Today

- Logistic equation in applications
- Trig review
Logistic equation in different contexts...
Rates of change that are proportional to two things

- Infectious disease: \( bSI \) (S=susceptible, I=infected)
- Spread of rumour: \( bNH \) (N = not heard rumour, H = heard rumour)
- Spread of new words: \( bNU \) (use word or not)
- Spread of new technologies: \( bNU \) (use tech or not)
- Active oil exploration sites: \( bUD \) (undiscovered and discovered)
- Waterlillies in a pond: \( bSW \) (waterlillies and space for waterwillies)
...two things that are just different forms of a single thing
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- When X meets Y, there's a chance Y turns into X.
...two things that are just different forms of a single thing

When \( X \) meets \( Y \), there’s a chance \( Y \) turns into \( X \).

Lose \( Y \): \[
\frac{dY}{dt} = -bXY
\]
and gain \( X \): \[
\frac{dX}{dt} = bXY
\]
...two things that are just different forms of a single thing

- When X meets Y, there's a chance Y turns into X.

- Lose Y: \( \frac{dY}{dt} = -bXY \) and gain X: \( \frac{dX}{dt} = bXY \)

- \( X+Y= \) constant = \( C \) so \( Y=C-X \).
...two things that are just different forms of a single thing

- When $X$ meets $Y$, there's a chance $Y$ turns into $X$.

- Lose $Y$: \[ \frac{dY}{dt} = -bXY \]
  and gain $X$: \[ \frac{dX}{dt} = bXY \]

- $X+Y$ = constant $= C$ so $Y=C-X$.

- \[ \frac{dX}{dt} = bX(C - X) \]
**Infectious disease**

Dr. Erin Mears: Once we know the $R_0$, we'll be able to get a handle on the scale of the epidemic.

**Minnesota Health #4:** So, it's an epidemic now. An epidemic of what?

**Dave:** We sent samples to the CDC.

**Dr. Erin Mears:** In seventy two hours, we'll know what it is, if we're lucky.

**Minnesota Health #4:** Clearly, we're not lucky.
Infectious disease
Infectious disease

N individuals, I of them have a flu, S=N−I do not.
Infectious disease

- $N$ individuals, $I$ of them have a flu, $S=N-I$ do not.

- If everyone interacts, new cases appear at a rate proportional to $SI$. 
Infectious disease

\( N \) individuals, \( I \) of them have a flu, \( S=N-I \) do not.

If everyone interacts, new cases appear at a rate proportional to \( SI \).

The DE describing the spread of disease:
Infectious disease

- $N$ individuals, $I$ of them have a flu, $S=N-I$ do not.

- If everyone interacts, new cases appear at a rate proportional to $SI$.

- The DE describing the spread of disease:

  (A) $\frac{dI}{dt} = -bI(N - I)$
  (B) $\frac{dI}{dt} = bI(N - I)$
  (C) $\frac{dS}{dt} = -bSI$
  (D) $\frac{dI}{dt} = bSI$
Infectious disease

N individuals, I of them have a flu, S=N−I do not.

If everyone interacts, new cases appear at a rate proportional to SI.

The DE describing the spread of disease:

\[ \frac{dI}{dt} = -bI(N - I) \]  \hspace{1cm}  \frac{dS}{dt} = -bSI \]  \hspace{1cm}  \frac{dI}{dt} = bI(N - I) \]  \hspace{1cm}  \frac{dI}{dt} = bSI \]

Compare this with \[ \frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right). \]
What is the carrying capacity?

\[ \frac{dI}{dt} = bI(N - I) \]

(A) \( \frac{b}{N} \)  
(B) \( \frac{N}{b} \)  
(C) \( I \)  
(D) \( N \)
Infectious disease

What is the carrying capacity?

\[ \frac{dI}{dt} = bI(N - I) \]

(A) \ b/N \hspace{2cm} (C) \ I

(B) \ N/b \hspace{2cm} (D) \ N

\[ \frac{dP}{dt} = rP \left( 1 - \frac{P}{K} \right) \]
Infectious disease

What is the carrying capacity?

\[
\frac{dI}{dt} = bI(N - I) = bNI \left(1 - \frac{I}{N}\right)
\]

(A) $b/N$

(B) $N/b$

(C) $I$

(D) $N$

\[
\frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right)
\]
Infectious disease

What is the carrying capacity?

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\frac{dI}{dt} = bI(N - I) = bNI \left(1 - \frac{I}{N}\right)
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(A) b/N  (C) I

(B) N/b  (D) N

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\frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right)
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Infectious disease

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\frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right)
\]
Infectious disease

What is the carrying capacity?

\[
\frac{dI}{dt} = bI(N - I) = bNI \left(1 - \frac{I}{N}\right)
\]

(A) b/N

(B) N/b

(C) I

(D) N

Everyone gets sick!

\[
\frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right)
\]
Infectious disease
Infectious disease

Suppose infected people recover at a rate proportional to how many there are.
Infectious disease

Suppose infected people recover at a rate proportional to how many there are.

The DE describing the spread of disease with recovery:

\[ \frac{dI}{dt} = bI(N - I) - \mu S \]  \hspace{1cm} (A)

\[ \frac{dI}{dt} = bI(N - I) - \mu I \]  \hspace{1cm} (B)

\[ \frac{dI}{dt} = -bI(N - I) + \mu I \]  \hspace{1cm} (C)

\[ \frac{dI}{dt} = bI(N - I) + \mu I \]  \hspace{1cm} (D)
Infectious disease

\[ \frac{dI}{dt} = bI(N - I) - \mu I \]
Infectious disease

\[ \frac{dI}{dt} = bI(N - I) - \mu I \]

\[ = bIN - bI^2 - \mu I \]
Infectious disease

\[ \frac{dI}{dt} = bI(N - I) - \mu I \]

\[ = bIN - bI^2 - \mu I \]

\[ = bI \left( N - \frac{\mu}{b} - I \right) \]
Infectious disease

\[
\frac{dI}{dt} = bI(N - I) - \mu I
\]

\[
= bIN - bI^2 - \mu I
\]

\[
= bI \left( N - \frac{\mu}{b} + I \right)
\]
Infectious disease

\[
\frac{dI}{dt} = bI(N - I) - \mu I
\]

\[
= bIN - bI^2 - \mu I
\]

\[
= bI \left( N - \frac{\mu}{b} - I \right)
\]

If \( \frac{\mu}{b} > N \) then

\[
K = N - \frac{\mu}{b} < 0
\]
Infectious disease

\[
\frac{dI}{dt} = bI(N - I) - \mu I
\]

\[
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\frac{dI}{dt} = bI(N - I) - \mu I
\]

\[
= bIN - bI^2 - \mu I
\]

\[
= bI \left(N - \frac{\mu}{b} + I\right)
\]

If \( \frac{\mu}{b} > N \) then

\[
K = N - \frac{\mu}{b} < 0
\]

and the disease dies out.
Infectious disease

\[
\frac{dI}{dt} = bI(N - I) - \mu I
\]

\[
= bIN - bI^2 - \mu I
\]

\[
= bI \left( N - \frac{\mu}{b} + I \right)
\]
Infectious disease

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\frac{dI}{dt} = bI(N - I) - \mu I
\]

\[
= bIN - bI^2 - \mu I
\]

\[
= bI \left( N - \frac{\mu}{b} + I \right)
\]

\[
K = N - \frac{\mu}{b} > 0
\]

If \( \frac{\mu}{b} < N \) then
Infectious disease

\[
\frac{dI}{dt} = bI(N - I) - \mu I
\]

\[
= bIN - bI^2 - \mu I
\]

\[
= bI \left( N - \frac{\mu}{b} - I \right)
\]

If \( \frac{\mu}{b} < N \) then

\[K = N - \frac{\mu}{b} > 0\]
Infectious disease

\[ \frac{dI}{dt} = bI(N - I) - \mu I \]
\[ = bIN - bI^2 - \mu I \]
\[ = bI \left( N - \frac{\mu}{b} - I \right) \]

If \( \frac{\mu}{b} < N \) then
\[ K = N - \frac{\mu}{b} > 0 \]
and the disease becomes an epidemic.

\( K \)
Infectious disease

\[
\frac{dI}{dt} = bI(N - I) - \mu I \\
= bIN - bI^2 - \mu I \\
= bI \left( N - \frac{\mu}{b} - I \right)
\]

If \( \frac{\mu}{b} < N \) then

\[
K = N - \frac{\mu}{b} > 0
\]

and the disease becomes an epidemic.

\[
R_0 = \frac{Nb}{\mu} > 1
\]
Infectious disease

If \[ \frac{\mu}{b} < N \] then

\[ K = N - \frac{\mu}{b} > 0 \]

and the disease becomes an epidemic.

\[ R_0 = \frac{Nb}{\mu} > 1 \]
Infectious disease

$$\frac{dI}{dt} = bI\left(\frac{N - I}{2}\right)$$

$$\mu I = bIN$$

$$\downarrow N\mu \iff \mu < N$$

If $\frac{\mu}{b} < N$ then

$$K = N - \frac{\mu}{b} > 0$$

and the disease becomes an epidemic.

$$R_0 = \frac{Nb}{\mu} > 1$$
If $\theta$ is measured counterclockwise from the positive x axis we define sin and cos so that

(A) $x = \sin(\theta)$, $y = \tan(\theta)$.

(B) $x = \tan(\theta)$, $y = \sin(\theta)$.

(C) $x = \sin(\theta)$, $y = \cos(\theta)$.

(D) $x = \cos(\theta)$, $y = \sin(\theta)$. 
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(D) $x=\cos(\theta)$, $y=\sin(\theta)$. 

Trig review
Trig review

Learn special angles in Quad I and modify signs for other Quads.

$(\cos\phi, \sin\phi)$
Trig review

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Trig review

Learn special angles in Quad I and modify signs for other Quads.

\[(\cos \phi, \sin \phi)\]

\[(\cos \theta, \sin \theta)\]

\[\cos \theta = -\cos \phi\]
Trig review

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\[(\cos \theta, \sin \theta)\]

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$$\cos \theta = -\cos \phi$$
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\cos \theta = -\cos \phi \\
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\cos \phi = -\cos \theta \\
\sin \phi = \sin \theta
\]
Trig review

Learn special angles in Quad I and modify signs for other Quads.

\[ \cos(\theta) = -\cos(\phi) \]
\[ \sin(\theta) = -\sin(\phi) \]

\[ \cos(\theta) = \cos(\phi) \]
\[ \sin(\theta) = -\sin(\phi) \]
Trig review

- The other trig functions:
Trig review

The other trig functions:

\[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]
Trig review

The other trig functions:

- \( \tan \theta = \frac{\sin \theta}{\cos \theta} \)
- \( \csc \theta = \frac{1}{\sin \theta} \)
Trig review

The other trig functions:

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\csc\theta = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
Trig review

The other trig functions:

- \( \tan \theta = \frac{\sin \theta}{\cos \theta} \)
- \( \csc \theta = \frac{1}{\sin \theta} \)
- \( \sec \theta = \frac{1}{\cos \theta} \)
- \( \cot \theta = \frac{1}{\tan \theta} \)
Which of the following is not a trig identity?

(A) \(1 + \cot^2 \theta = \csc^2 \theta\)

(B) \(\tan^2 \theta + 1 = \sec^2 \theta\)

(C) \(\sin(2 \theta) = 2 \sin \theta \cos \theta\)

(D) \(\cos(\theta) = \sin(\theta - \pi/2)\)

(E) \(\sin(\theta) = \cos(\theta - \pi/2)\)
Trig review

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$\cos(A+B) = \cos A \cos B - \sin A \sin B$
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Trig review

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$\sin^2 \theta + \cos^2 \theta = 1$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$
Trig review

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$\sin \theta \cos \theta = \sin(A+B) = \cos A \cos B - \sin A \sin B$
**Trig review**

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\[\sin^2 \theta + \cos^2 \theta = 1\]

\[\cos^2 \theta \div \cos^2 \theta \div \cos^2 \theta\]
Trig review

Which of the following is not a trig identity?

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(D) $\cos(\theta) = \sin(\theta-\pi/2)$

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$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta \quad \cos^2\theta \quad \cos^2\theta$

$\sin\theta \quad \cos\theta \quad 1$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$
Trig review

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(D) \(\cos(\theta) = \sin(\theta - \pi/2)\)

(E) \(\sin(\theta) = \cos(\theta - \pi/2)\)

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

\[
\frac{\cos^2 \theta}{\cos^2 \theta} \quad \frac{\cos^2 \theta}{\cos^2 \theta} = 1
\]

\[
\cos(A+B) = \cos A \cos B - \sin A \sin B
\]

Use \(\sin(A+B)\) (watch today’s 2\(^{nd}\) video)

Know graphs, how to shift or use \(\sin(A+B), \cos(A+B)\)
Trig review

$\cos(2\pi/3) =$

(A) $\frac{\sqrt{3}}{2}$

(B) $-\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$
Trig review

\[ \cos\left(\frac{2\pi}{3}\right) = \]

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Trig review

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Trig review

\[ \cos \left( \frac{2\pi}{3} \right) = \]

(A) \[ \frac{\sqrt{3}}{2} \]

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(C) \[ \frac{1}{2} \]

(D) \[ -\frac{1}{2} \]
Trig review

\[
\cos(2\pi/3) = 
\]

(A) \( \frac{\sqrt{3}}{2} \)

(B) \( -\frac{\sqrt{3}}{2} \)

(C) \( \frac{1}{2} \)

(D) \( -\frac{1}{2} \)

And \( 2\pi/3 \) is in Quad II so \( \cos(2\pi/3) < 0 \).
Trig review

\[ \tan \left( \frac{\pi}{4} \right) = \]

(A) \( \frac{1}{\sqrt{2}} \)

(B) 1

(C) \( \sqrt{2} \)

(D) \( \frac{1}{2} \)
Trig review

\[ \tan \left( \frac{\pi}{4} \right) = \]

(A) \( \frac{1}{\sqrt{2}} \)

(B) 1

(C) \( \sqrt{2} \)

(D) \( \frac{1}{2} \)