Today

- Logistic equation in applications
- Trig review

Logistic equation in different contexts...

Rates of change that are proportional to two things

- Infectious disease: bSI (S=susceptible, I=infected)
- Spread of rumour: bNH (N = not heard rumour, H = heard rumour)
- Spread of new words: bNU (use word or not).
- Spread of new technologies: bNU (use tech or not).
- Active oil exploration sites: bUD (undiscovered and discovered).
- Waterlillies in a pond: bSW (waterlillies and space for waterwillies).

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- X+Y= constant = C so Y=C-X.
- $$\frac{dX}{dt} = bX(C - X)$$

Infectious disease

Dr. Erin Mears: Once we know the R_0, we'll be able to get a handle on the scale of the epidemic.
Minnesota Health #4: So, it's an epidemic now. An epidemic of what?
Dave: We sent samples to the CDC.
Dr. Erin Mears: In seventy two hours, we'll know what it is, if we're lucky.
Minnesota Health #4: Clearly, we're not lucky.

Infectious disease

Infectious disease

- N individuals, I of them have a flu, $S=N-I$ do not.

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- The DE describing the spread of disease:

(A) $\frac{dI}{dt} = -bI(N-I)$

(B) $\frac{dI}{dt} = bI(N-I)$

(C) $\frac{dS}{dt} = -bSI$

(D) $\frac{dI}{dt} = bSI$

Infectious disease

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(A) $\frac{dI}{dt} = -bI(N-I)$ (C) $\frac{dS}{dt} = -bSI$

(B) $\frac{dI}{dt} = bI(N-I)$ (D) $\frac{dI}{dt} = bSI$

- Compare this with $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$.

Infectious disease

What is the carrying capacity?

$$\frac{dI}{dt} = bI(N - I)$$

(A) b/N

(B) N/b

(C) I

(D) N

Infectious disease

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Infectious disease

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Infectious disease

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Infectious disease

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Infectious disease

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Everyone gets sick!

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

Infectious disease

Infectious disease

- Suppose infected people recover at a rate proportional to how many there are.

Infectious disease

- Suppose infected people recover at a rate proportional to how many there are.
- The DE describing the spread of disease with recovery:

(A) $\frac{dI}{dt} = bI(N - I) - \mu S$

(B) $\frac{dI}{dt} = bI(N - I) - \mu I$

(C) $\frac{dI}{dt} = -bI(N - I) + \mu I$

(D) $\frac{dI}{dt} = bI(N - I) + \mu I$

Infectious disease

$$\frac{dI}{dt} = bI(N - I) - \mu I$$

Infectious disease

$$\frac{dI}{dt} = bI(N - I) - \mu I$$

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Infectious disease

$$\frac{dI}{dt} = bI(N - I) - \mu I$$

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Infectious disease

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$$K = N - \frac{\mu}{b} < 0$$

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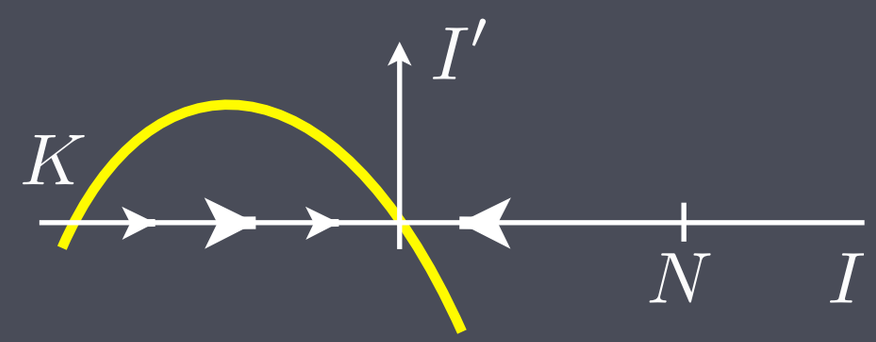

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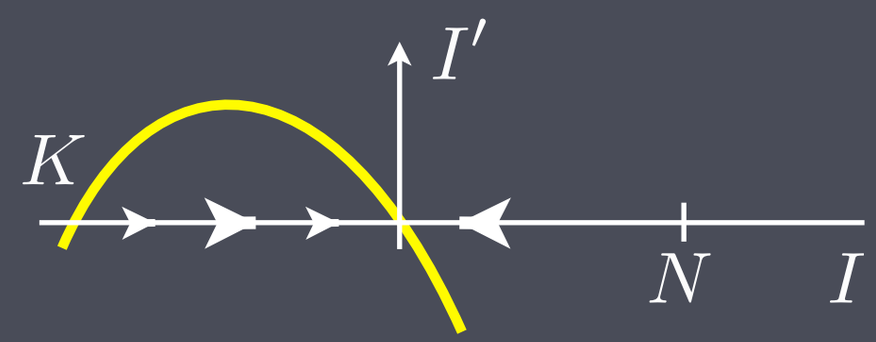

and the disease dies out.

Infectious disease

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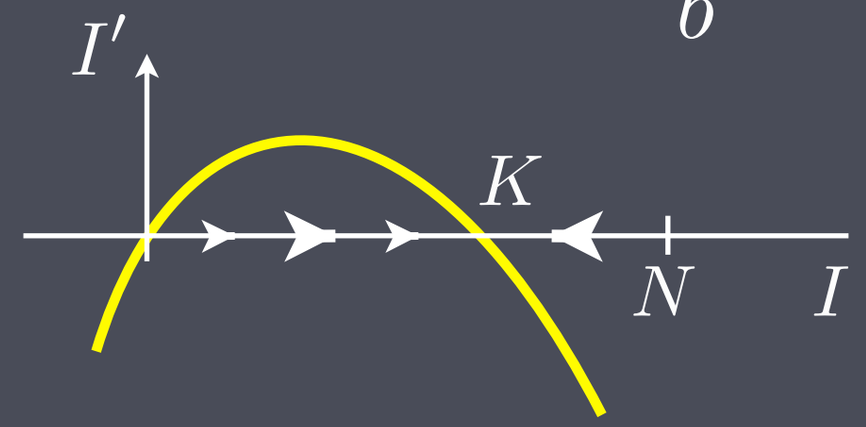

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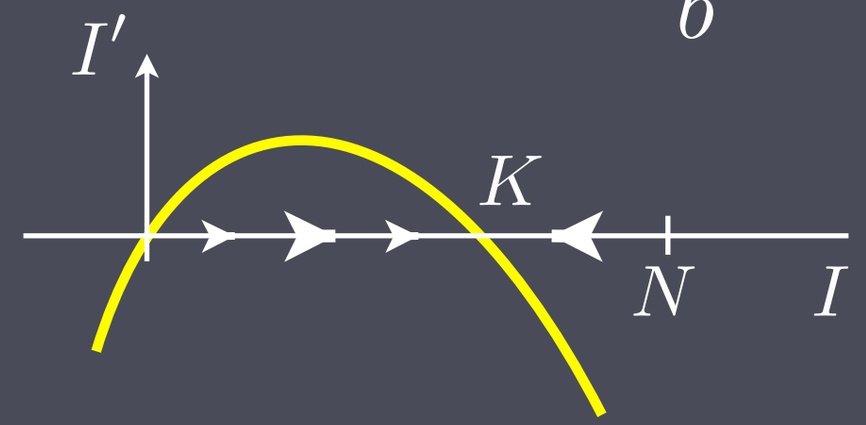

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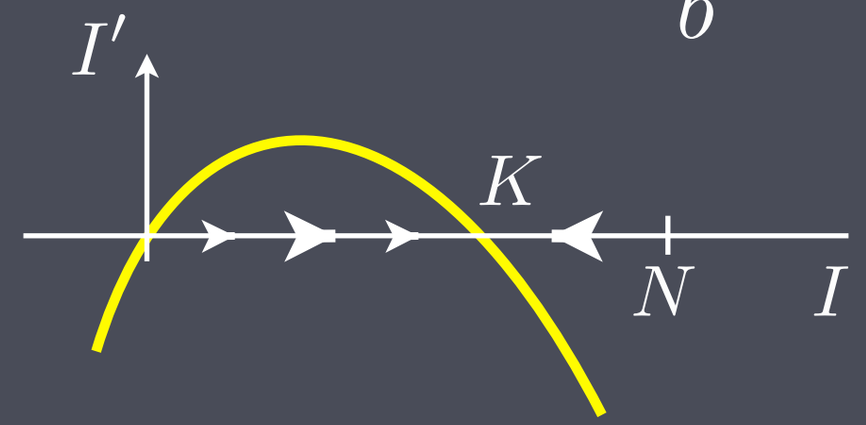

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$$R_0 = \frac{Nb}{\mu} > 1$$

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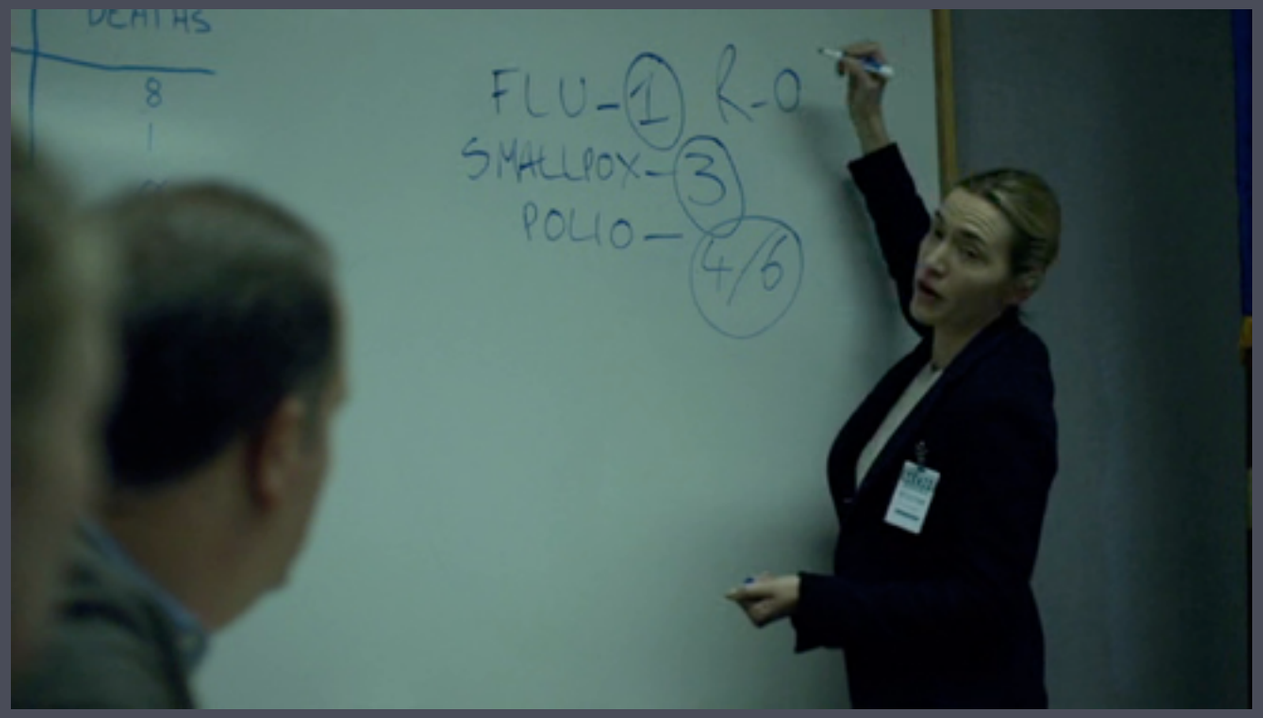

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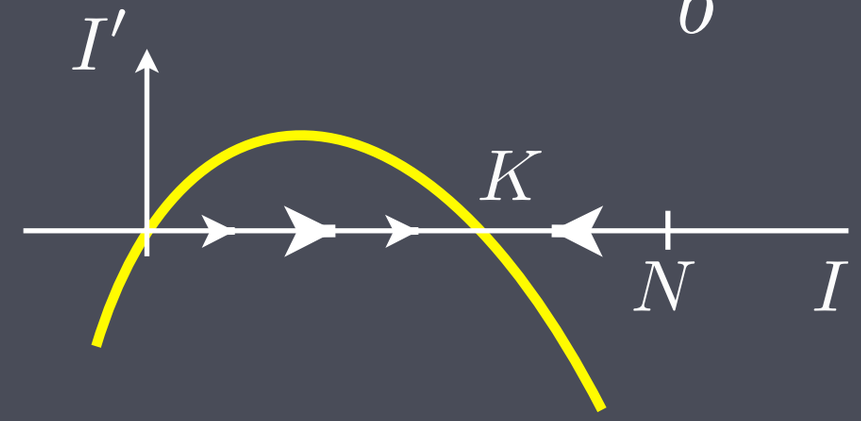

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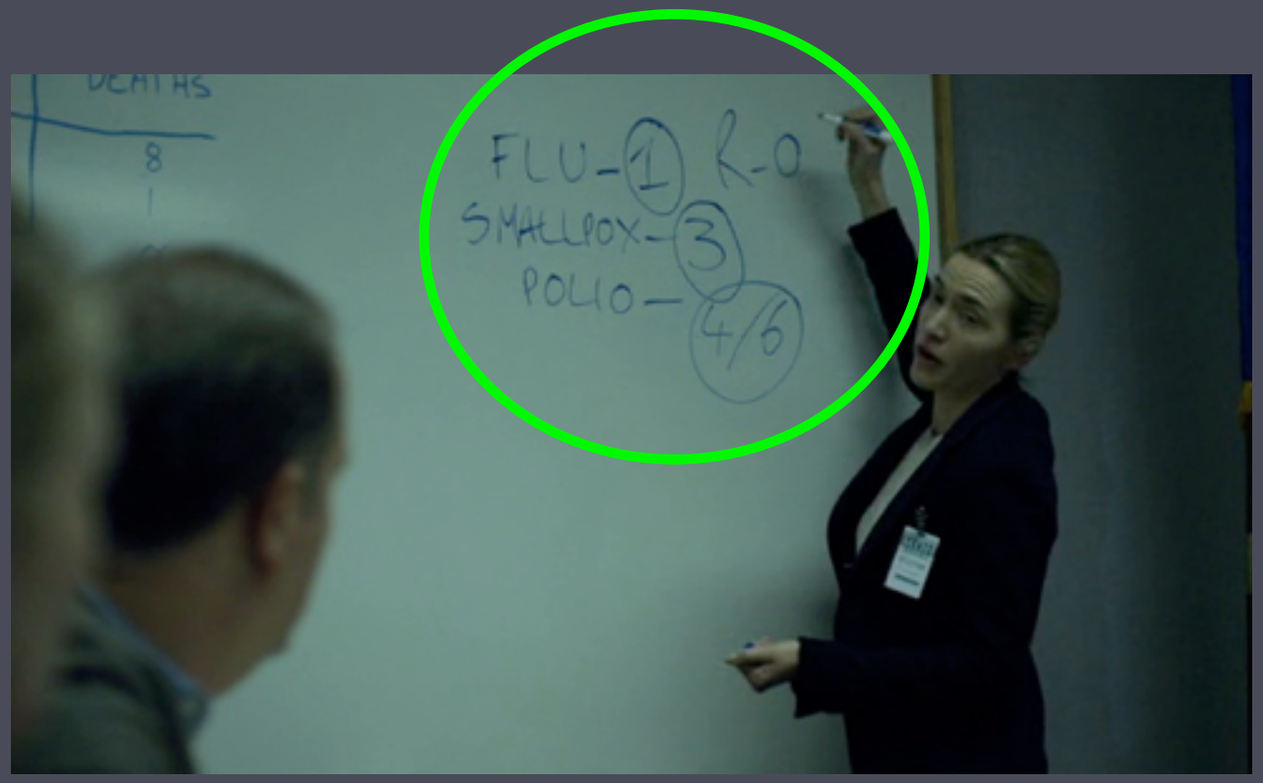

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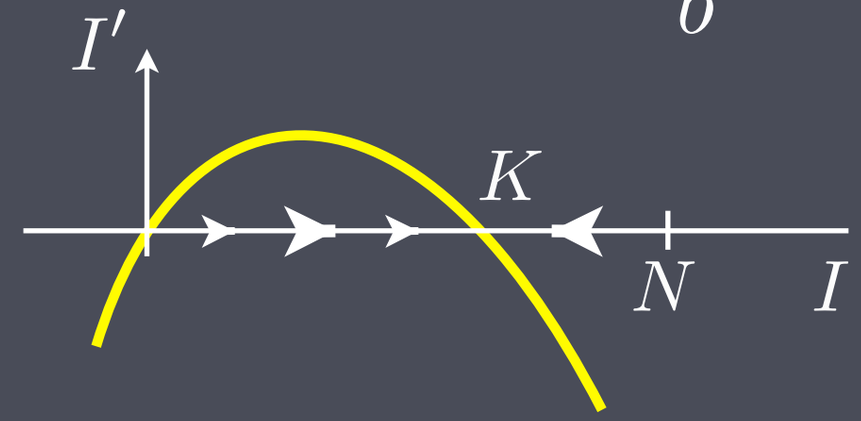

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Trig review

- If θ is measured counterclockwise from the positive x axis we define sin and cos so that

  (A) $x=\sin(\theta)$, $y=\tan(\theta)$.

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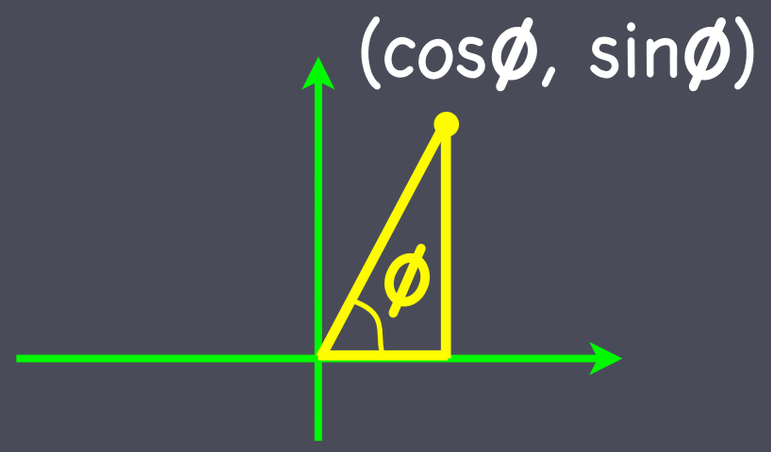

Learn special angles in Quad I
and modify signs for other Quads.

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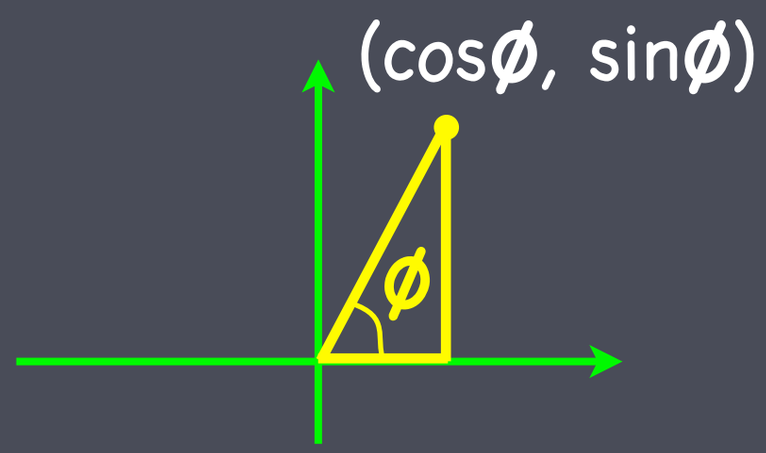

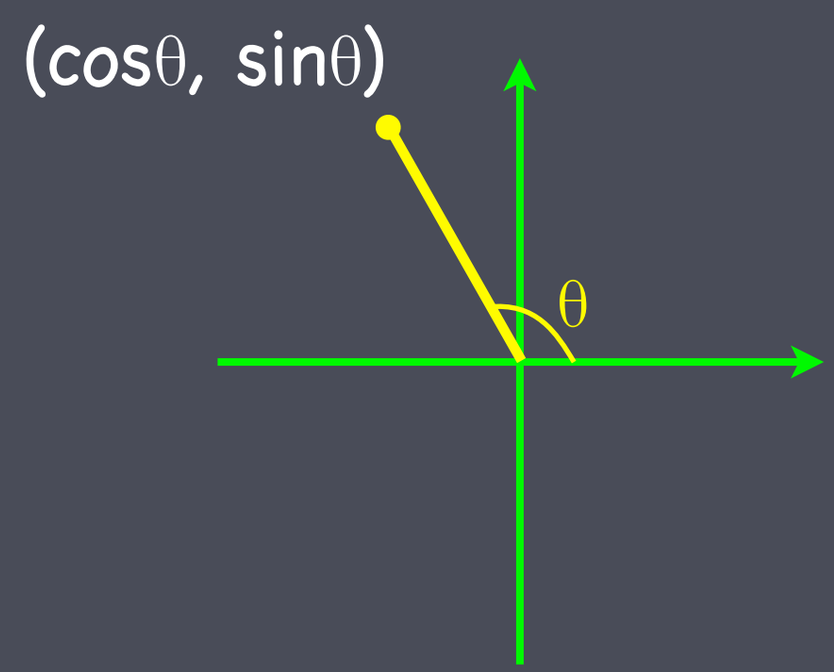

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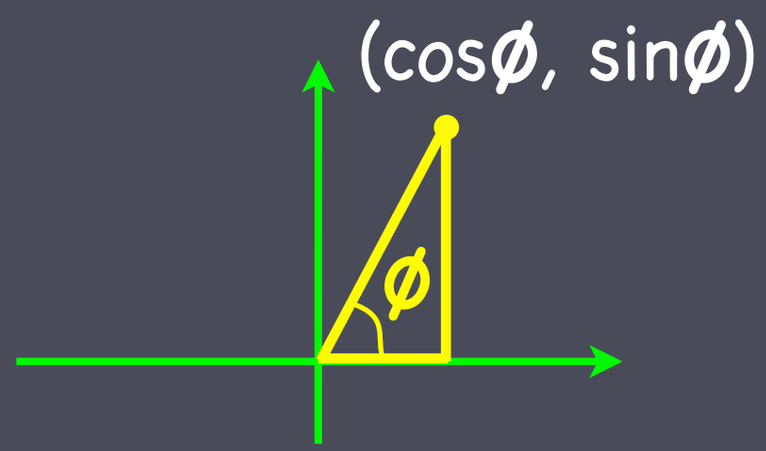

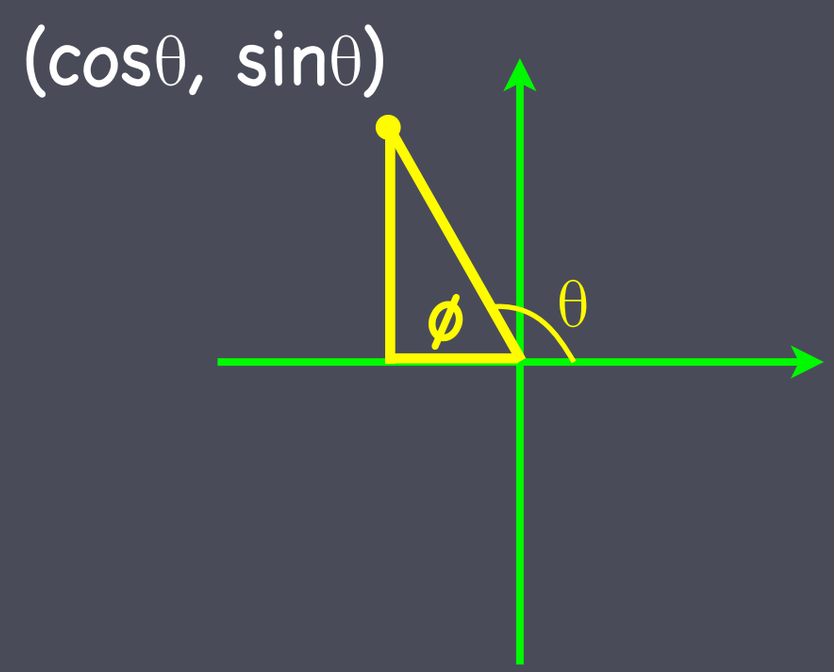

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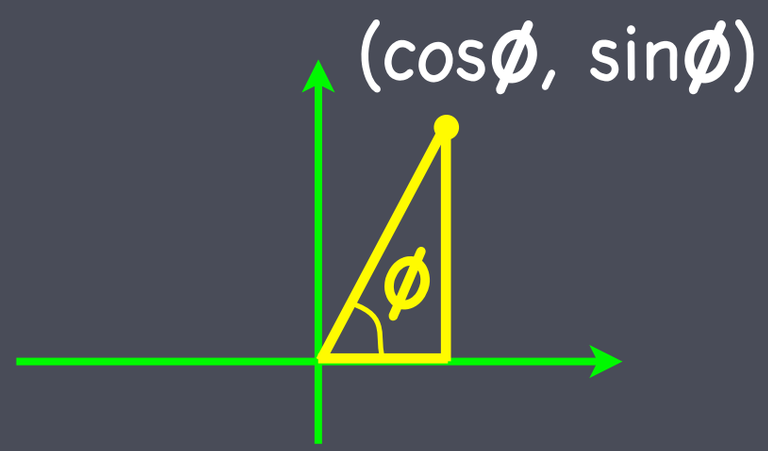

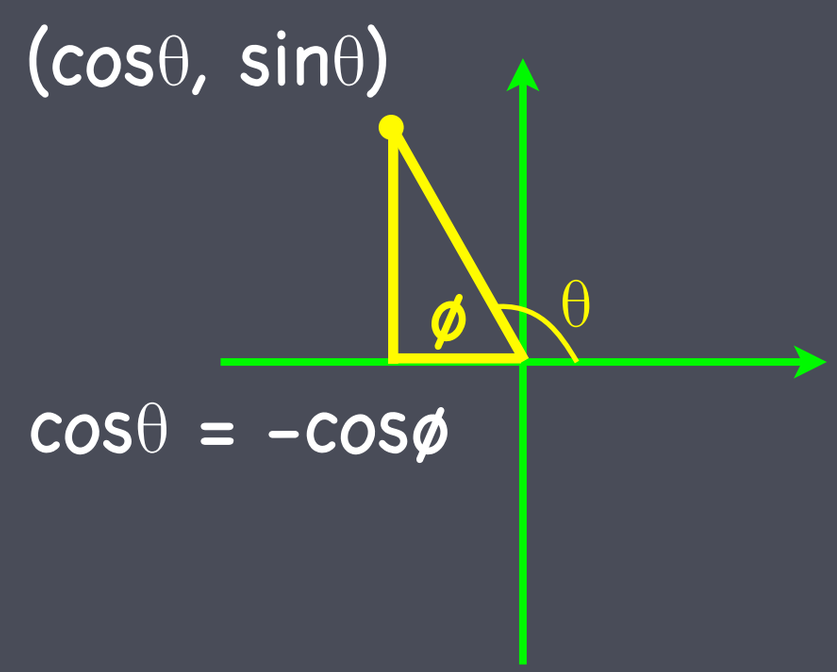

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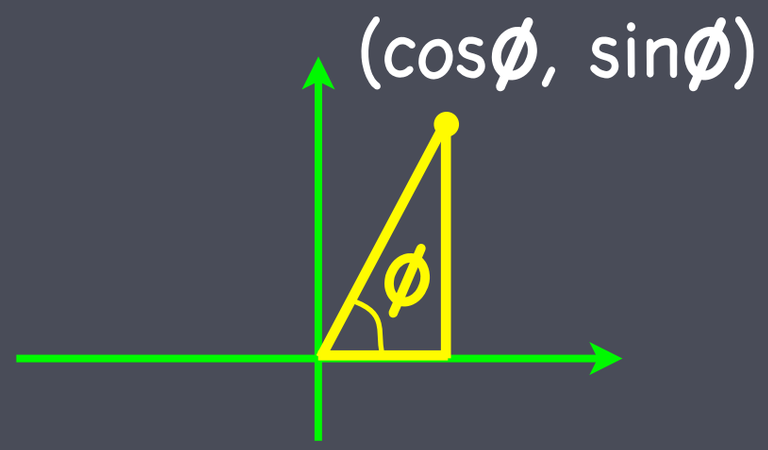

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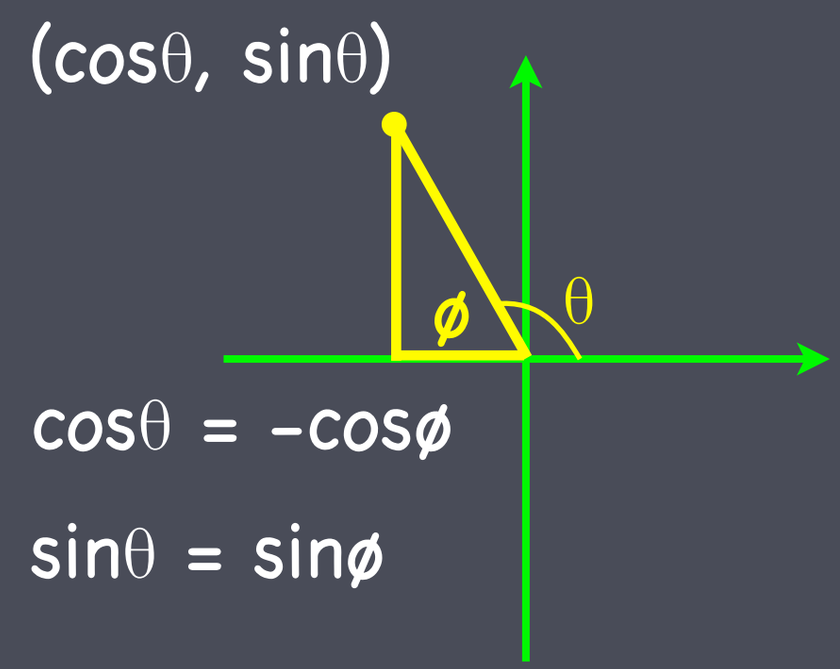

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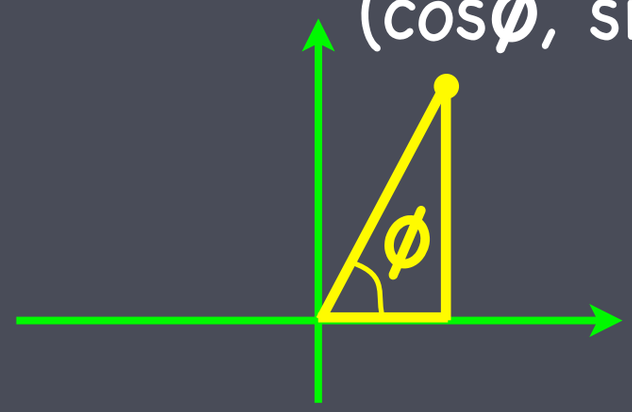

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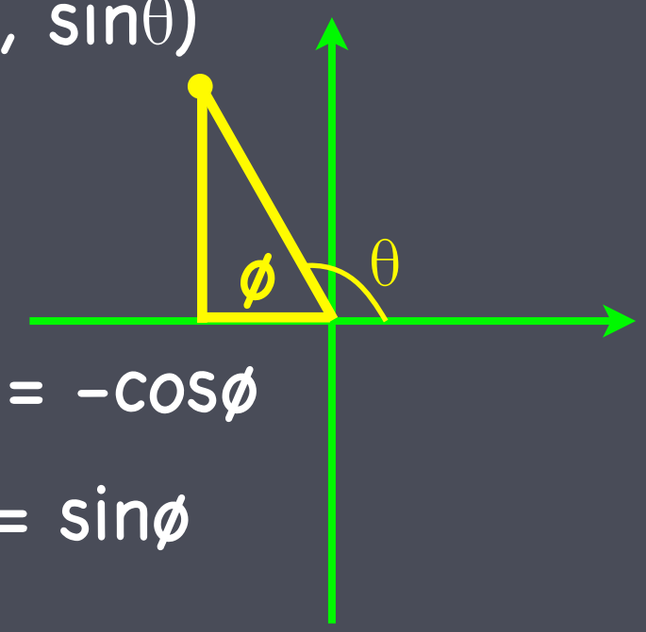

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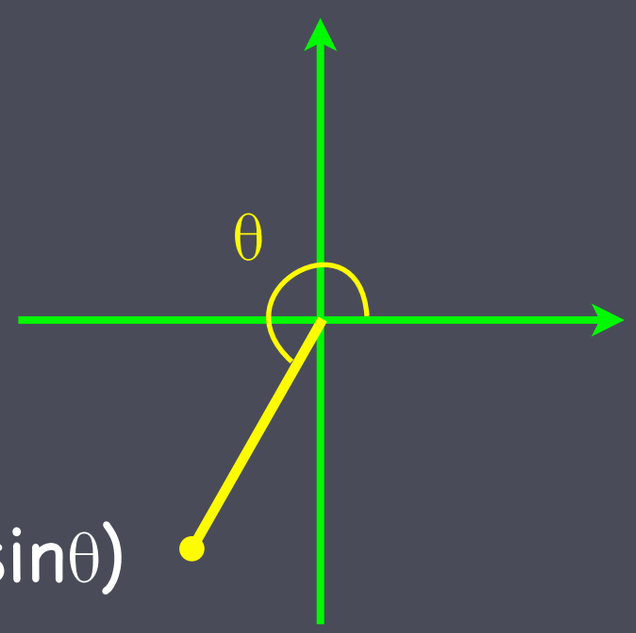

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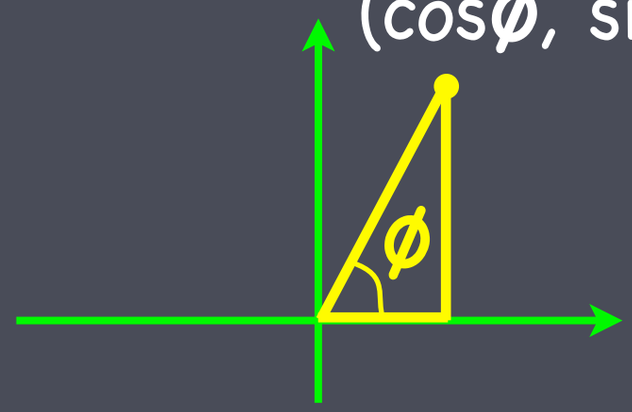

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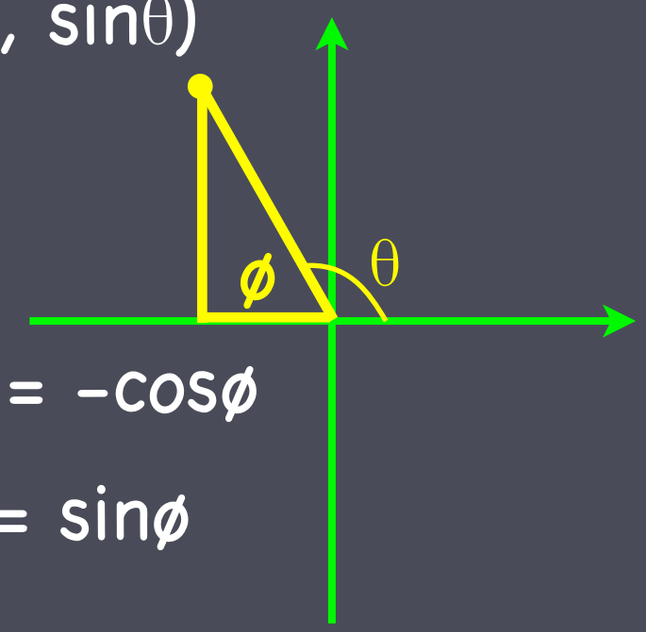

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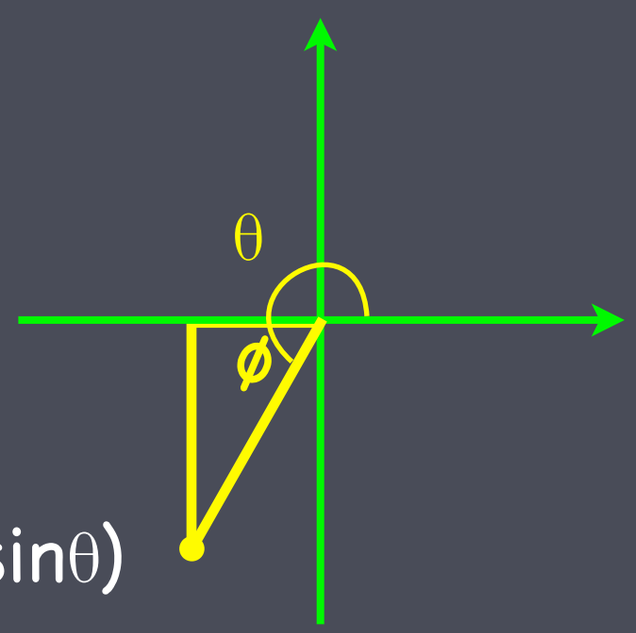

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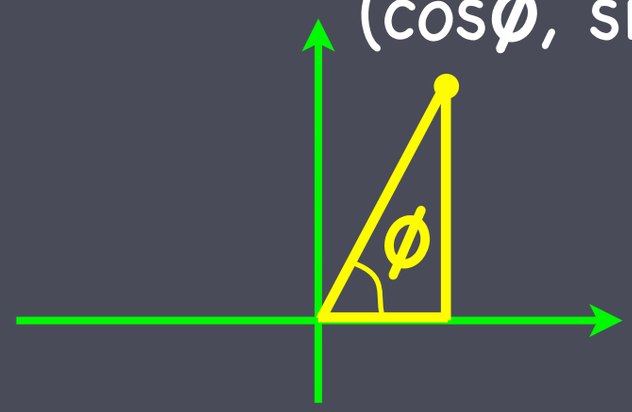

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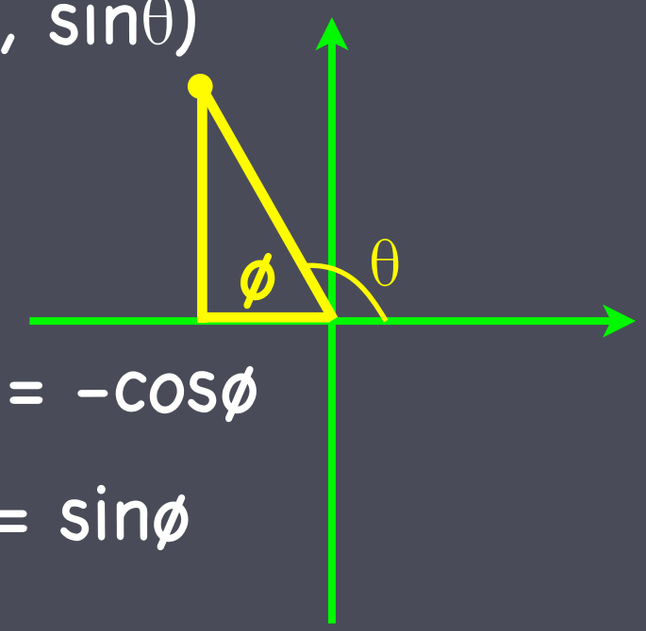

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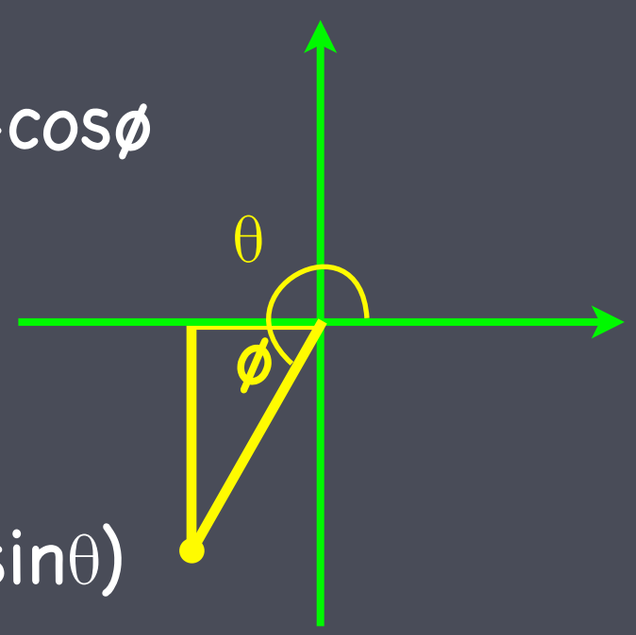

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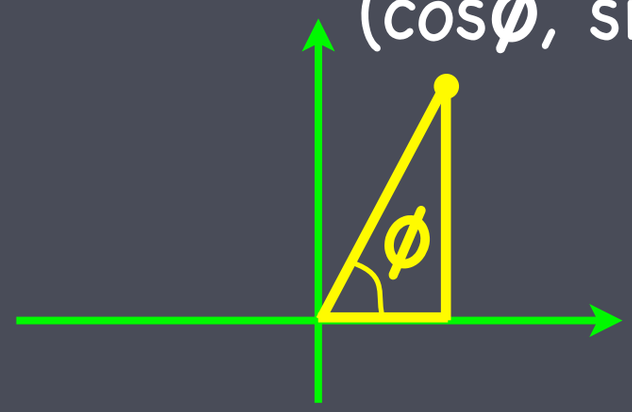

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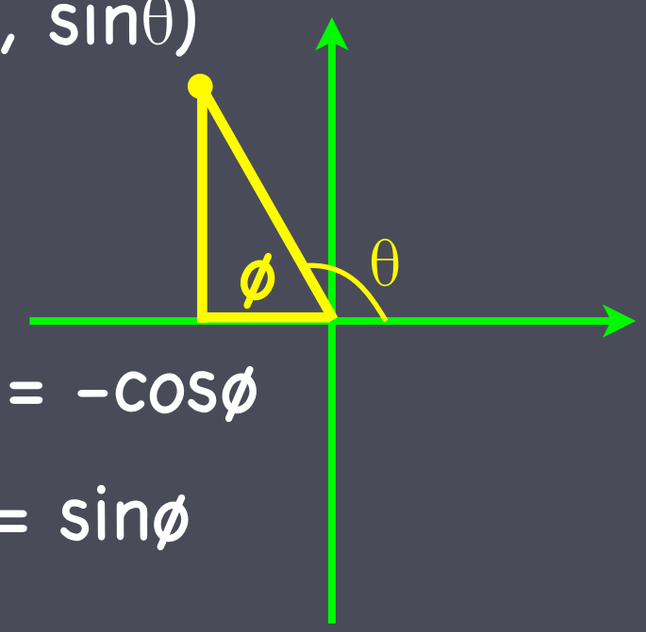

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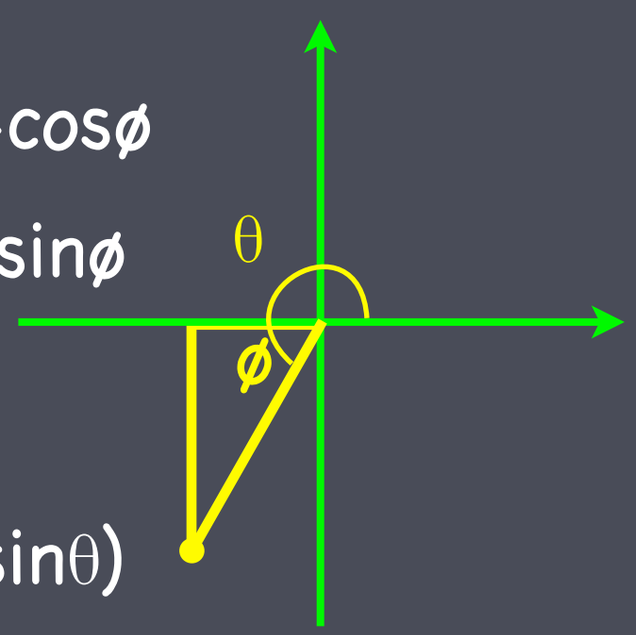

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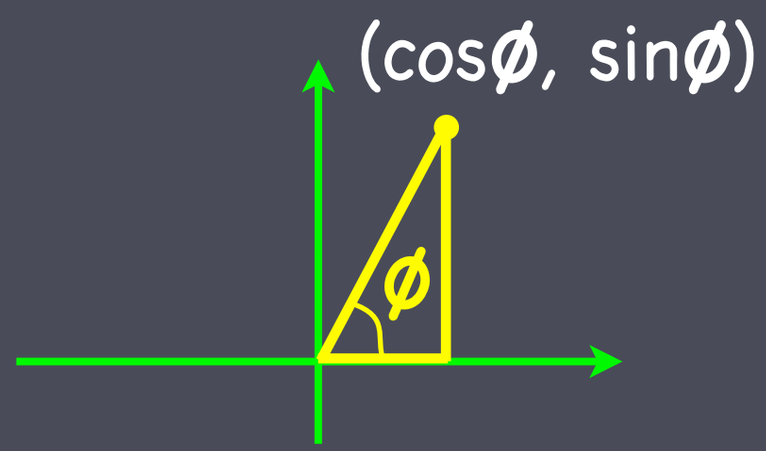

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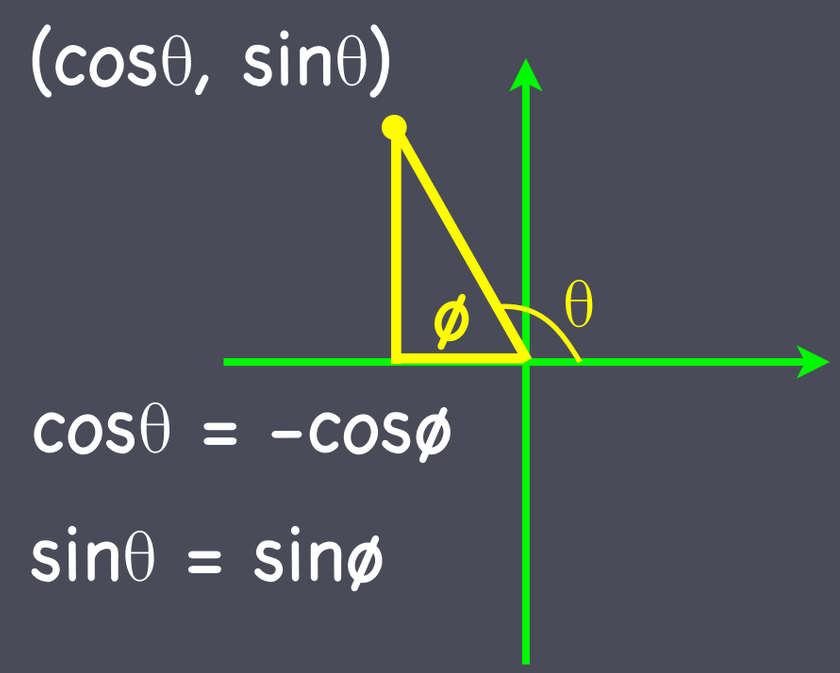

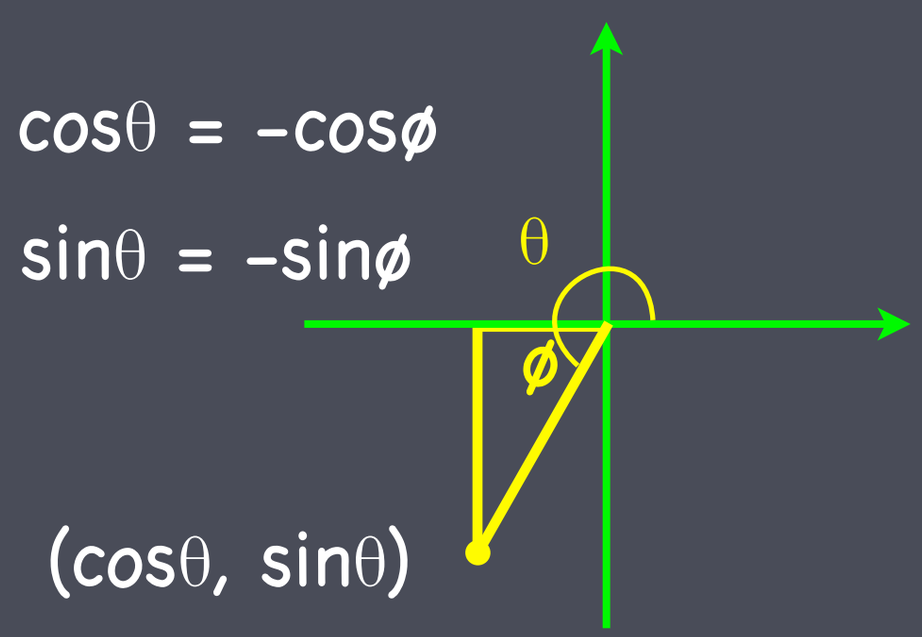

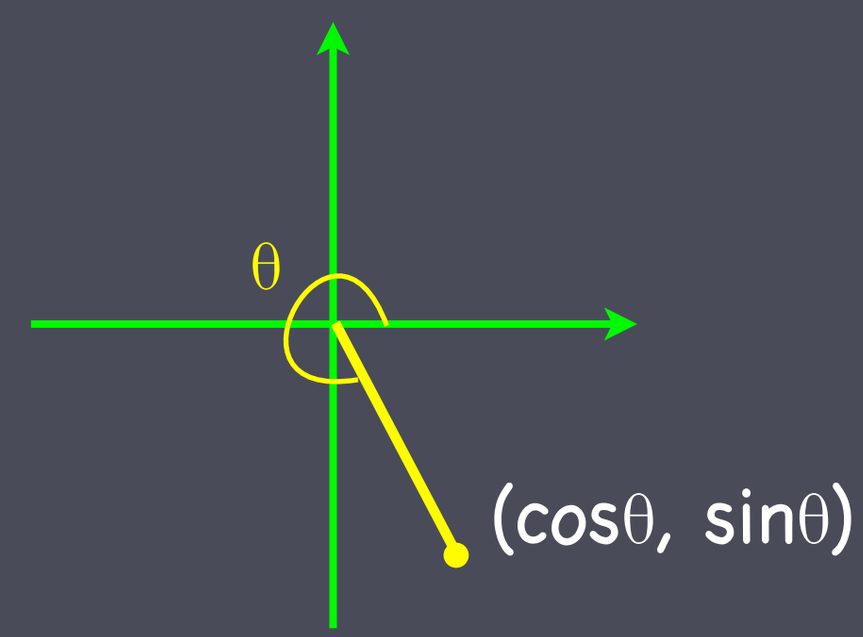

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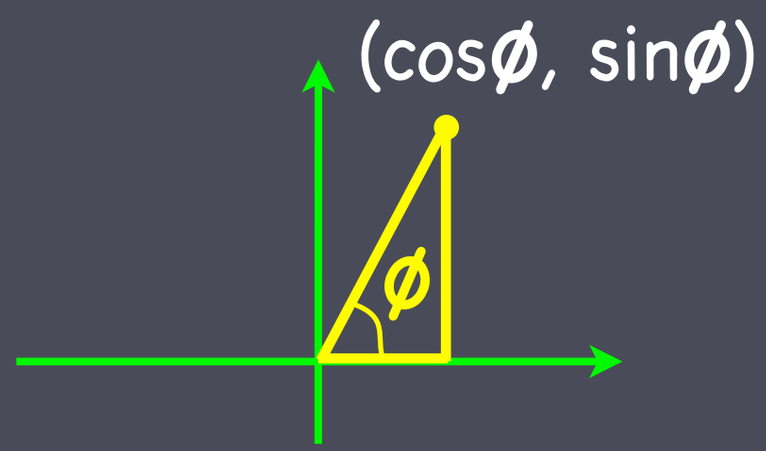

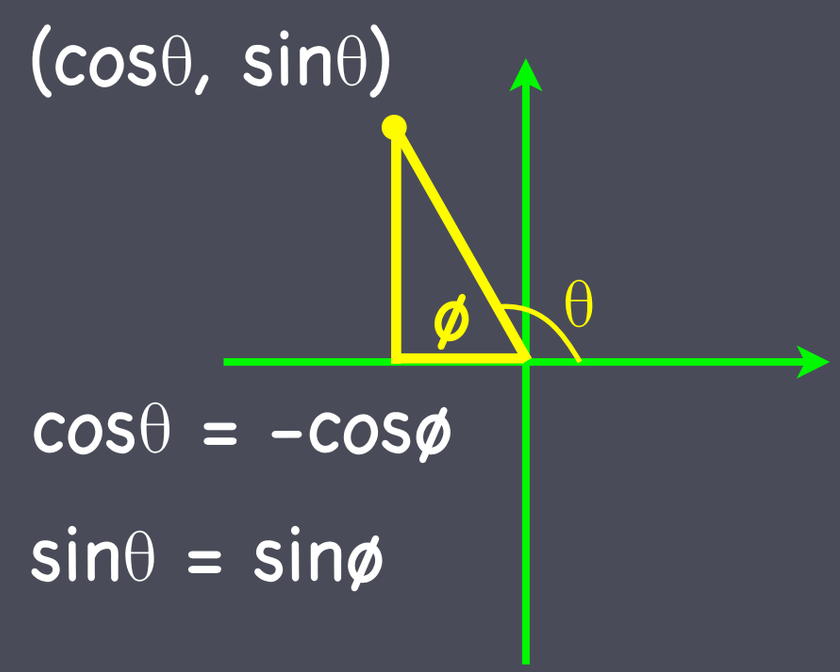

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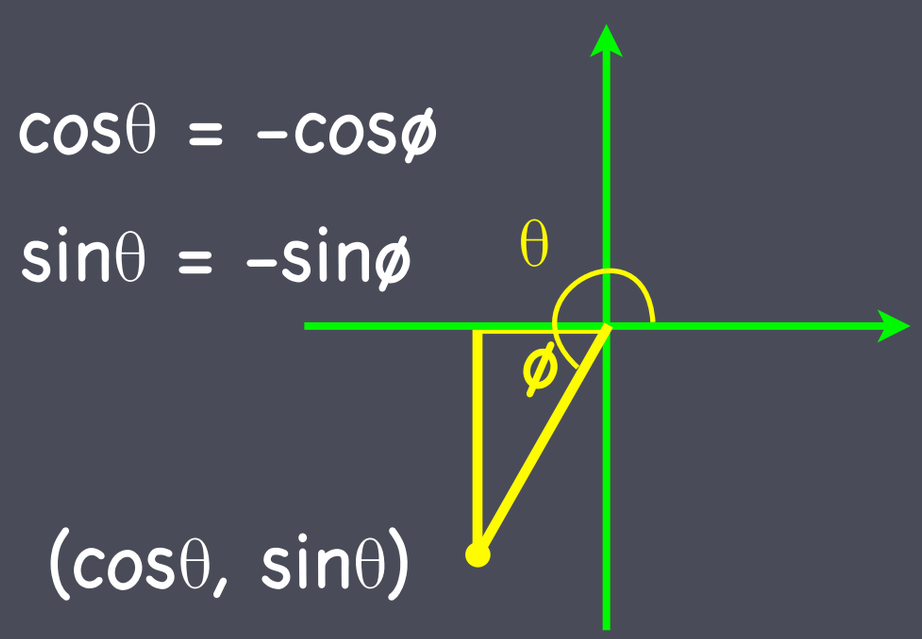
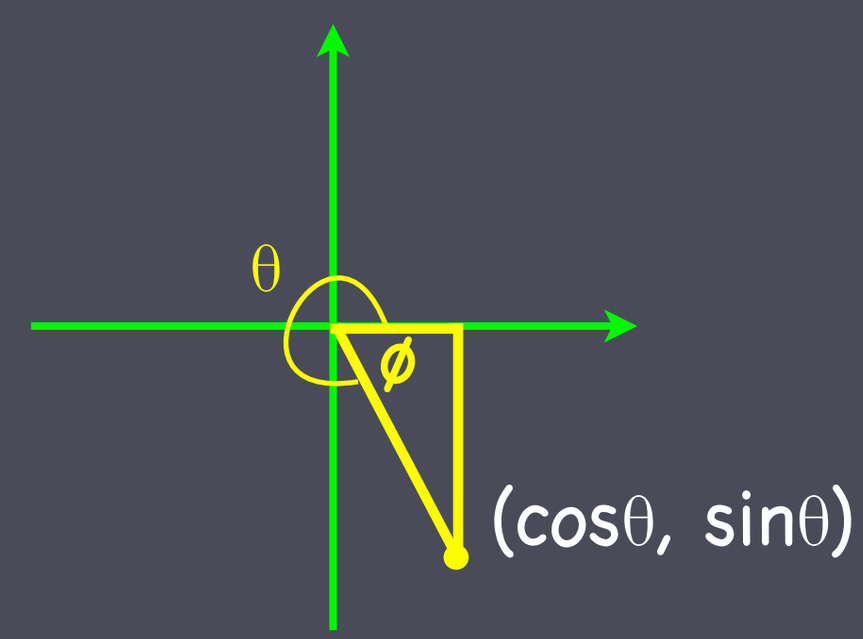
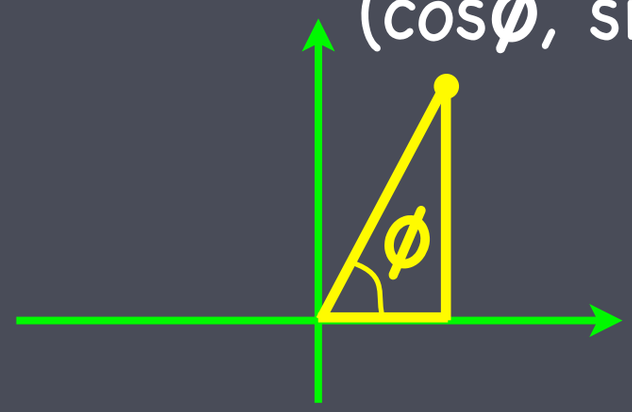
and modify signs for other Quads.





Trig review



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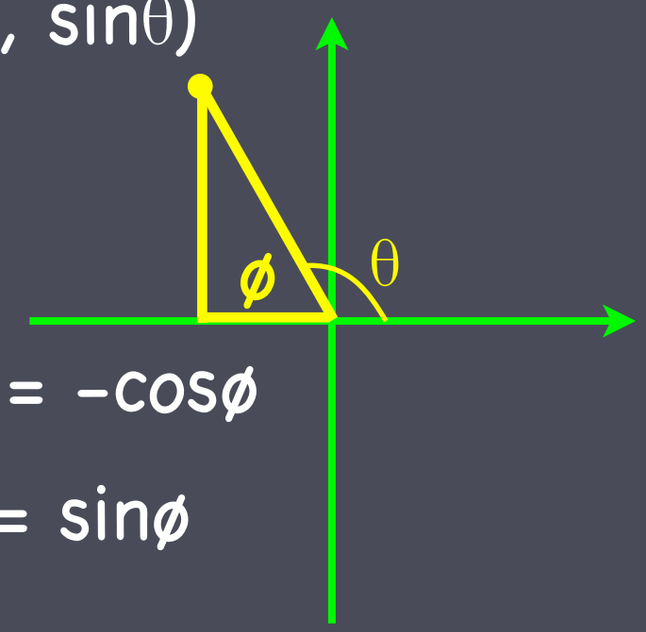

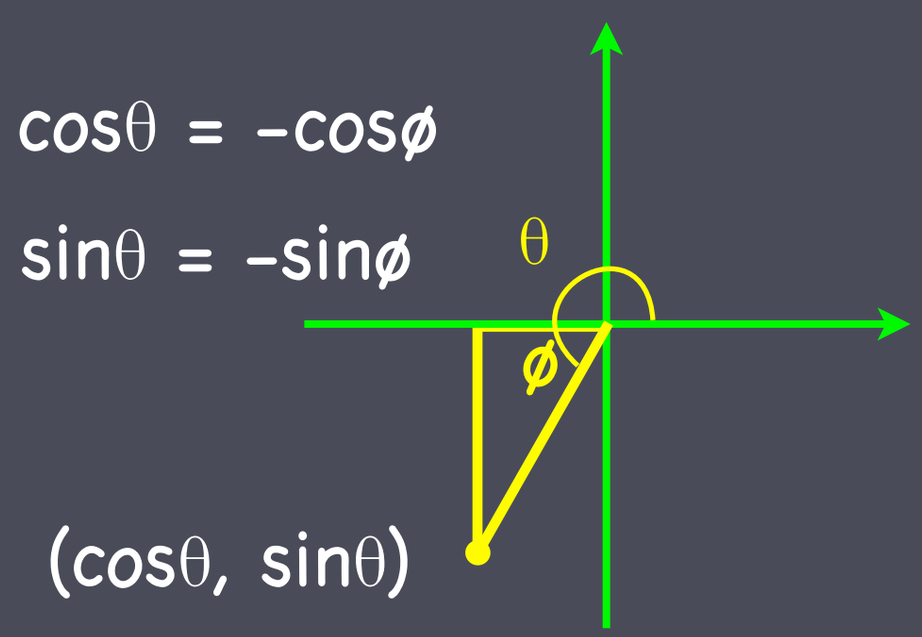

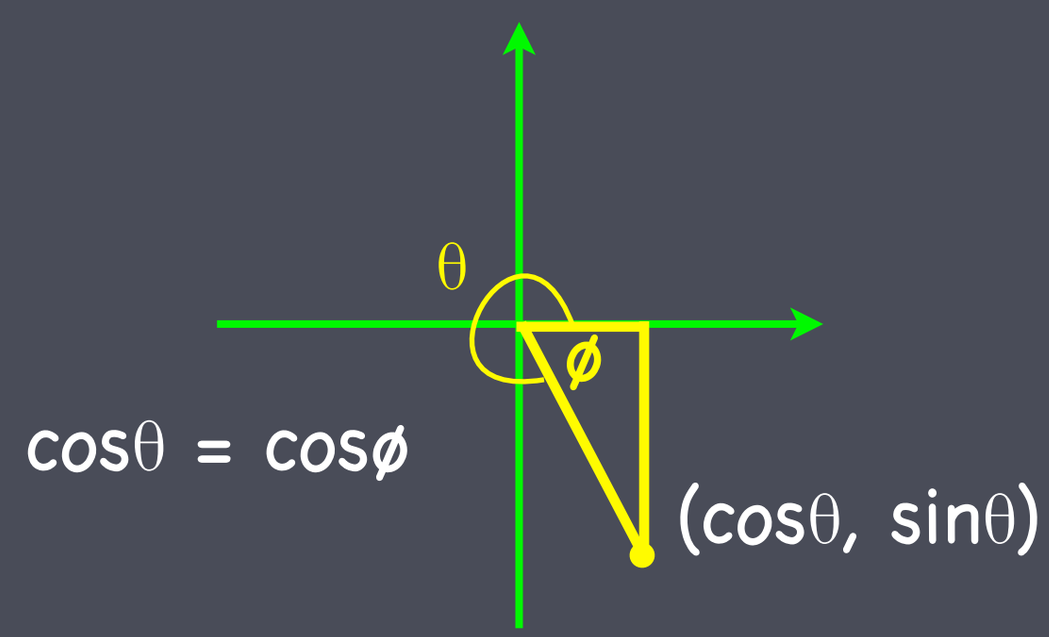

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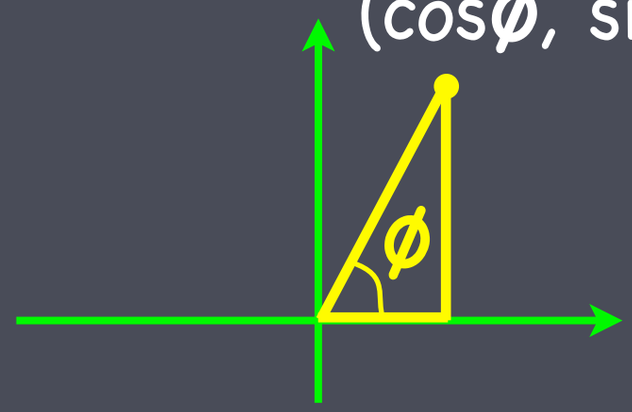

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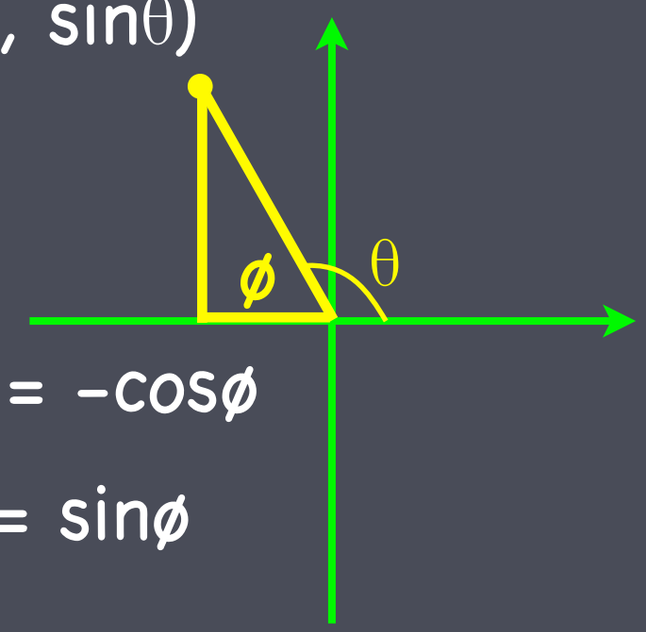

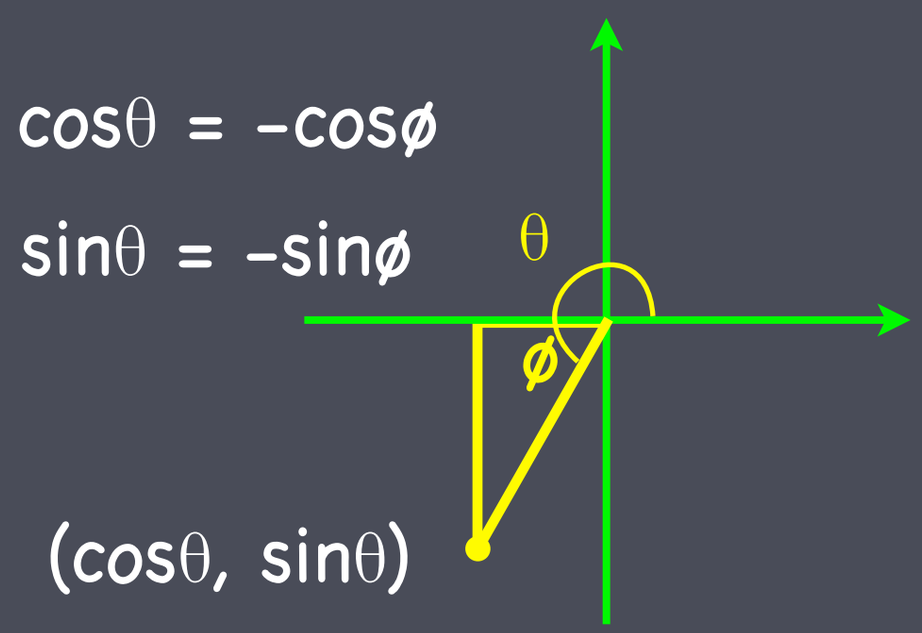

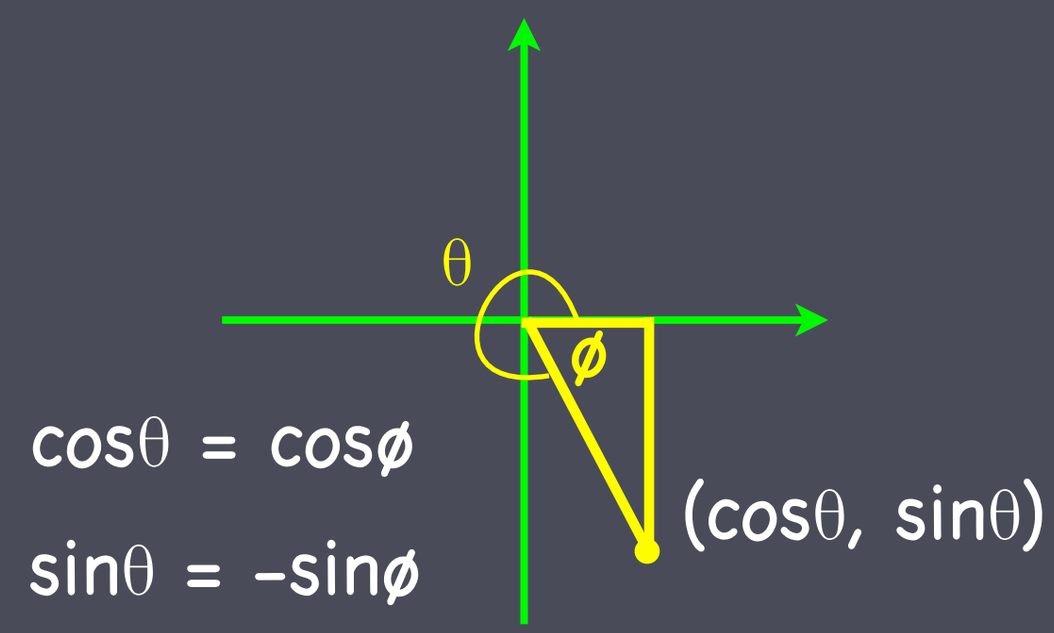

Trig review

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- Which of the following is not a trig identity?

(A) $1 + \cot^2\theta = \csc^2\theta$

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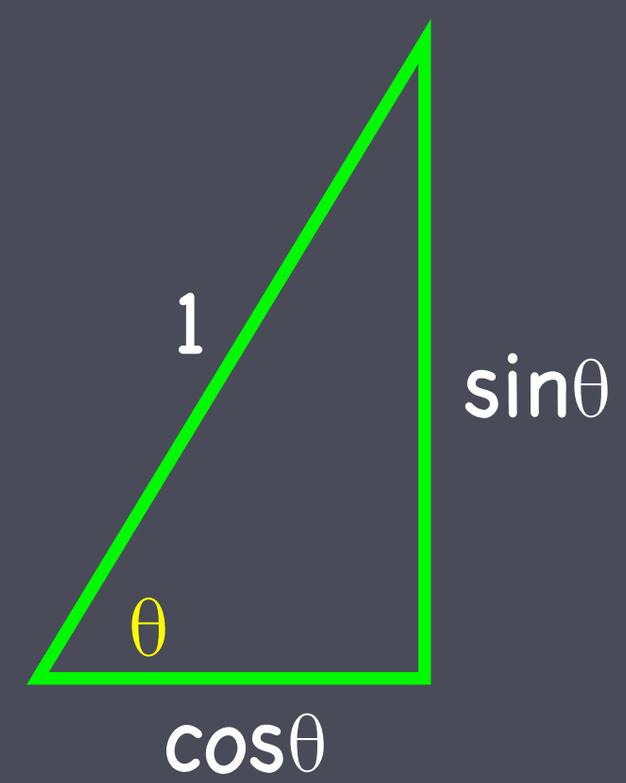

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$\sin^2\theta + \cos^2\theta = 1$

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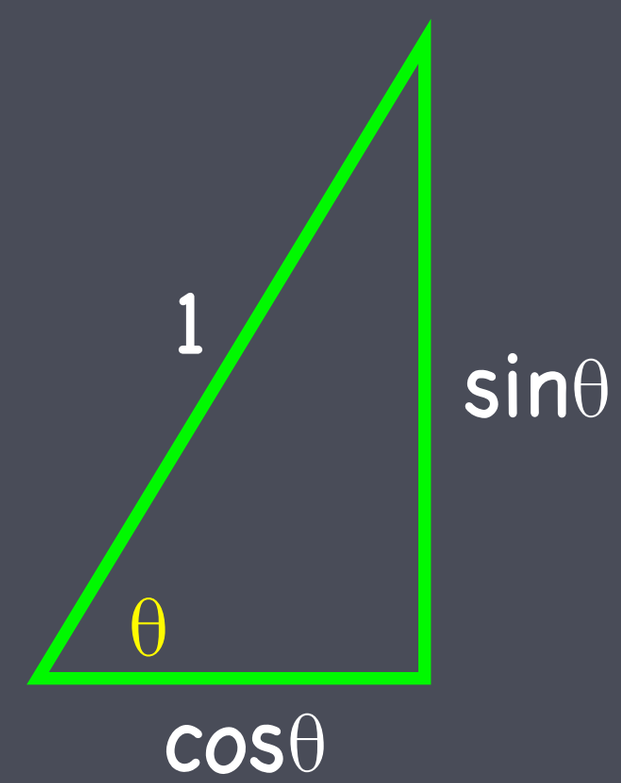

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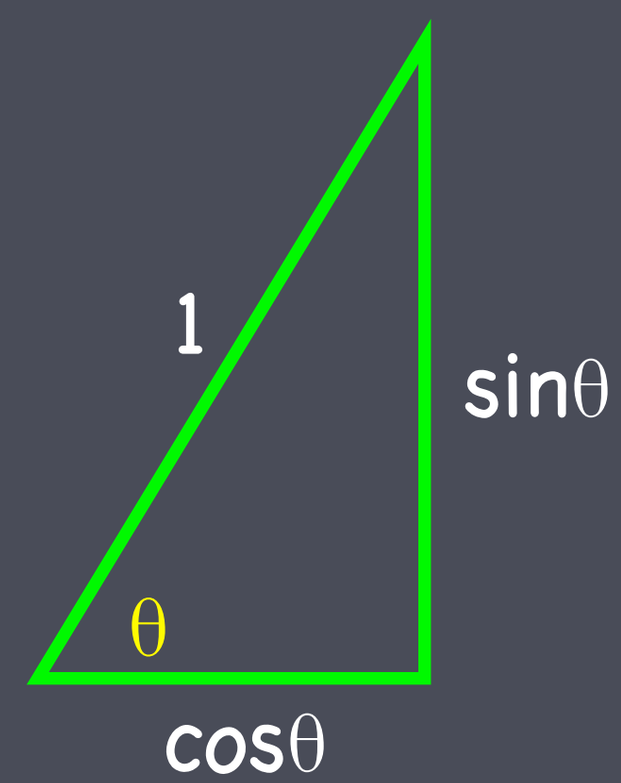

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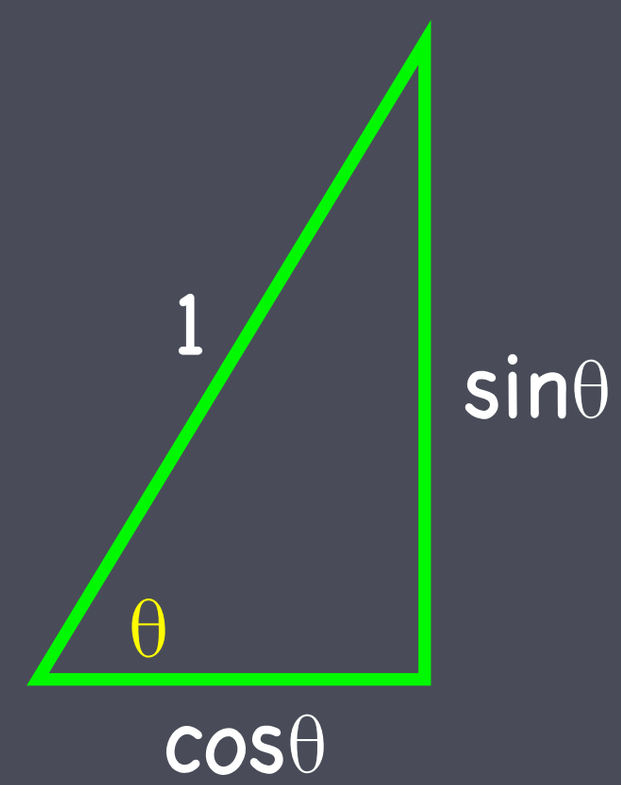

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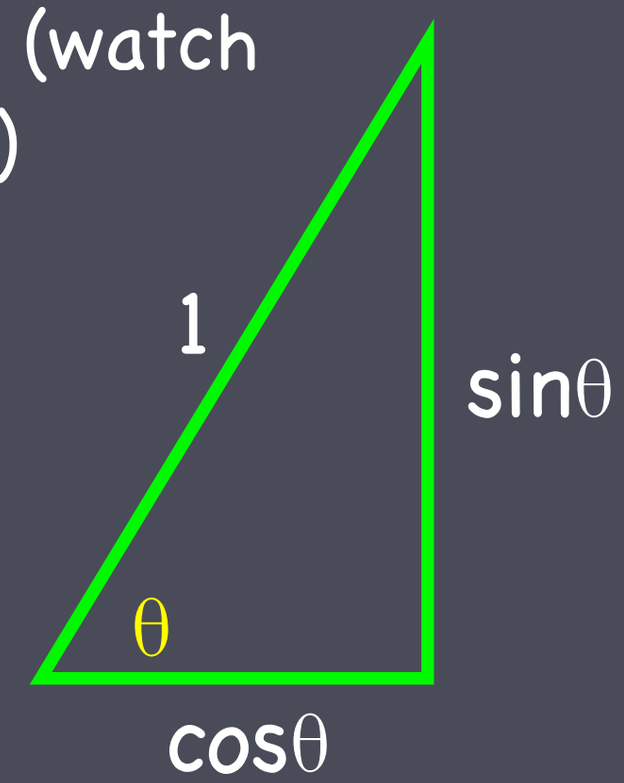

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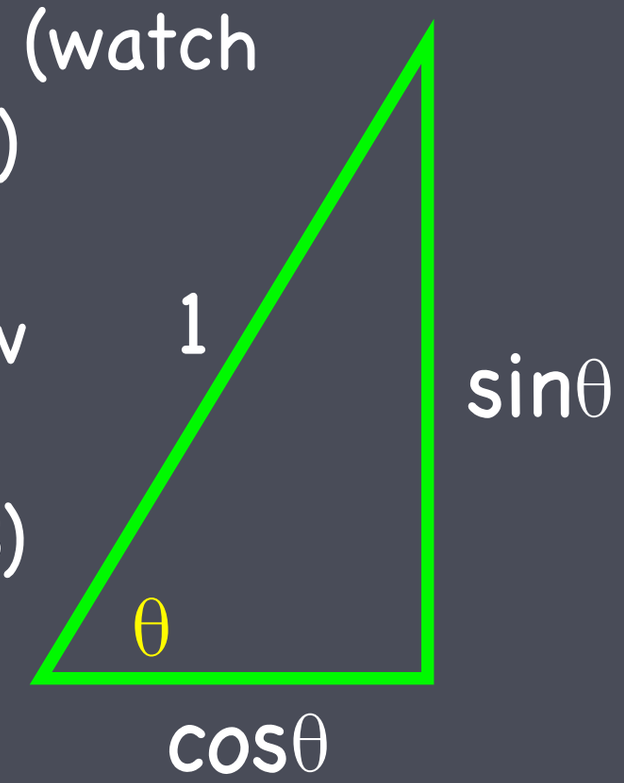
Know graphs, how to shift or use sin(A+B), cos(A+B)

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$



$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Trig review

cos(2π/3) =

(A) $\frac{\sqrt{3}}{2}$

(B) $-\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

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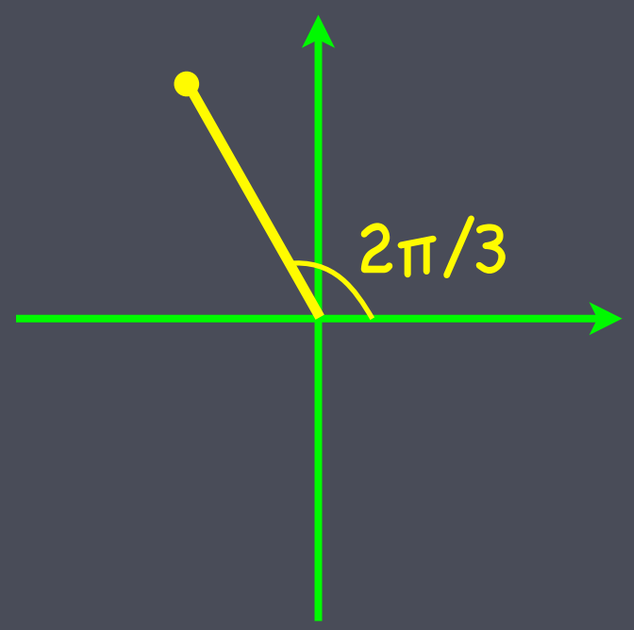

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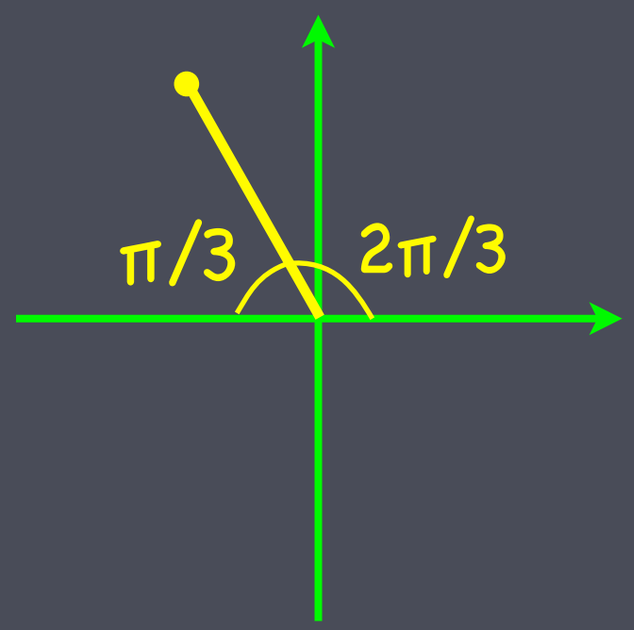

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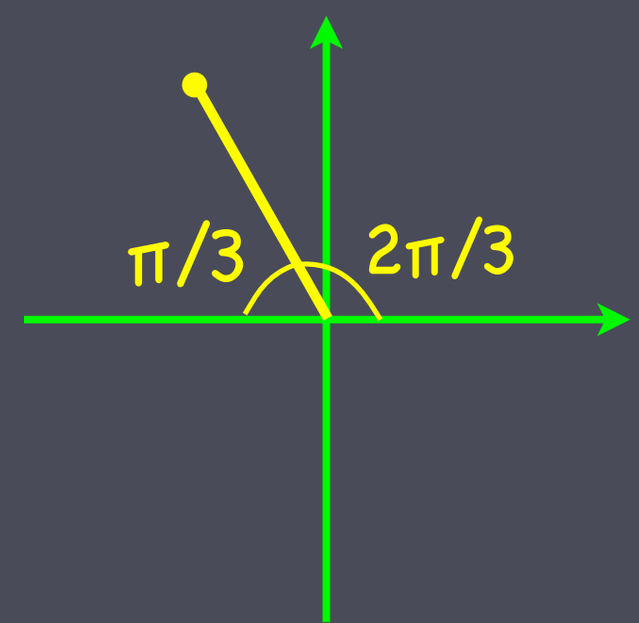

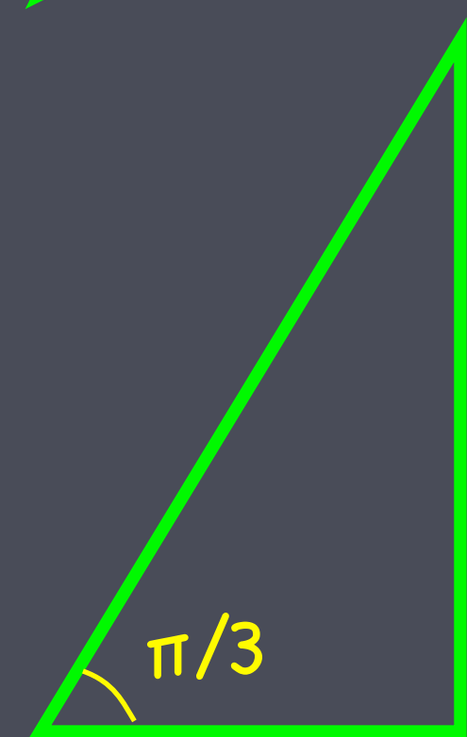

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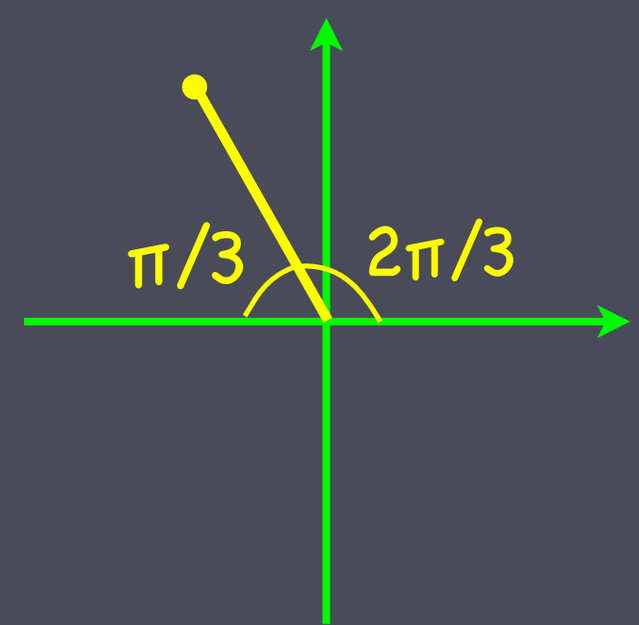

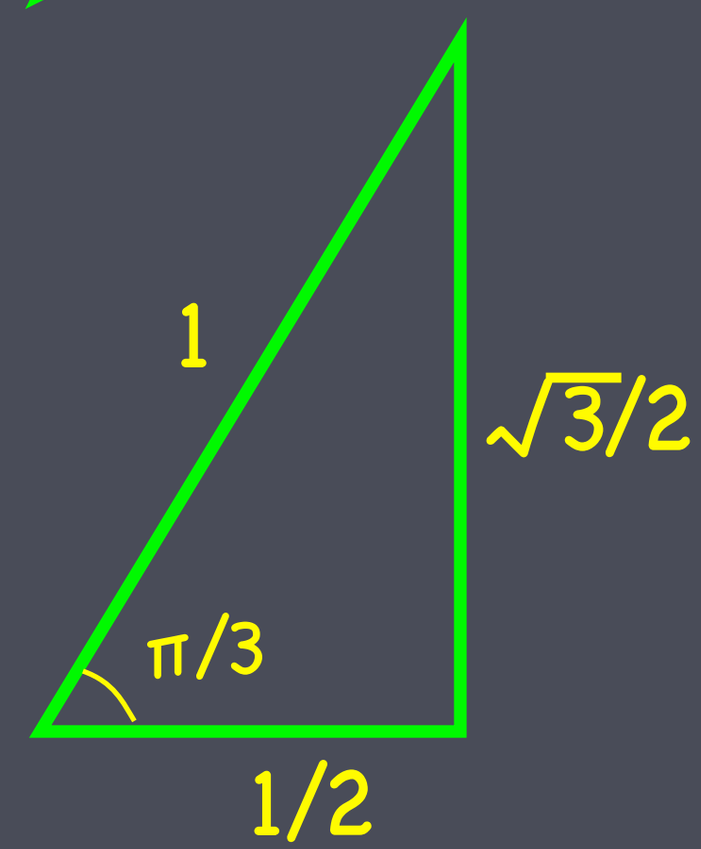

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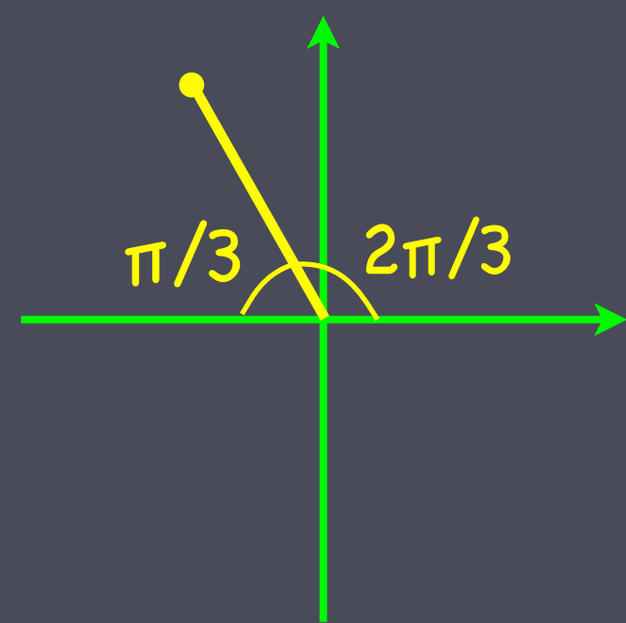
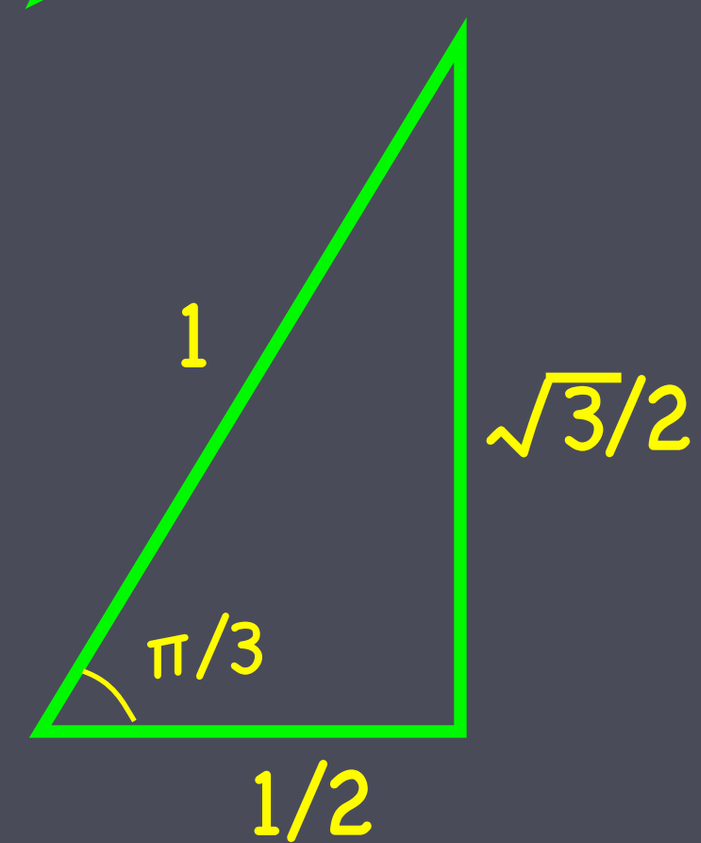


And 2π/3 is in Quad II so cos(2π/3) < 0.



Trig review

$$\tan(\pi/4) =$$

(A) $\frac{1}{\sqrt{2}}$

(B) 1

(C) $\sqrt{2}$

(D) $\frac{1}{2}$

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