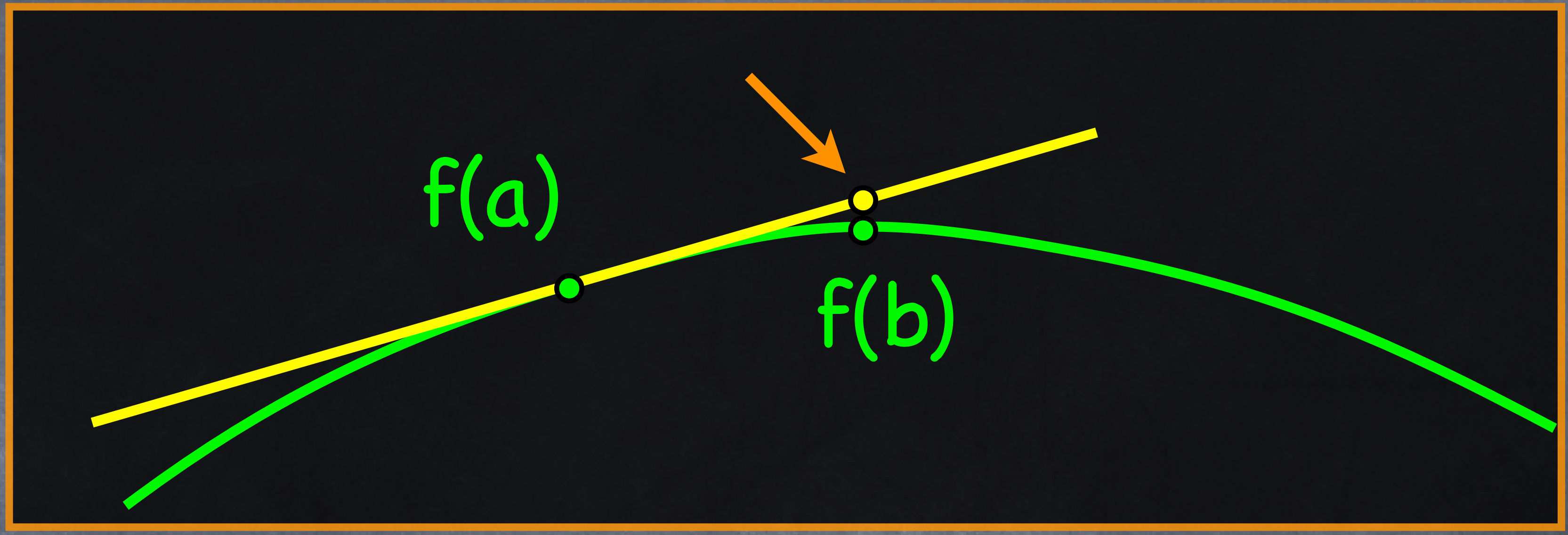
$$L(x) = f(a)+f'(a)(x-a)$$

Linear approximation



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Use linear approximation to estimate $\sqrt{99}$

(A) 9.94

(B) 9.95

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- $f(x) = x^{1/2}$.

Use linear approximation to estimate $\sqrt{99}$

(A) 9.94

(B) 9.95

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(E) 9.98

- $f(x) = x^{1/2}$.
- b=99.

Use linear approximation to estimate $\sqrt{99}$

(A) 9.94

(B) 9.95

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- $f(x) = x^{1/2}$.
- b=99.
- a=100.

Use linear approximation to estimate $\sqrt{99}$

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- $f(x) = x^{1/2}$.
- b=99.
- a=100.
- $f(b) \approx f(a) + f'(a)(b-a)$

Use linear approximation to estimate $\sqrt{99}$

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- $f(x) = x^{1/2}$.
- b=99.
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- $f(b) \approx f(a) + f'(a)(b-a)$

  $\approx 10 + 1/20\ (99-100) = 9.95$.

Use linear approximation to estimate sin(3)

(A) 0

(B) 3

(C) π

(D) 0.141120...

(E) 0.14159...

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- $f(x) = \sin(x)$.
- $b = 3$.
- $a = \pi$.
- $f(b) \approx f(a) + f'(a)(b-a)$

  $\approx 0 + (-1)(3-\pi) = 0.14159...$

Use linear approximation to estimate $0.03^{1/3}$

(A) 0

(B) 28/90

(C) 79/240

(D) 0.310723

(E) infinity

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Newton's method

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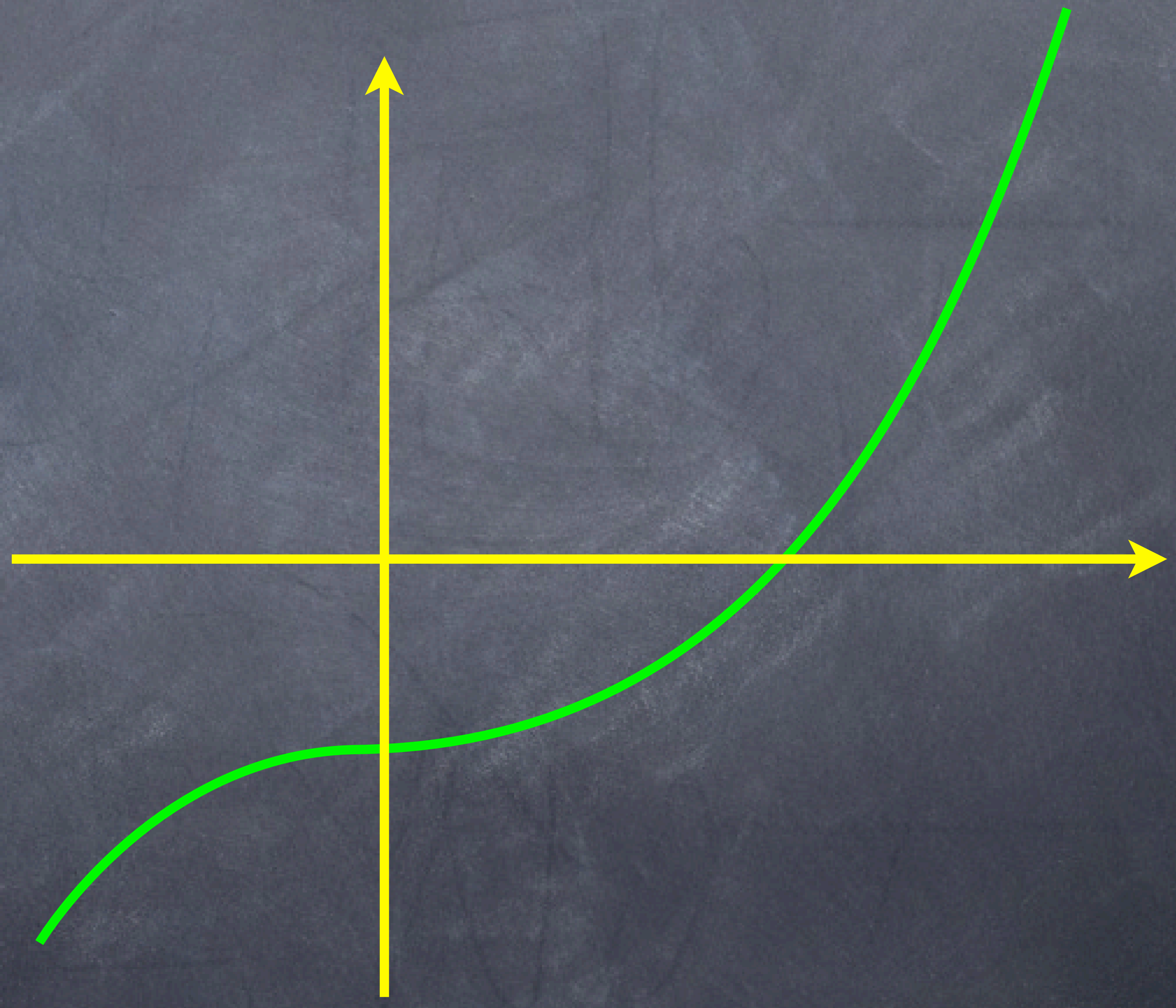
    - define $f(x)=x^2-2$.

Find the zero of $f(x)=x^3-2$.



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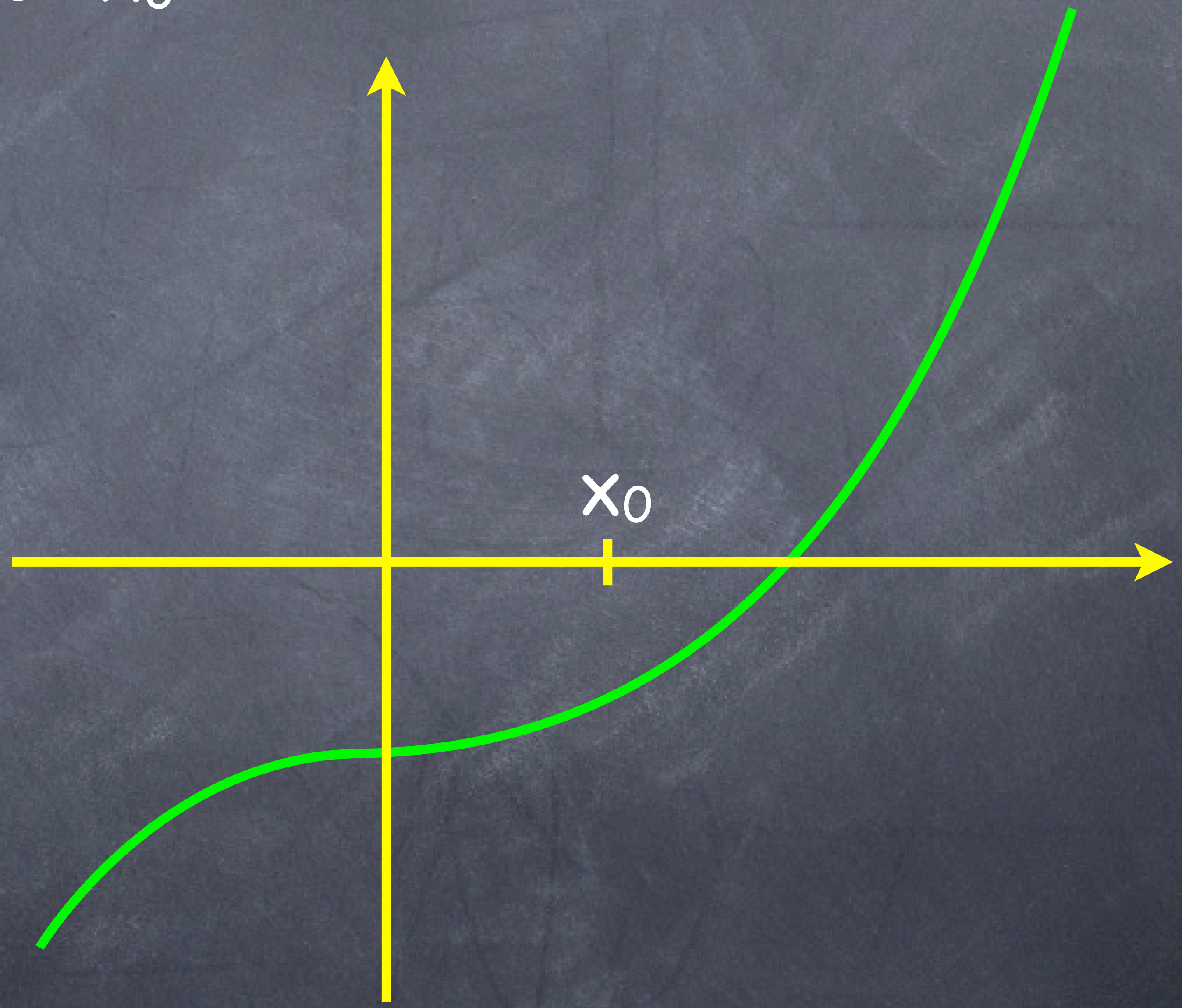

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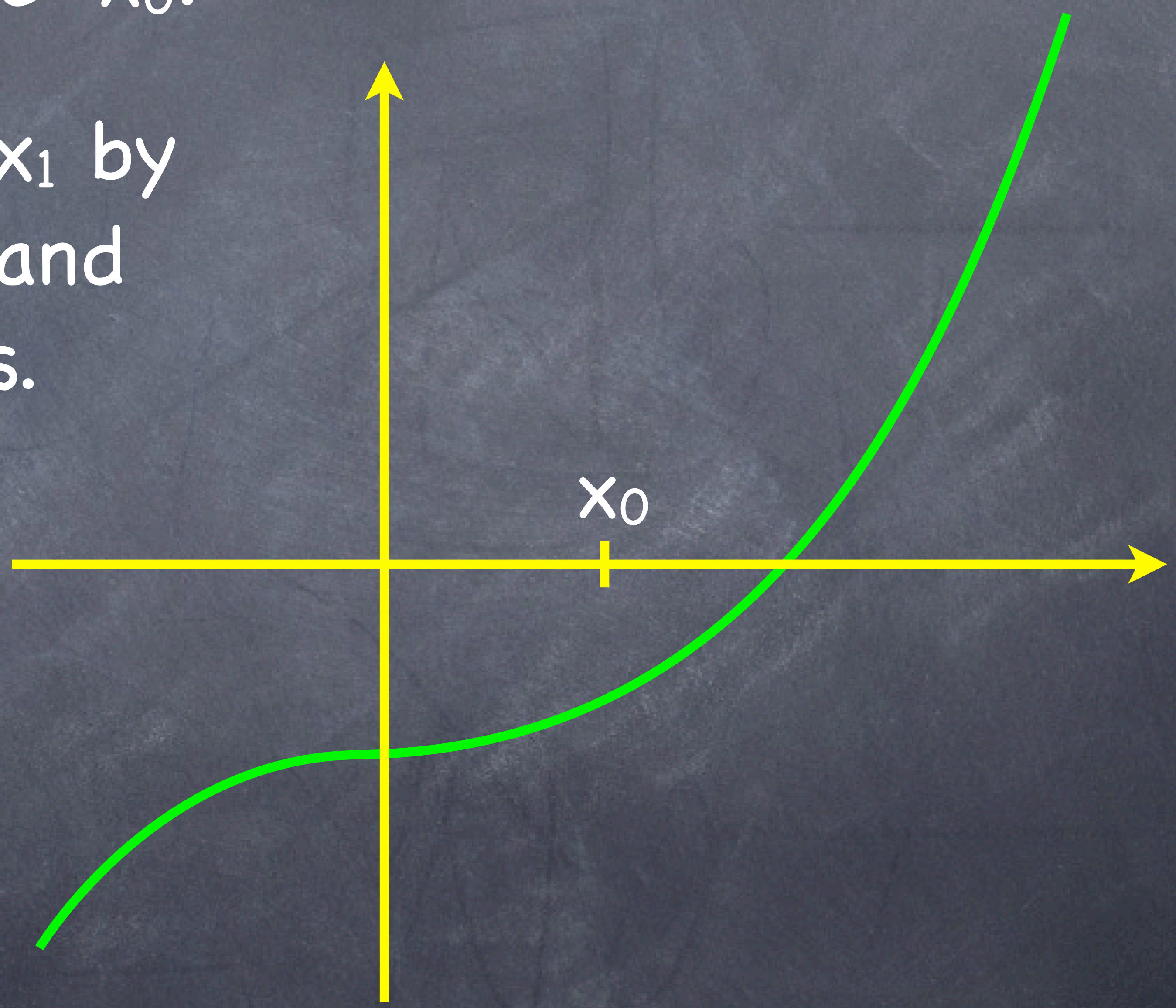

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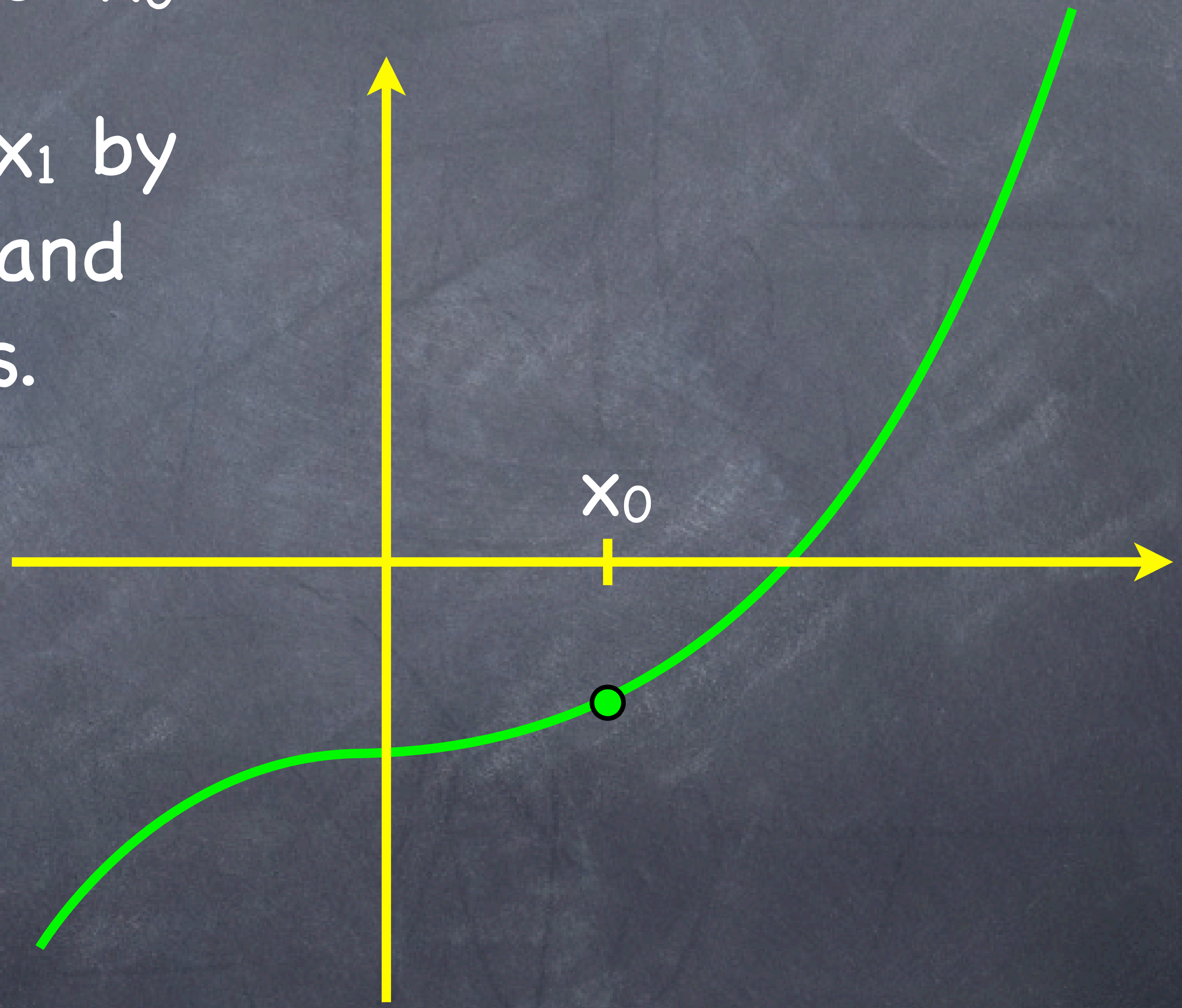

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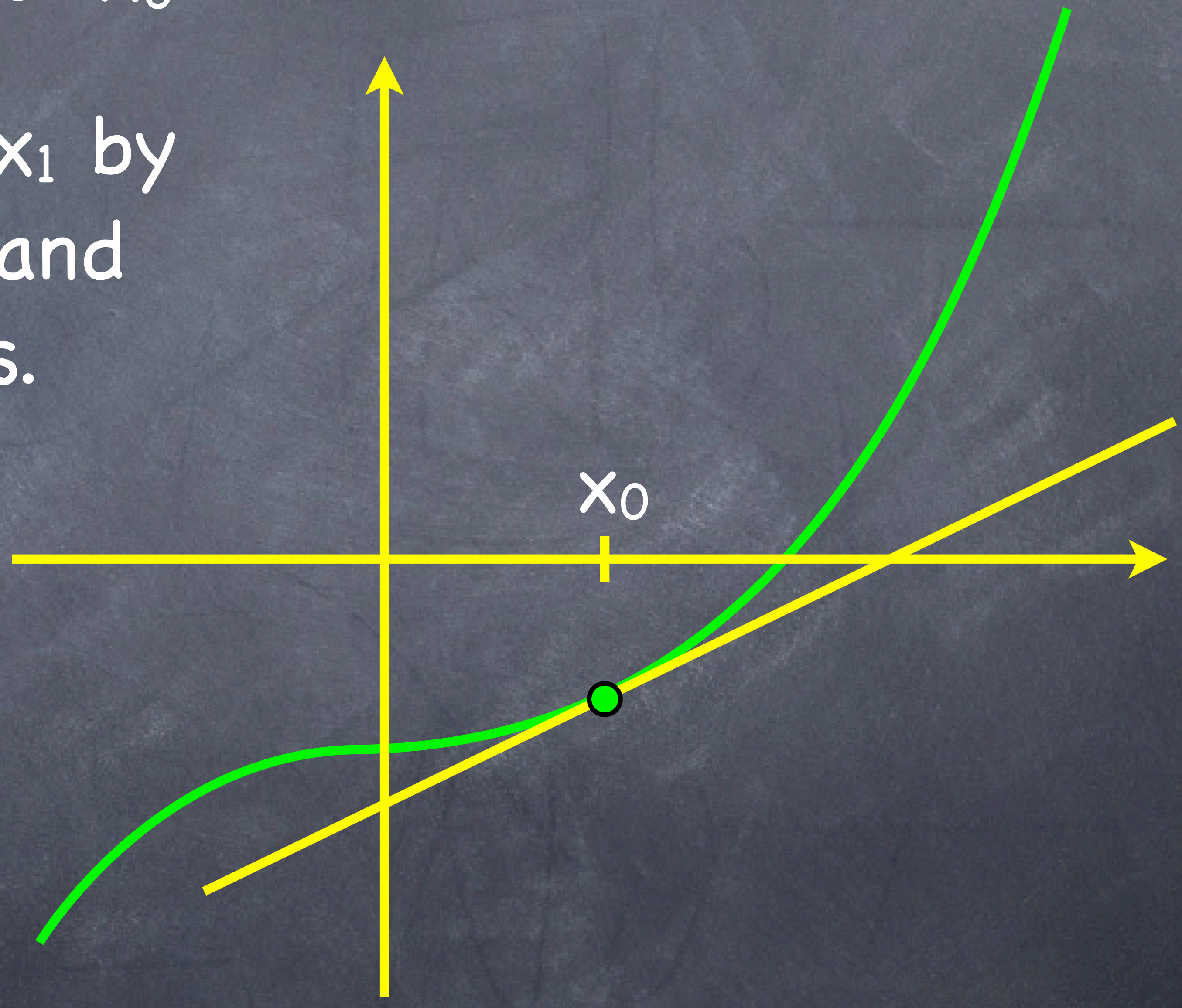

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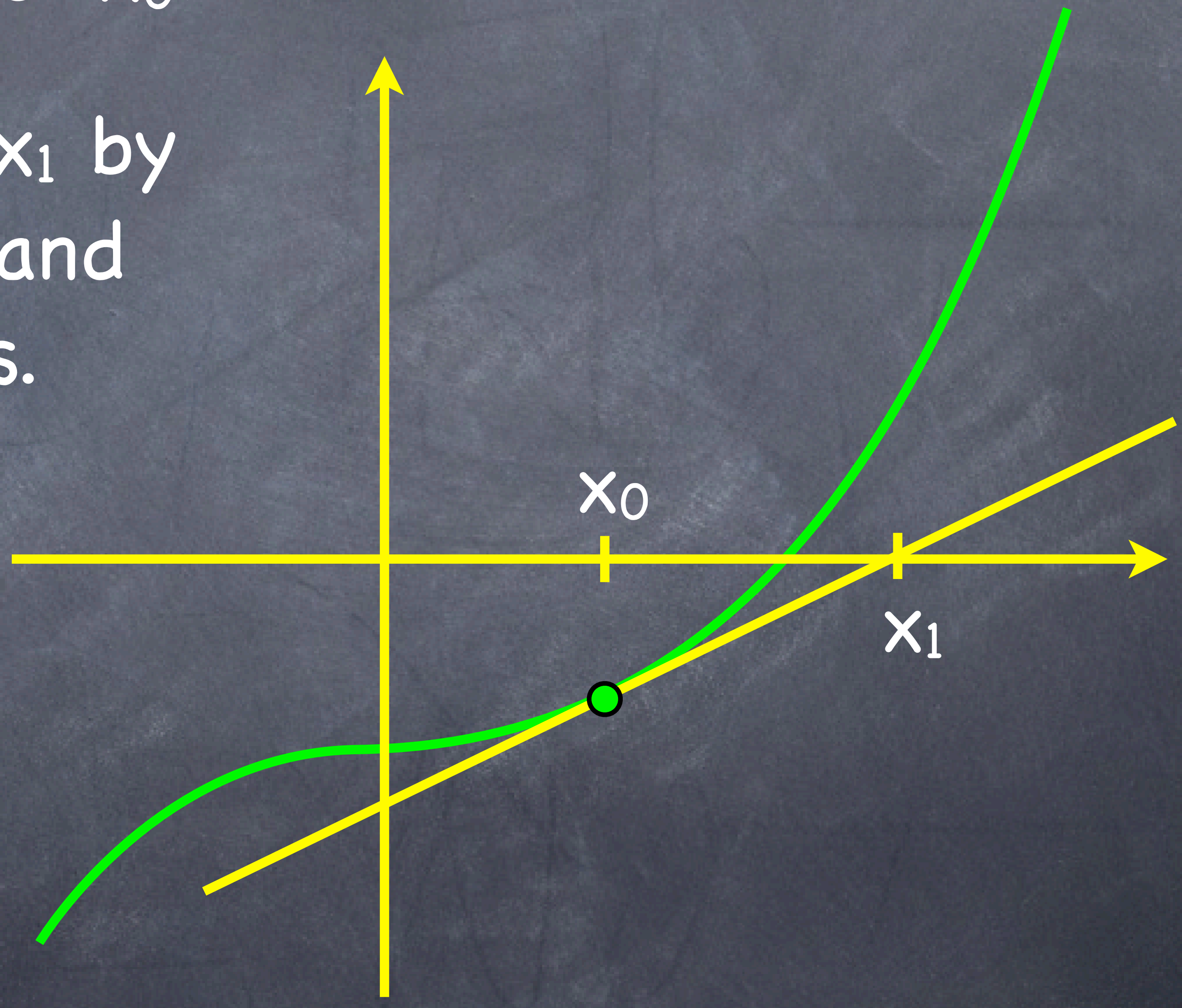

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- Start with a "guesstimate" x_0.
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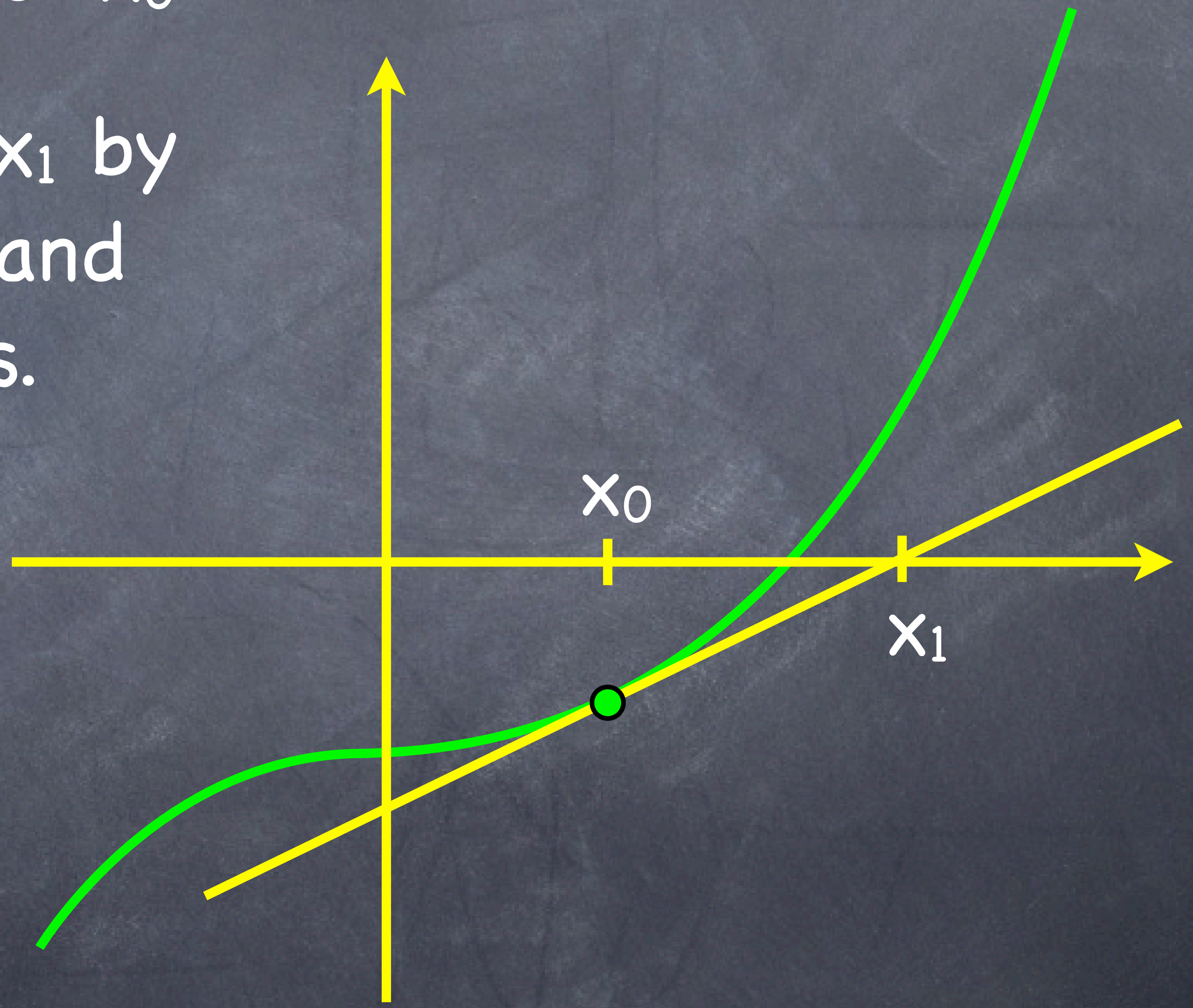
- Repeat to get x_2...



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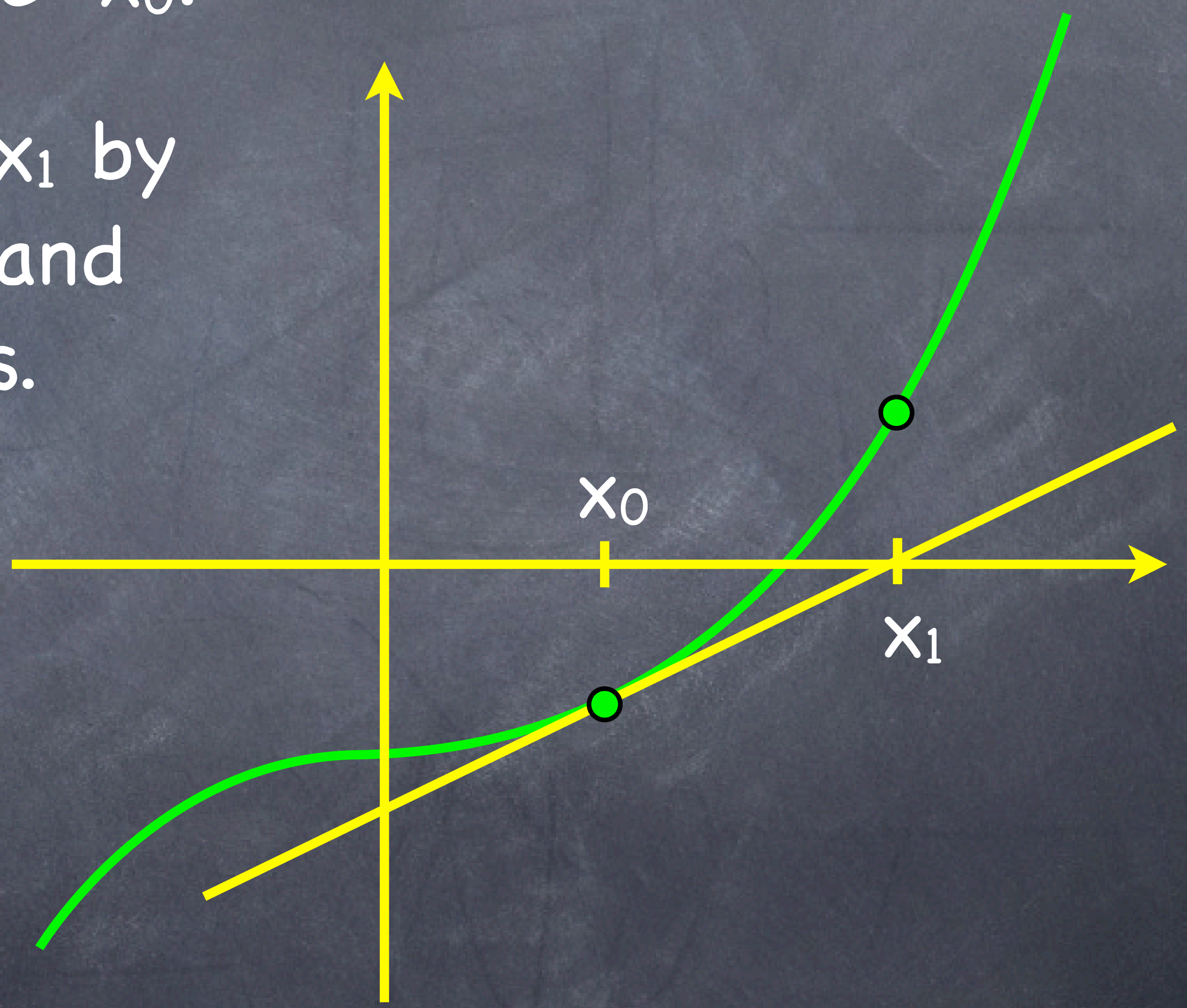

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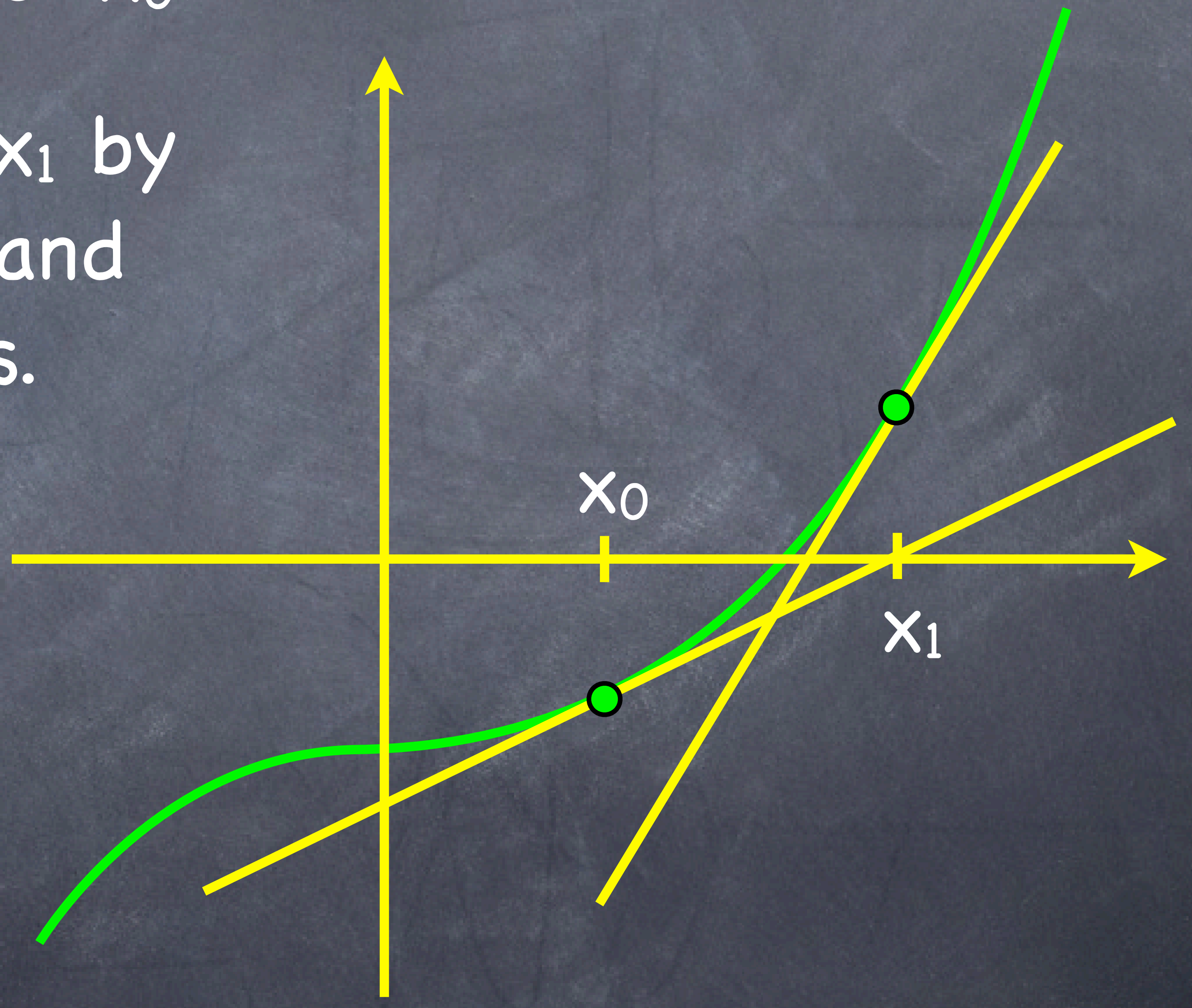

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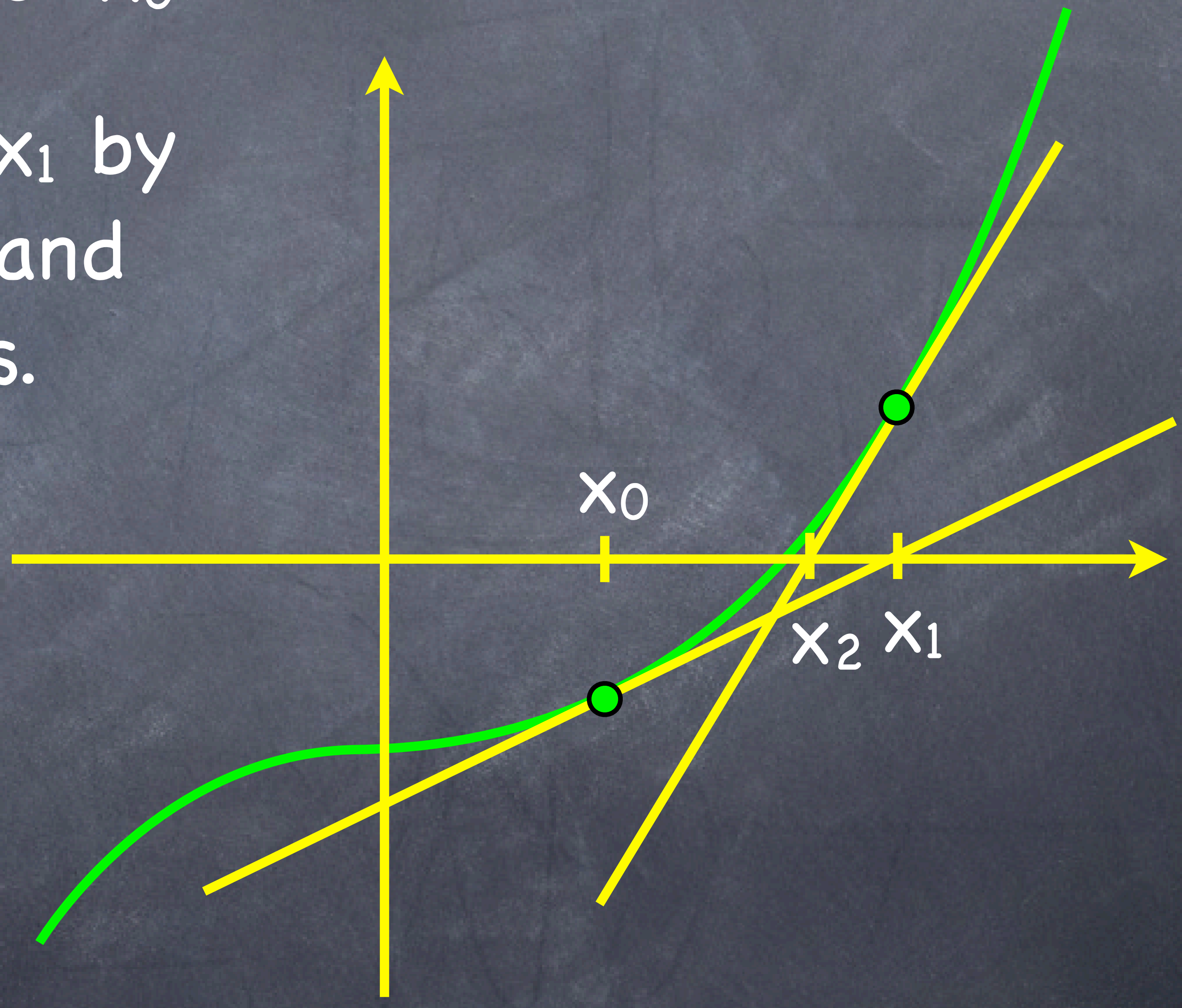
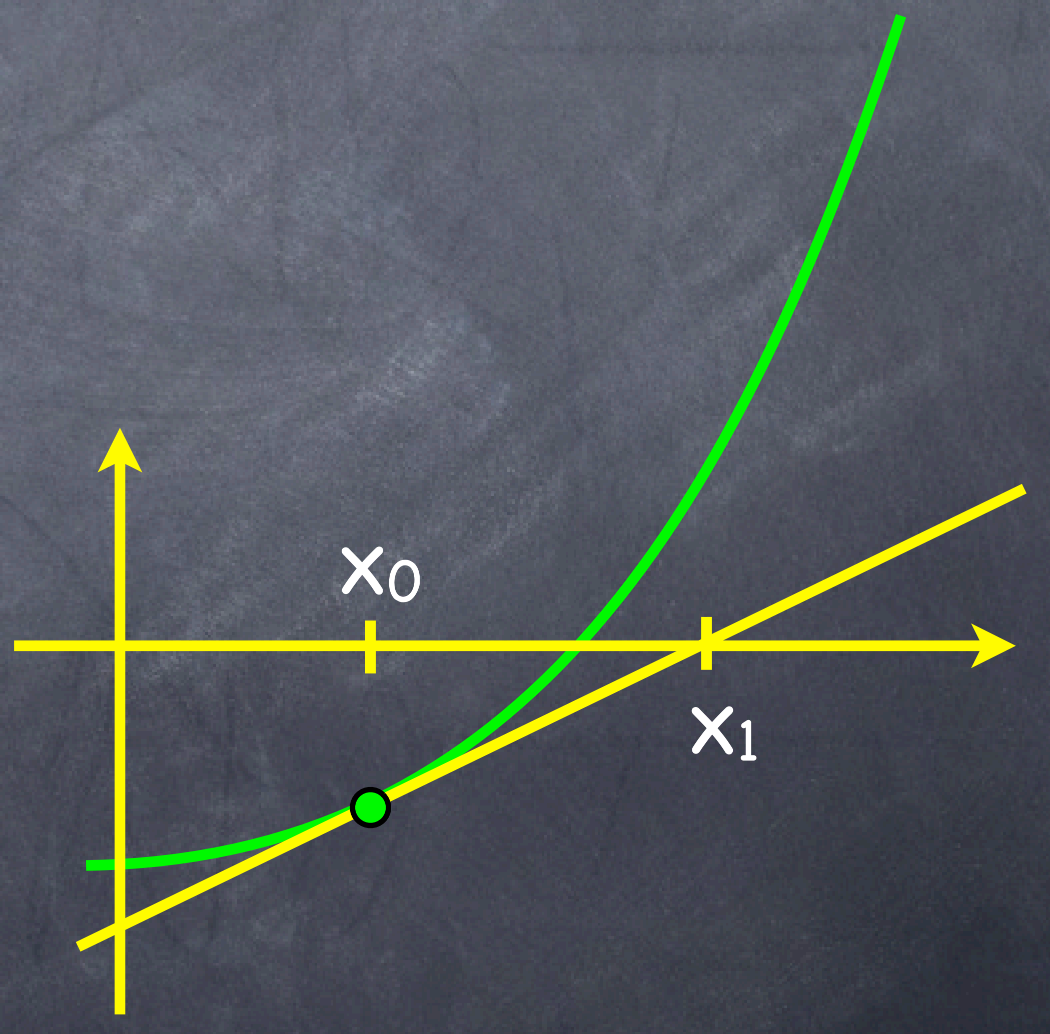


Calculating successive estimates



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- First, find tangent line at x_n:
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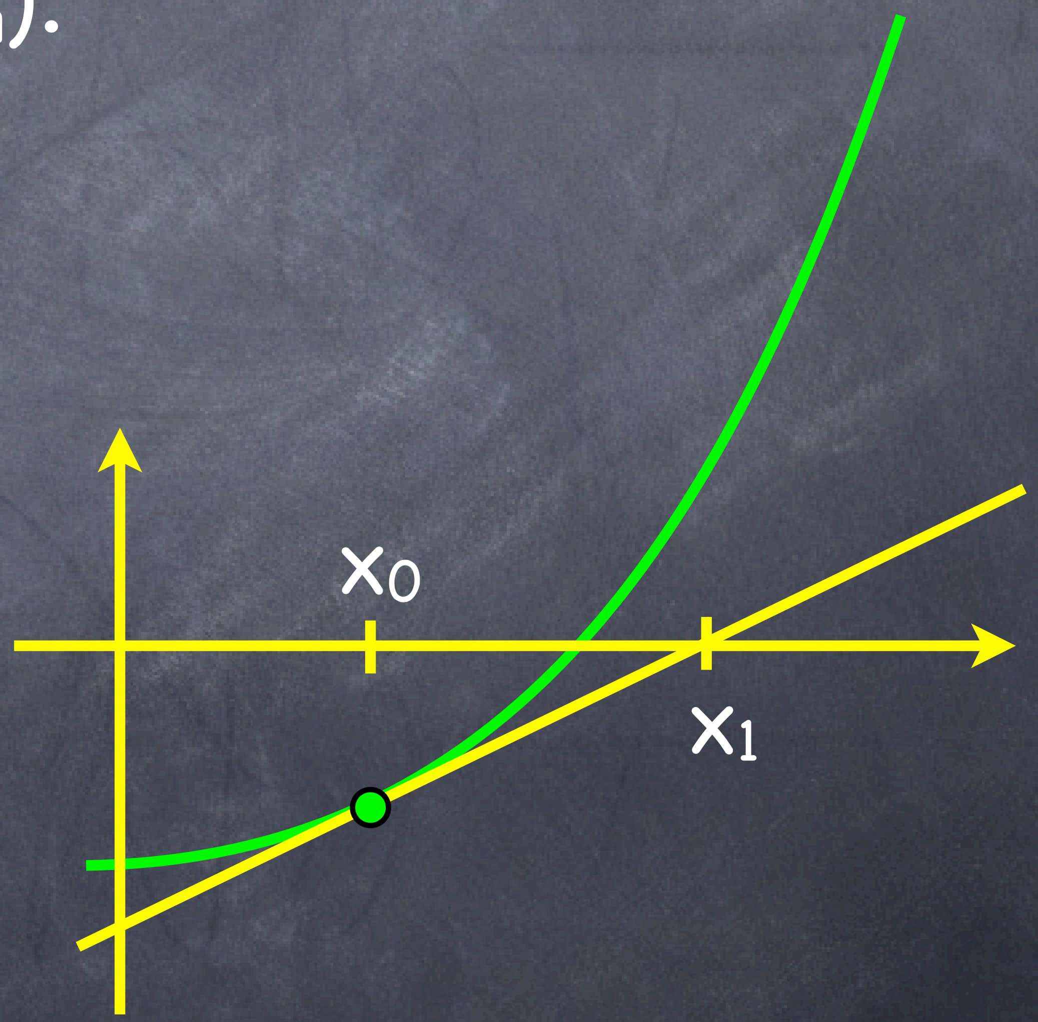

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- Find x-intercept, that will be x_{n+1}:

(A) $x_{n+1} = x_n + f(x_n) / f'(x_n)$.

(B) $x_{n+1} = x_n - f(x_n) / f'(x_n)$.

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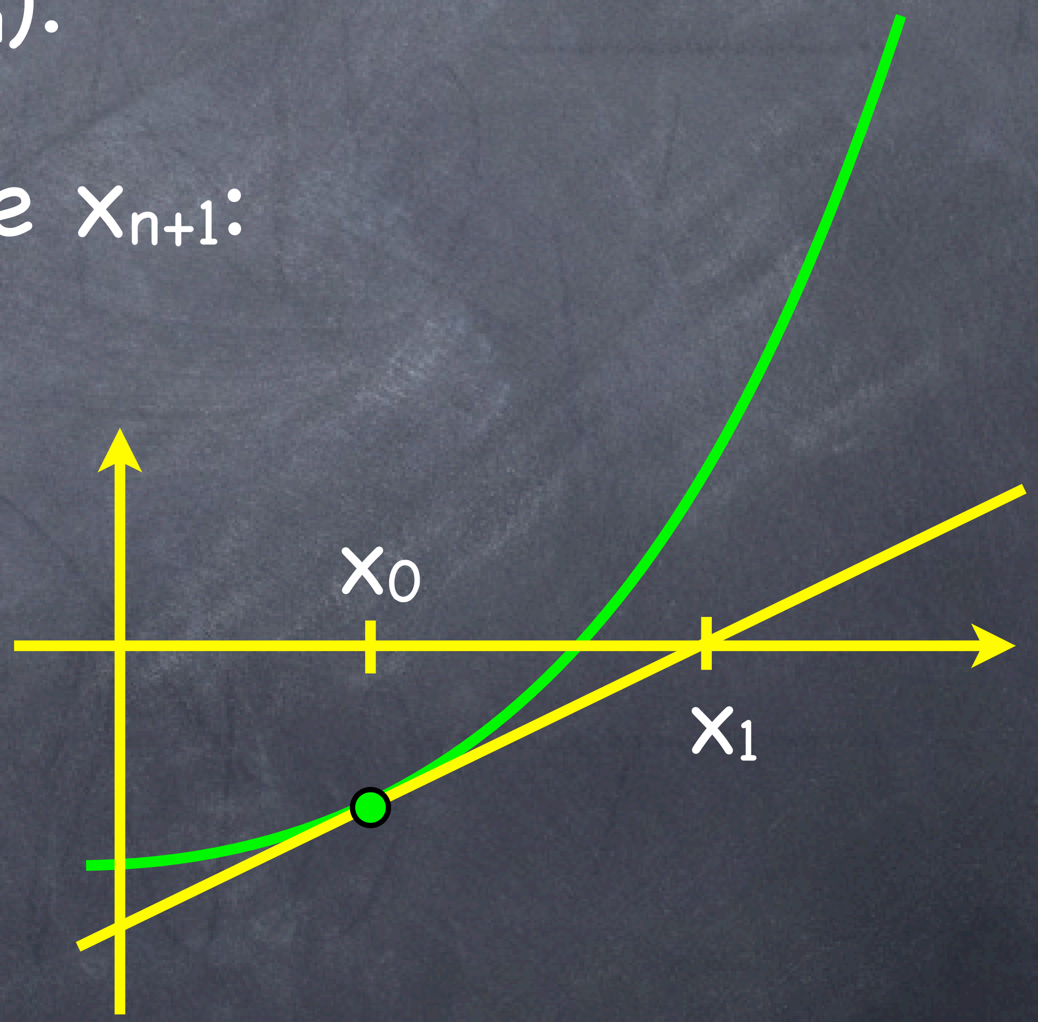

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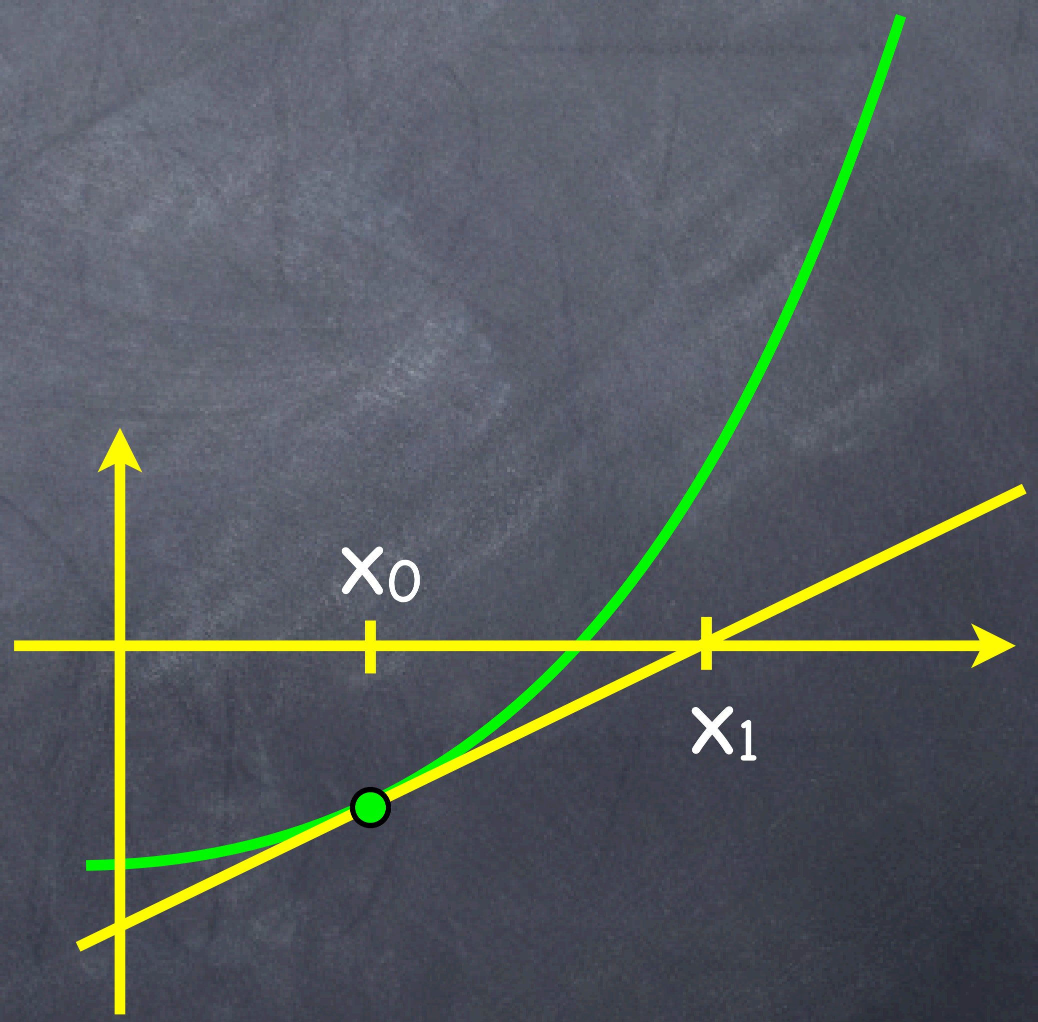

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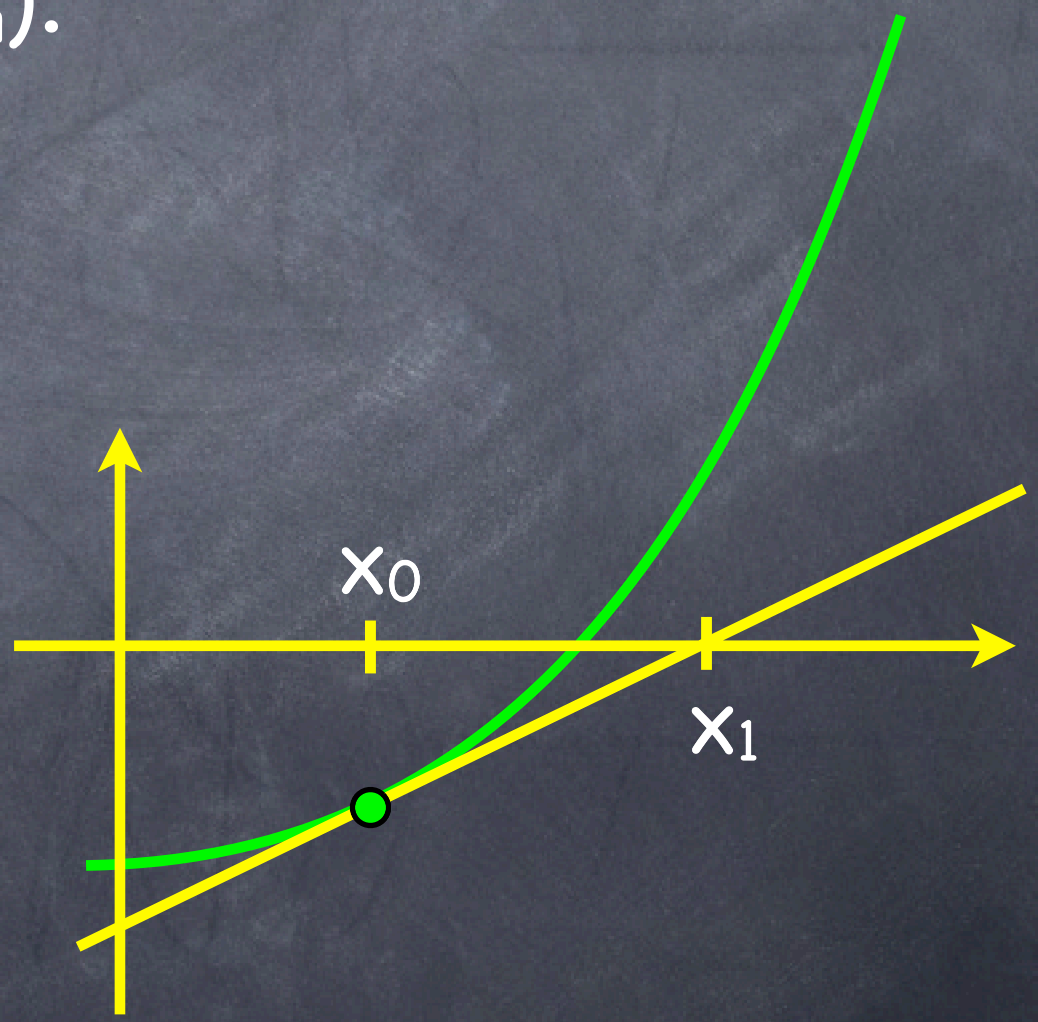

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