Today

- Newton’s method.
- Inc/dec, critical points and extrema.
Quiz 2 Friday - Linear approx and Newton's method only have one prob each on Assignment 4, should have been more. See "ExtraPractice" set for more.
Newton's method

NM is used to find zeros of a function: \( f(x) = 0 \).

It can be applied to finding approximates of

- **critical points** of a function \( g(x) \)
- (i.e. zeros of \( g'(x) \)):
  - define \( f(x) = g'(x) \),

- **intersections** of functions, \( g(x) = h(x) \):
  - define \( f(x) = g(x) - h(x) \),

- **irrational numbers**: e.g. \( \sqrt[3]{2} \):
  - define \( f(x) = x^3 - 2 \).
Find the zero of \( f(x) = x^3 - 2 \).

- Start with a “guesstimate” \( x_0 \).
- Get a “better” estimate \( x_1 \) by finding the tangent line and following it to the \( x \)-axis.
- Repeat to get \( x_2 \)…
Calculating successive estimates

First, find tangent line at \( x_n \):

\[
L(x) = f(x_n) + f'(x_n)(x-x_n).
\]

Find x-intercept, that will be \( x_{n+1} \):

(A) \( x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \).
(B) \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \).
(C) \( x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)} \).
(D) \( x_{n+1} = x_n + \frac{f'(x_n)}{f(x_n)} \).
Calculating successive estimates

First, find tangent line at $x_n$:

- $L(x) = f(x_n) + f'(x_n)(x-x_n)$.

Find $x$-intercept, that will be $x_{n+1}$:

(A) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$.
(B) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.
(C) $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$.
(D) $x_{n+1} = x_n + \frac{f'(x_n)}{f(x_n)}$.
To estimate $\sqrt{3}$, which function would you apply Newton's method to?

(A) $f(x) = x^{1/2}$
(B) $f(x) = x^{1/2} - 3$
(C) $f(x) = x - 3^{1/2}$
(D) $f(x) = x^2 - 3$
(E) $f(x) = (x-3)^{1/2}$
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This one has a zero at $\sqrt{3}$ but requires that you know $\sqrt{3}$ to use NM to estimate $\sqrt{3}$.  

$\sqrt{3}$  

$\sqrt{3}$
Estimate $\sqrt{3}$ using Newton's method with initial guess $x_0=2$.

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

(A) 7/4
(B) 97/56
(C) 1.7
(D) 1.73205080757

Finished already? Now use linear approximation. Which approach is better?
Estimate $\sqrt{3}$ using Newton's method with initial guess $x_0=2$.

(A) $\frac{7}{4} = 1.75 \quad \text{--- } x_1$

(B) $\frac{97}{56} = 1.73214 \quad \text{--- } x_2$

(C) 1.7

(D) 1.73205080757 \quad \text{--- first 11 digits of } \sqrt{3}$.
How (not) to choose $x_0$

Desmos:

- https://www.desmos.com/calculator/nyquvpptn5
- https://www.desmos.com/calculator/ahkrp3jamj
Increasing/decreasing

We say a function is **increasing** on some interval if for any points $a$ and $b$ with $a < b$ we have that $f(a) < f(b)$.

When $f'$ exists, same as $f'(x) > 0$.

We say a function is **decreasing** on some interval if for any points $a$ and $b$ with $a < b$ we have that $f(a) > f(b)$.

When $f'$ exists, same as $f'(x) < 0$.

Notice - no reference to $f'(x)$!!
Local extrema (min/max)

A point \( a \) is a **local minimum** of a function \( f(x) \) provided that \( f(x) > f(a) \) for all \( x \) on an interval around \( a \) (excluding \( a \), of course).

Which of the following is a local minimum?

If the function is differentiable at the minimum, then it must look like (A).