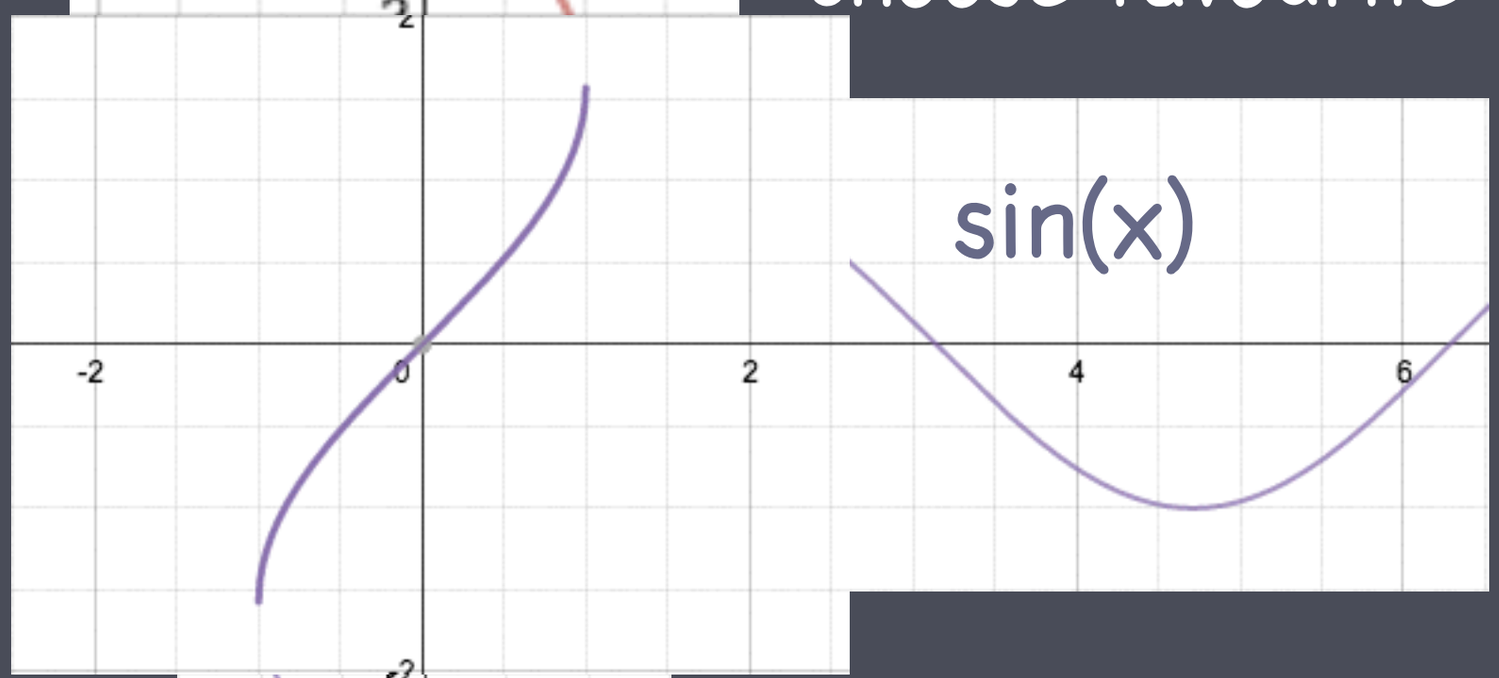
Today

- Inverse trig review
- Derivatives of inverse (trig) functions
- Mono-mono twins

Inverse trig

- $f(x) = \sin(x) \dashrightarrow f^{-1}(x) = \arcsin(x)$
- Flip sin(x) about y=x.
- Many angles (x) have the same sin(x) so mirror image is not a function - must choose favourite values for arcsin.



arcsin(x)

The domain of arcsin is...

(A) $(-\pi/2, \pi/2)$

(B) $[-\pi/2, \pi/2]$

(C) $[0, \pi]$

(D) $(-1, 1)$

(E) $[-1, 1]$

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The range for arcsin(x) is...

(A) [-1, 1]

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(E) (-infinity, infinity)

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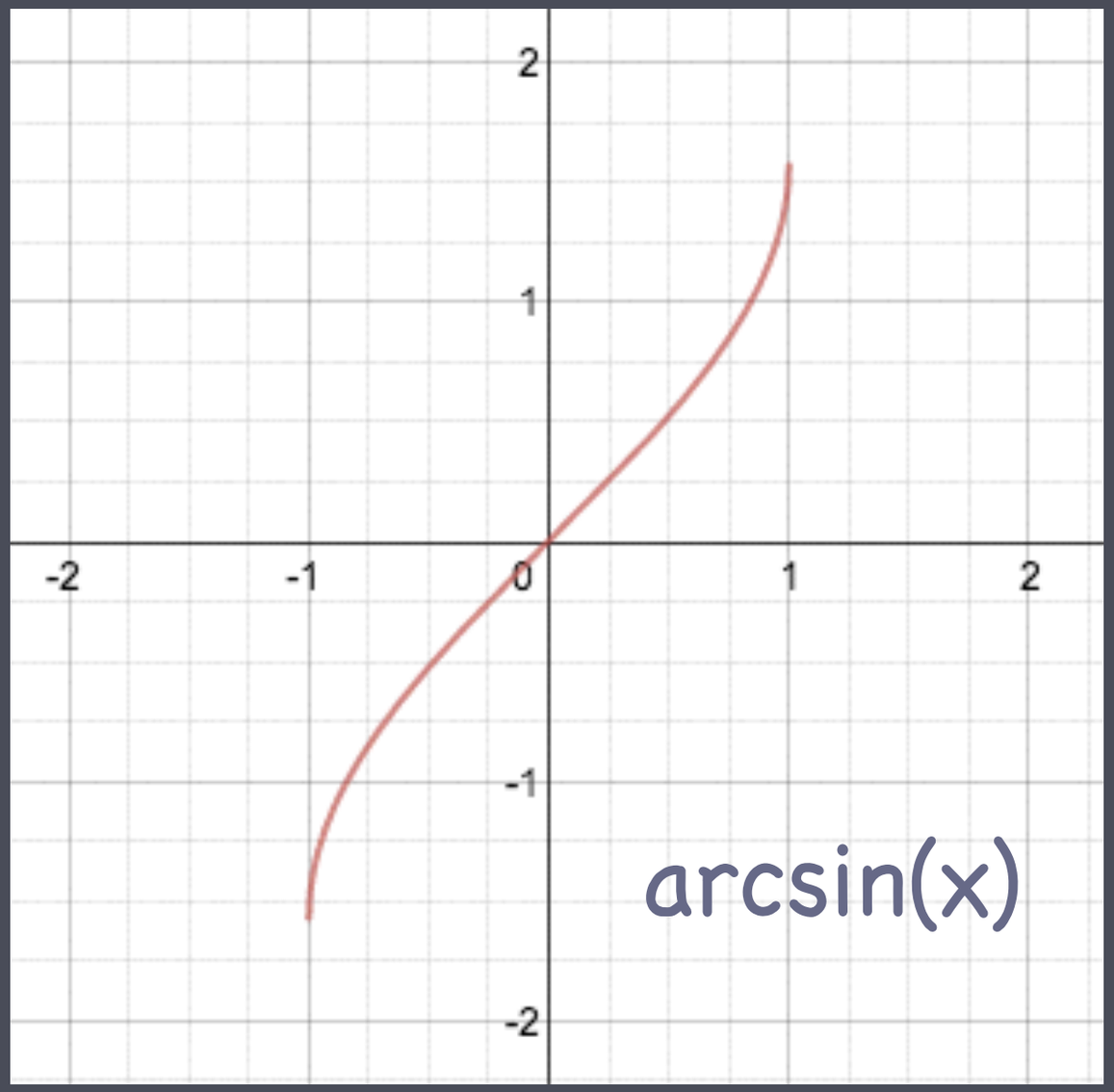


Derivatives of inverse trig functions

- For ANY inverse function, find its derivative implicitly...
- $y = \arcsin(x)$
- $\sin(y) = x$ <--- rewrite in inverted mode
- $\cos(y)\, y' = 1$ <--- take implicit derivative
- $y' = 1/\cos(y) = 1/\cos(\arcsin(x))$ <--- solve for y'

$\cos(\arcsin(x)) = \ldots$

(A) $\mathrm{sqrt}(1-x^2)$

(B) $1/\mathrm{sqrt}(1-x^2)$

(C) $x/\mathrm{sqrt}(1-x^2)$

(D) $\mathrm{sqrt}(1-x^2)/x$

(E) $1/x$

cos(arcsin(x)) = . . .

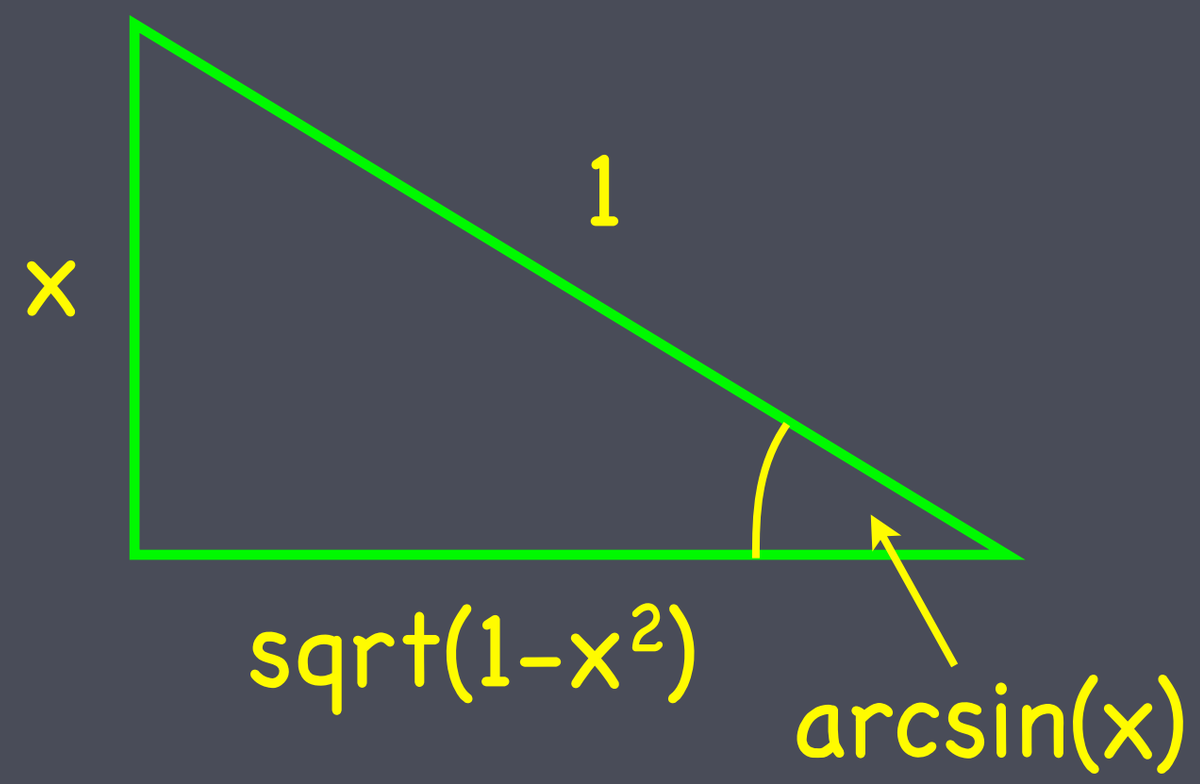
(A) sqrt($1-x^2$)

(B) 1/sqrt($1-x^2$)

(C) x/sqrt($1-x^2$)

(D) sqrt($1-x^2$)/x

(E) 1/x



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- $y' = 1/\cos(y) = 1/\cos(\arcsin(x))$ <--- solve for y'
- $y' = 1/\mathrm{sqrt}(1-x^2)$

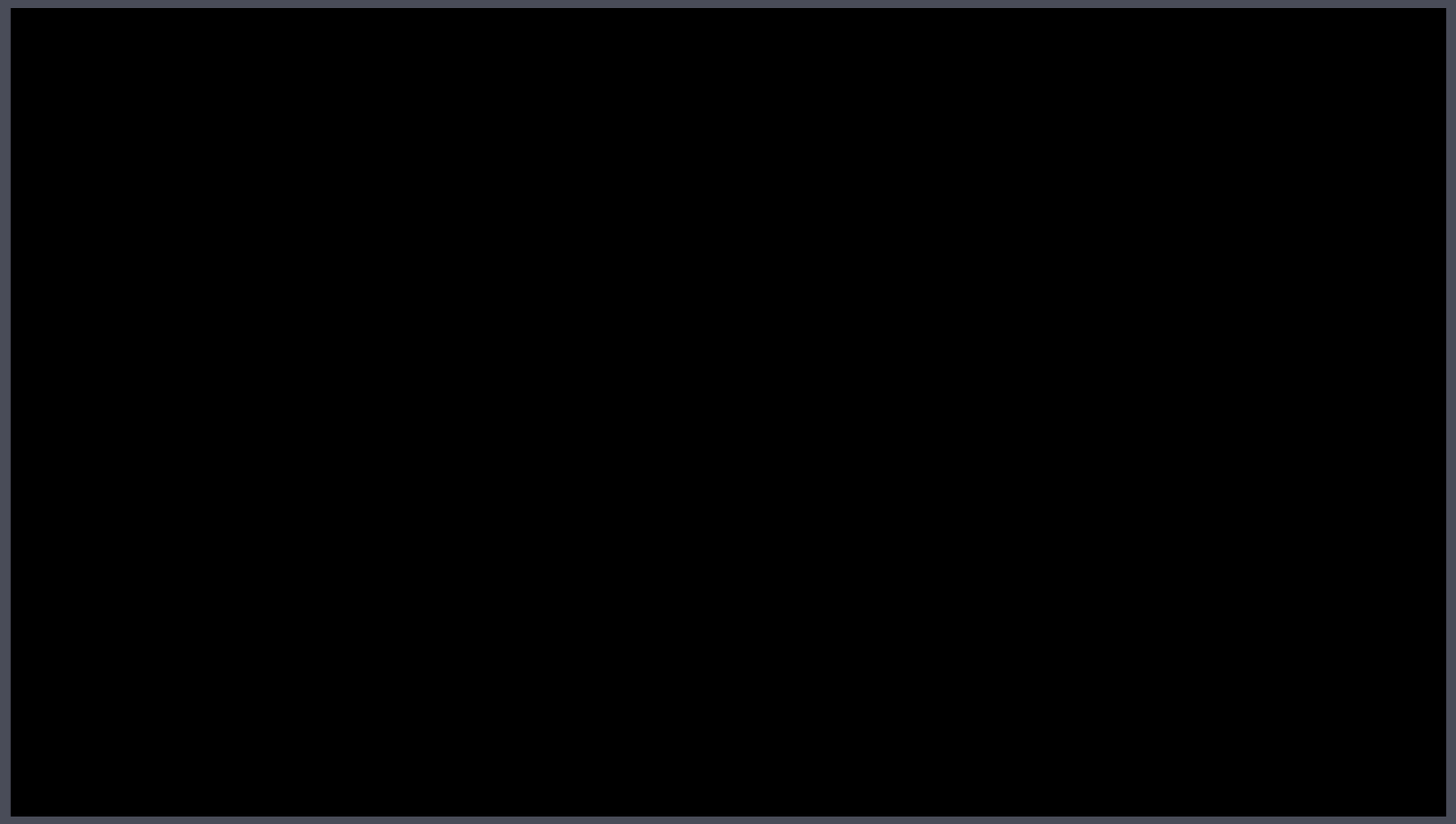
Wrap up example...

- A "real-life" optimization problem starring
  - least squares fitting,
  - Hill functions,
  - exponential decay,
  - two very small people,
  - a very serious question about survival.

Why I teach calculus

- People often make decisions
  - based on emotional responses,
  - with insufficient information
  - without thinking carefully and
  - without even knowing that they could do otherwise.
- Sometimes this is just fine.
- Other times, they should have used math.
- Quantitative thinking (not necessarily calculus) is my target (even though calculus is the vehicle).

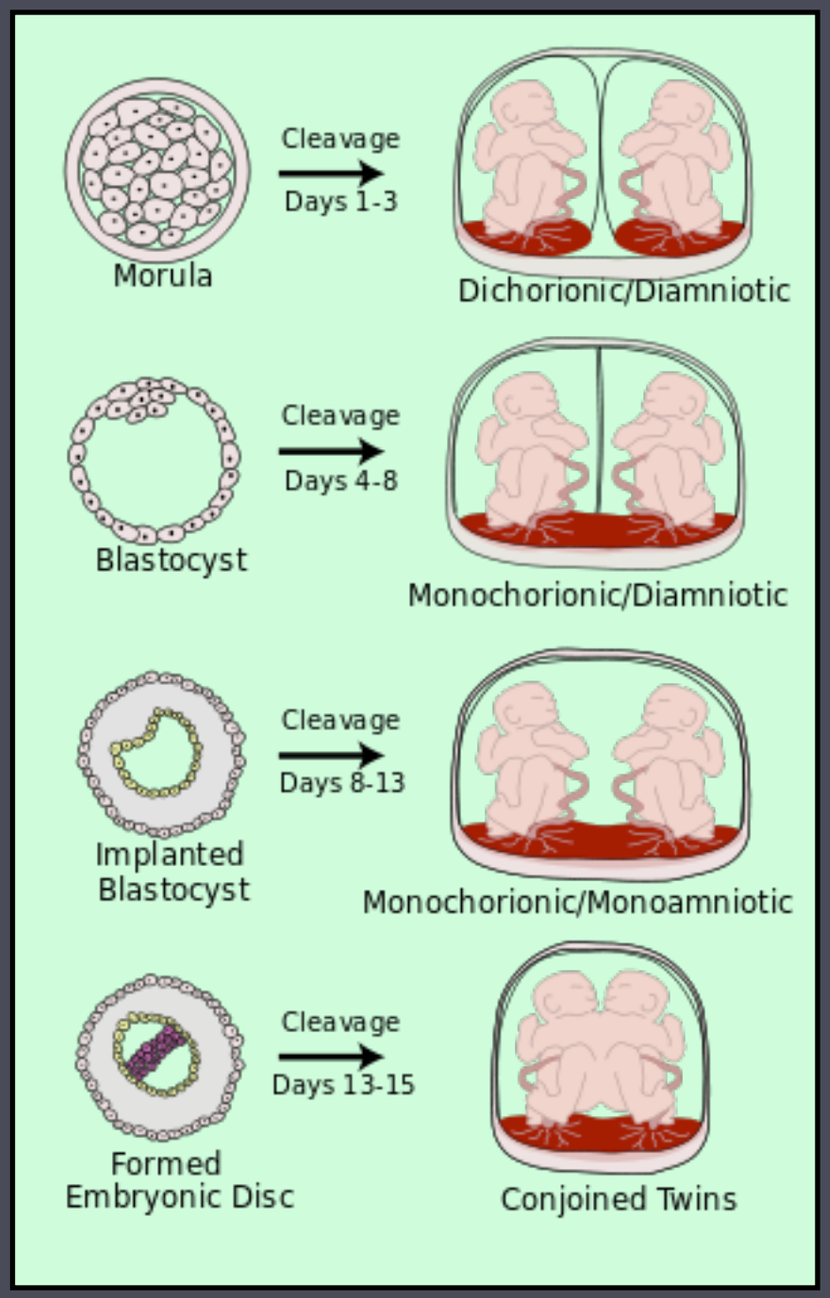
The main characters



Pregnancy

- For singletons, full term is 40 weeks.
- For twins, "full term" is 37 weeks.
- Twins are often premie. Premie survival is uncertain.
  - Babies delivered at 22 weeks rarely survive.
  - About 50% of babies delivered at 24 weeks survive.
  - By 32 weeks, prospects are very good.
  - 34 weeks is almost the same as full term.

Monochorionic/ Monoamniotic twins



- ~1 in 50,000 pregnancies
- 1% of twins
- ~8/yr in Canada

Complications

- Cord entanglement
- Cord compression
- Twin-to-twin transfusion syndrome

Risks and interventions

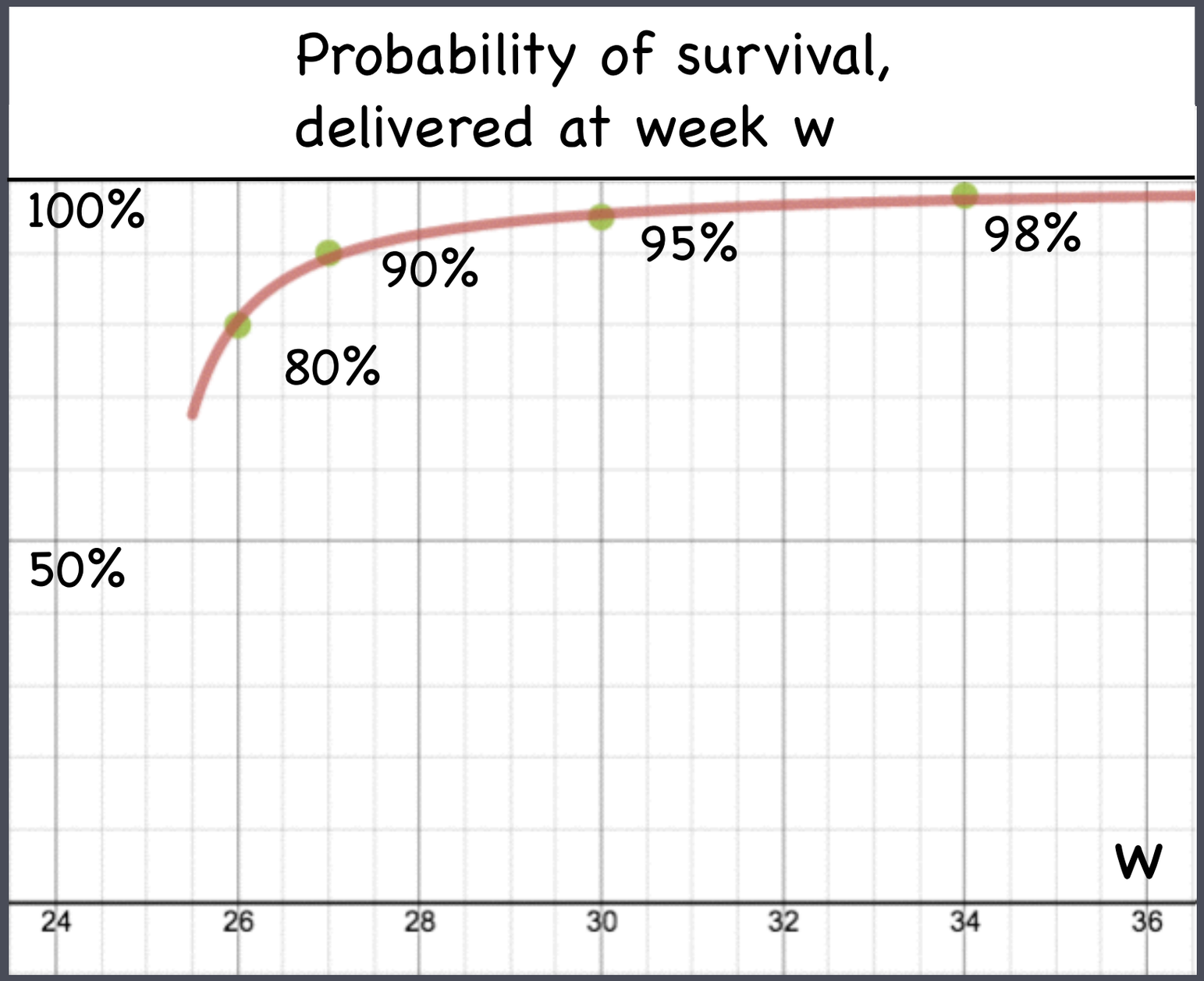
- 50% of unmonitored mono-mono twins survive to birth.
- Survival goes up to 81-95% with monitoring.
- Monitoring involves twice-daily ultrasounds, listening to heart beats for irregularities.
- Delivery is by scheduled c-section at 32-34 weeks or earlier by emergency c-section if monitoring reveals problems.

When to schedule a c-section

- Schedule it too late, risk of mono-mono complications goes up.
- Schedule it too early, risk of death due to prematurity goes up.
- Consider a simple scenario for which analysis is possible (by MATH 102 students)
  - Without monitoring, when should you schedule a c-section?

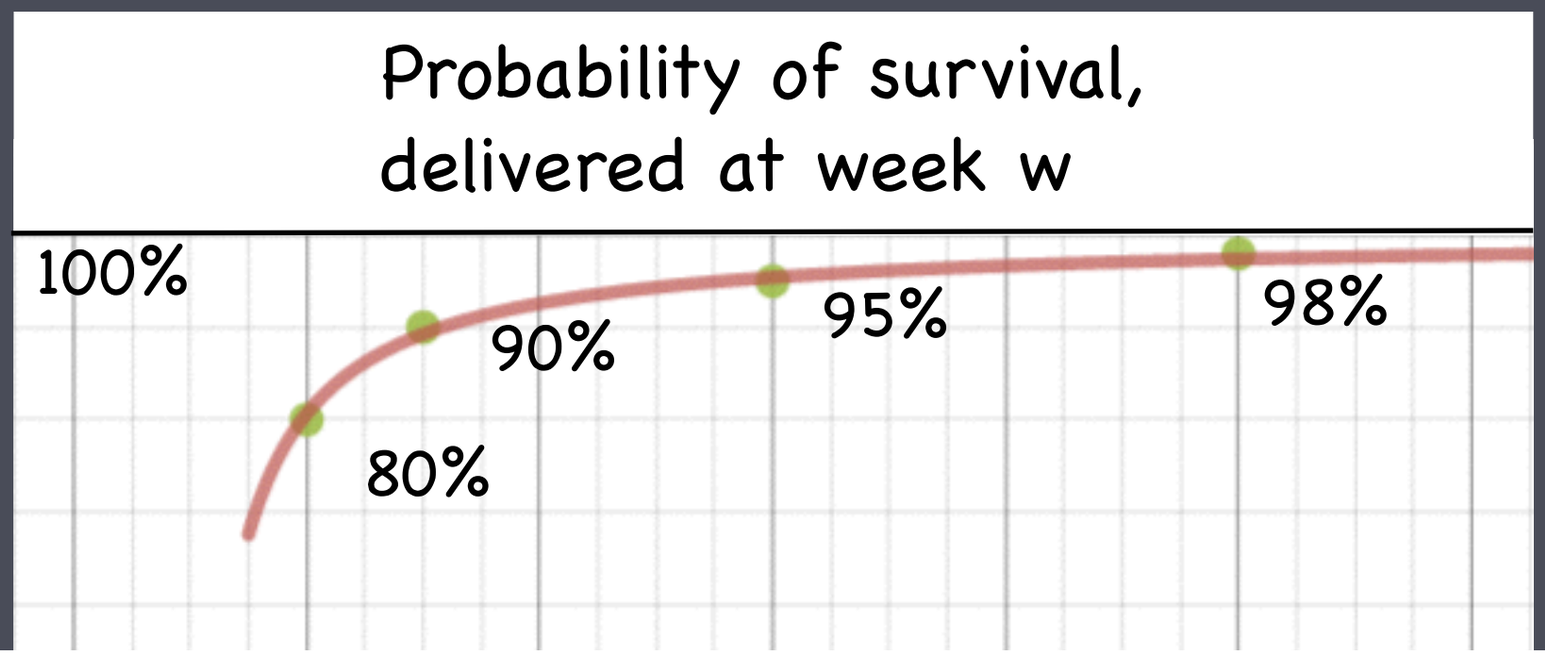
When to schedule a c-section

- Collect premie survival data.
- Fit a smooth curve (Hill function).



When to schedule a c-section

- Solve #5a, MT2 2012.



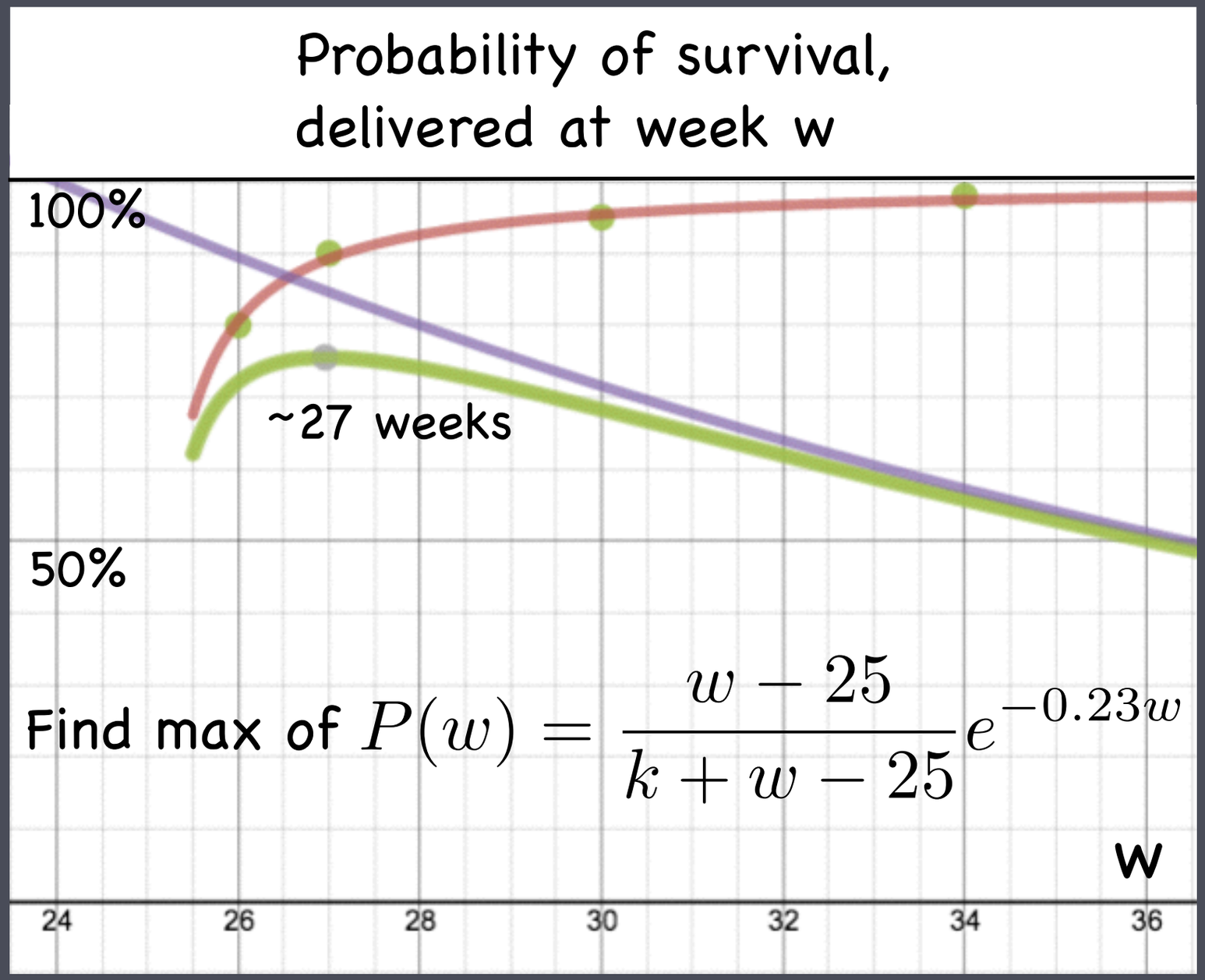
Long Answer Problems

5. During pregnancy, a fraction of twins are at risk of death due to entanglement of their umbilical cords. To prevent such deaths, the twins are removed early by surgery (Caesarean section). However, removing them earlier puts them at risk of death due to premature birth. In this problem, you will determine when to schedule delivery so as to maximize their chance of survival in the face of these opposing risk factors.

   (a) **[5 pt]** Out of 25 such pregnancies that were identified at 24 weeks into the pregnancy, 16 were still alive at 32 weeks. Write down an exponential function that describes the number of surviving twins, T(t), as a function of the time (measured in weeks) into the pregnancy.

When to schedule a c-section

- Solve #5a, MT2 2012.
- Want prob of "surviving until delivery AND surviving as a premie from delivery":
- Red x purple
- Optimum at 27 weeks
- Delivery one week later means 2% lower survival probability. That's 4-6 fewer babies per year in NA.



The more complicated model - with monitoring

- The question: when to start monitoring, when to schedule a (no-later-than) c-section.
- Must account for
  - emergency c-section at any time during monitoring,
  - false alerts, missed events.
- Requires integration and much more data.