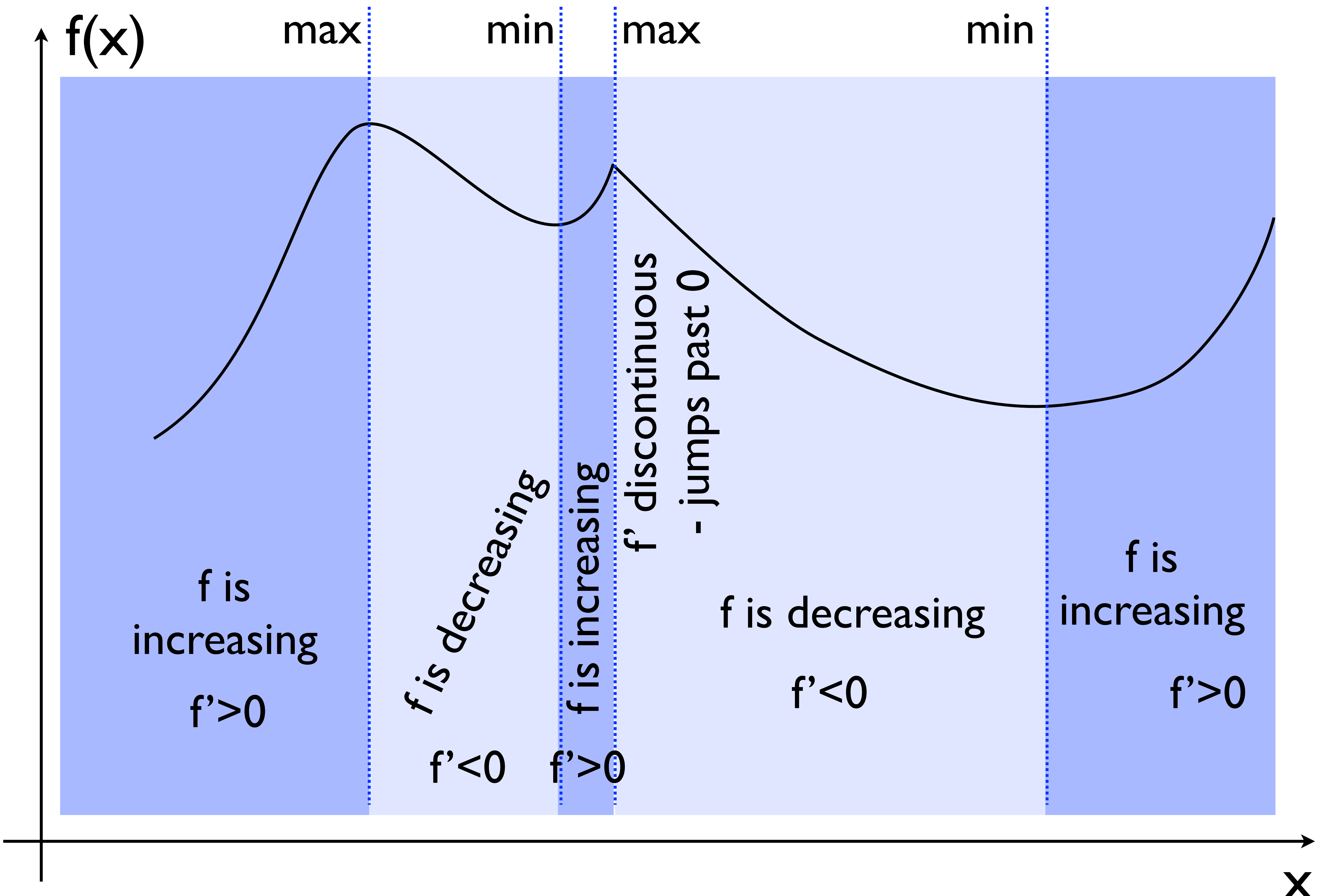
Today

- Putting it all together - using f, f′ and f″ to sketch a graph.
- Absolute extrema
- Intro to optimization

Using f, f′ and f″ to graph f

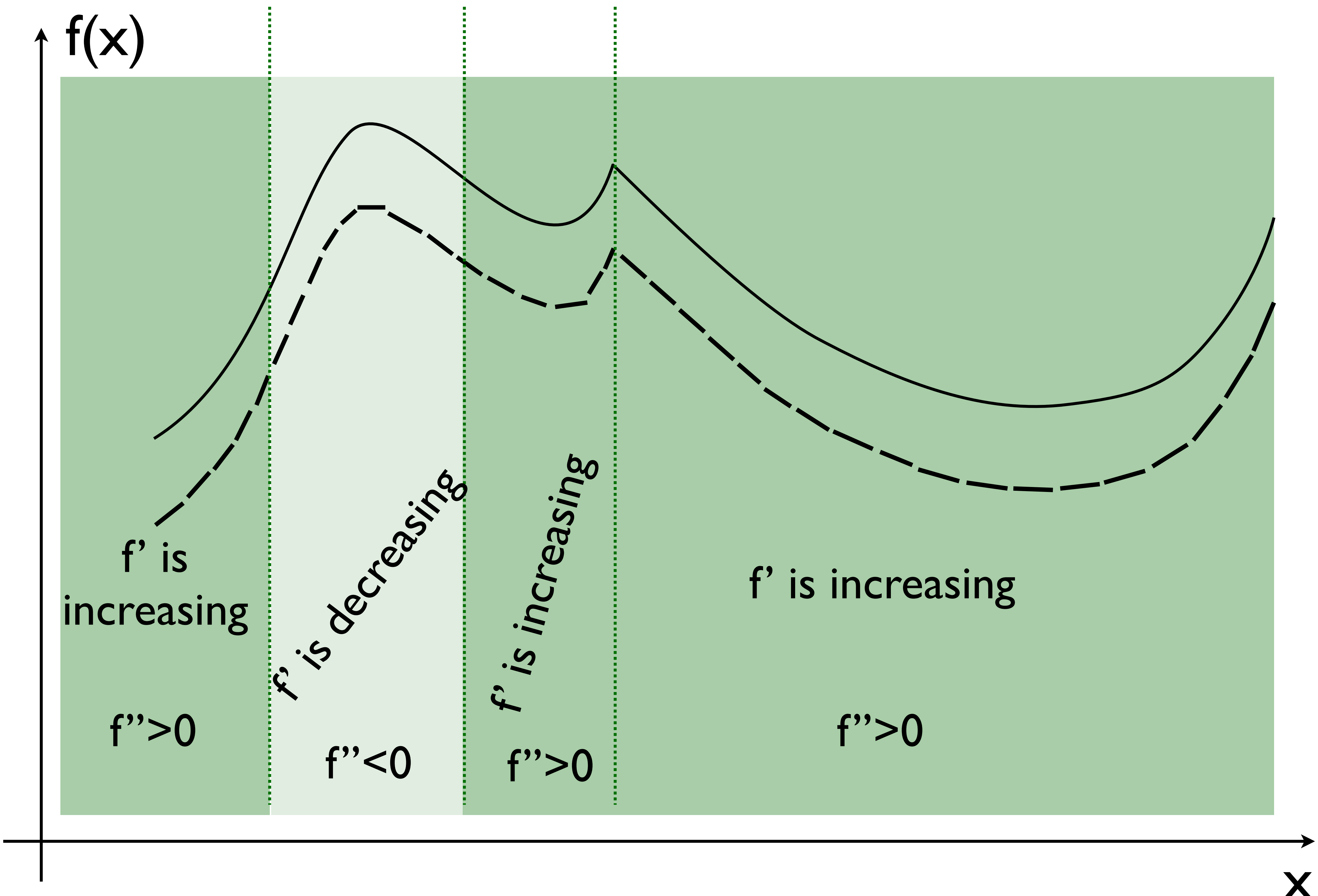
Annotating the graph of f(x) with f'(x) info



What does that mean for the first derivative?

What is the function doing?

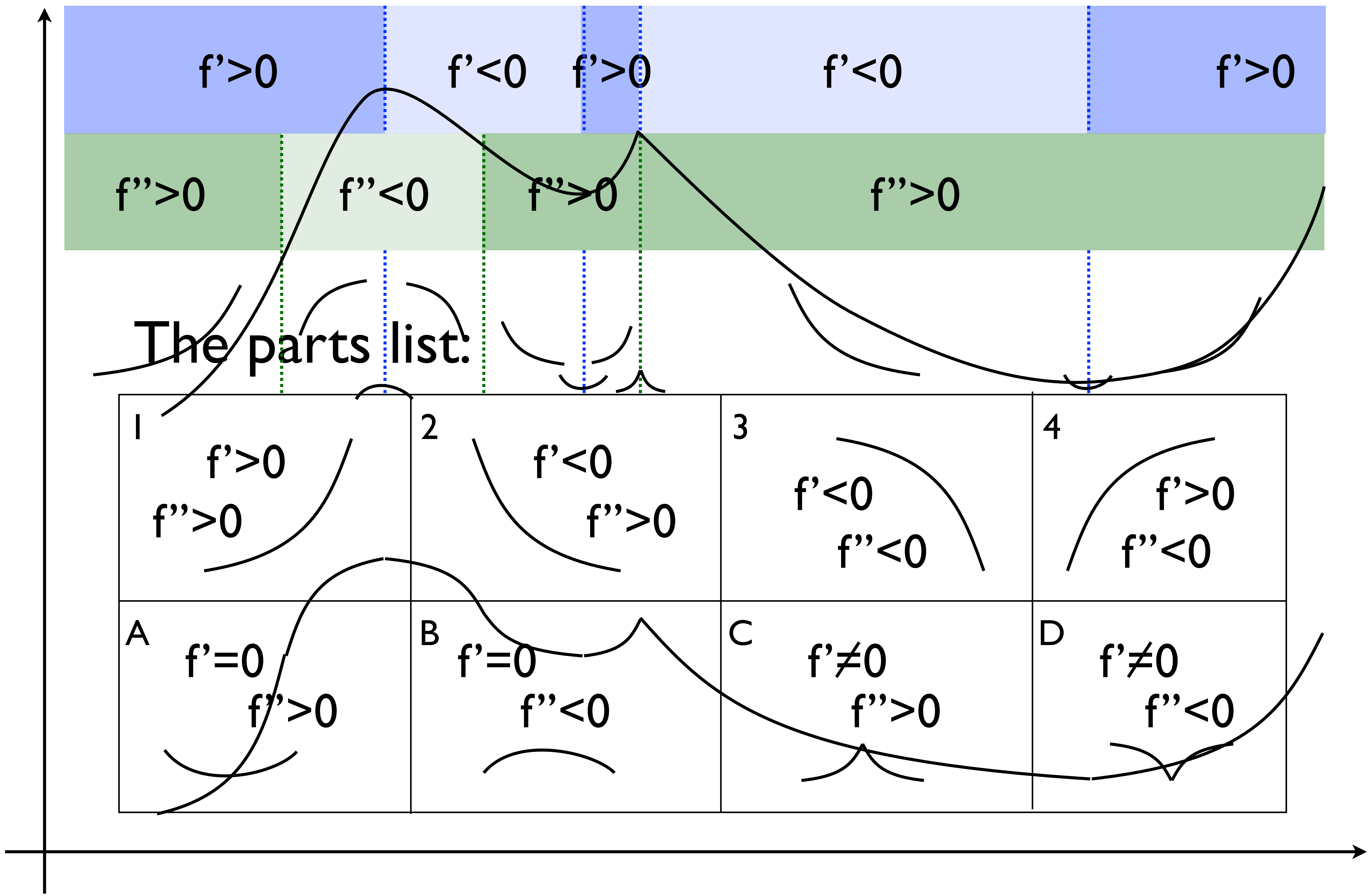
Annotating the graph of f(x) with f''(x) info



What is the first derivative doing?

What does this mean for the second derivative?

What you have to do to graph it.



$$f''(x) = 12(3x^2-2x)$$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	0	+

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
f′(x)	-	0	-	0	+

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,\infty)$
f″(x)	+	0	-	0	+

The whole table

x	(-∞,0)	0	(0,2/3)	2/3	(2/3,1)	1	(1,4/3)	4/3	(4/3,∞)
f(x)	+	0	-	-	-	-	-	0	+
f′(x)	-	0	-	-	-	0	+	+	+
f″(x)	+	0	-	0	+	+	+	+	+

Not a min/max

minimum

$-4x^3$

inflection inflection point

Absolute extrema

- A continuous function on a closed interval [a,b] takes on its highest (lowest) value either at a local maximum (minimum) or at an end point (x=a or x=b). Call this an absolute maximum (minimum).
- When looking for absolute extrema, check critical points AND end points!

Where does $f(x)=x^3-x^2$ take on its absolute minimum on the interval [-1,2]?

(A) $x=-1$

(B) $x=0$

(C) $x=2/3$

(D) $x=2$

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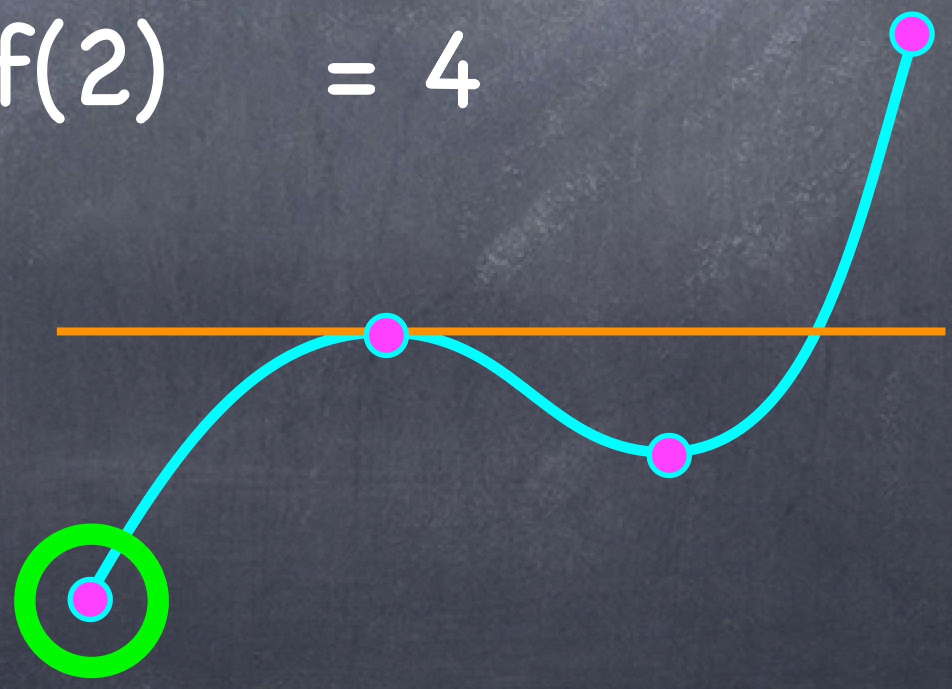
$f(-1) = -2$

$f(0) = 0$

$f(2/3) = -4/27$

$f(2) = 4$



Optimization

- Given a scenario involving a choice of some number, use calculus to find the best value.
    - Translate scenario into a mathematical problem.
    - Solve the problem.
    - Translate back (make sure it makes sense).

I have 10 meters of fence. I want the biggest enclosure possible for my goat. I only know how to make rectangular enclosures.

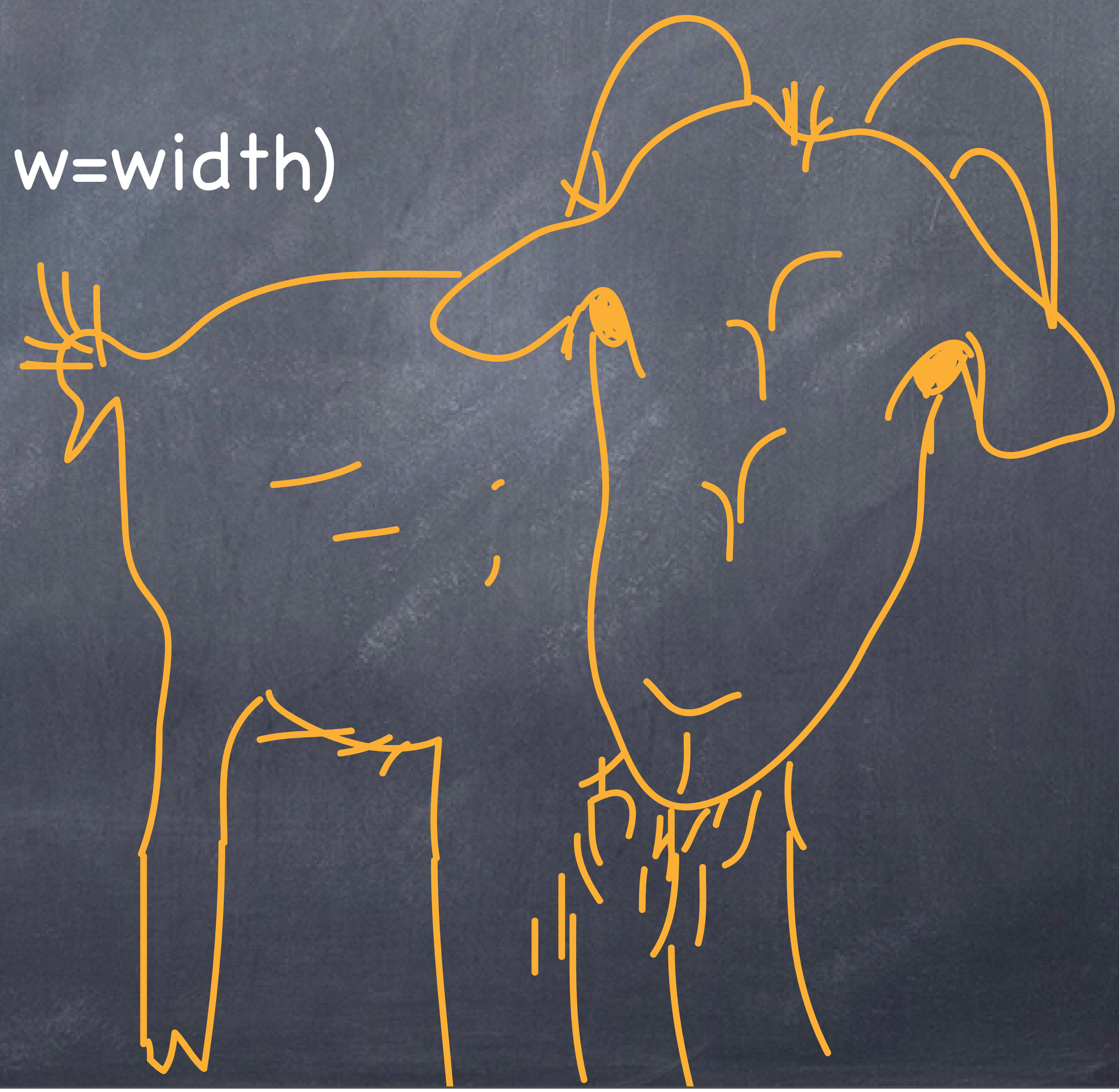
Find the max of

(A) A(w) = lw. (l=length, w=width)

(B) A(w) = w(10-w)

(C) A(w) = w(5-2w)

(D) A(w) = w(5-w)



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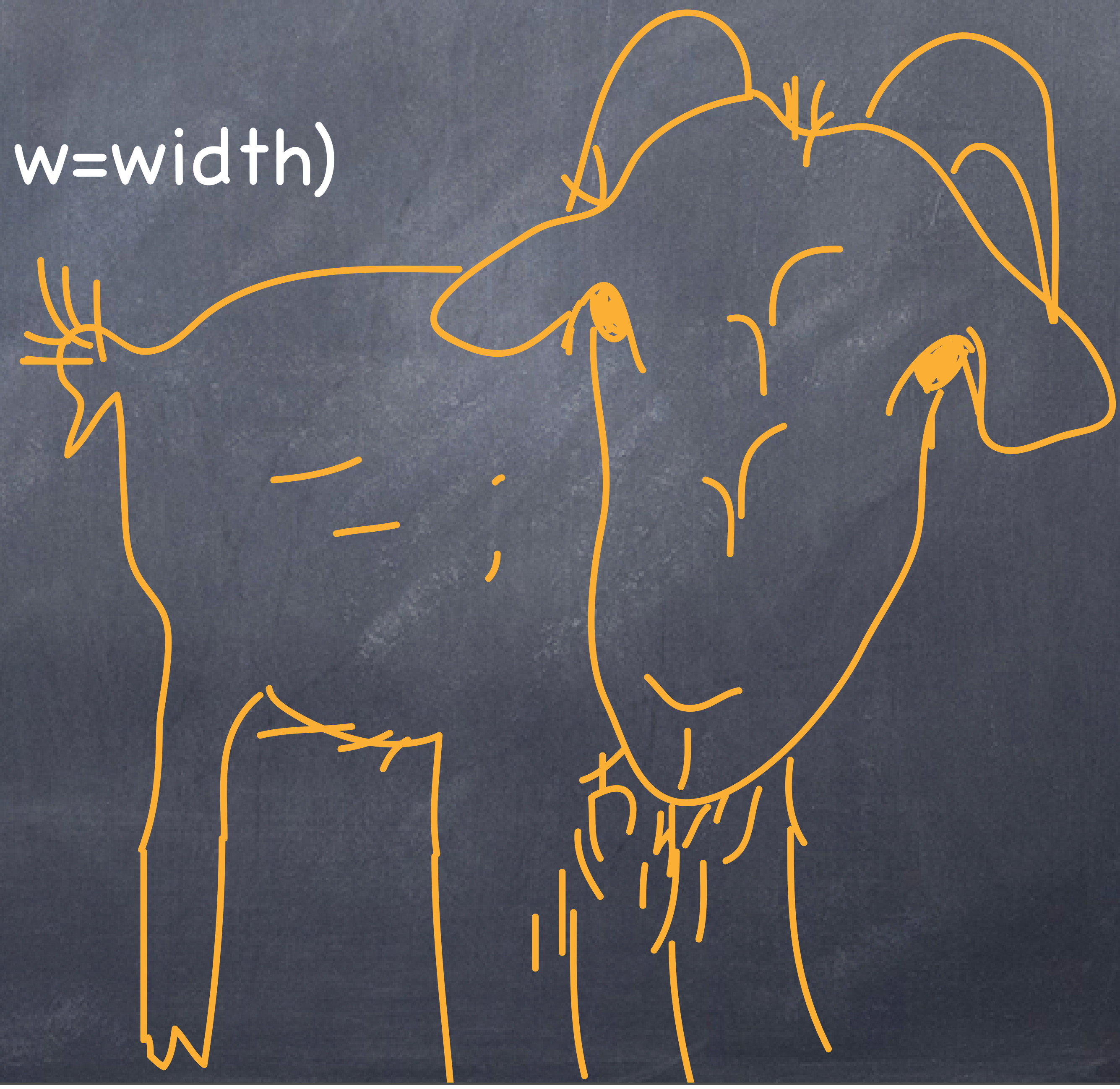

I have 10 meters of fence. I want the enclosure to be as small as possible but it can't be narrower than my goat (1/2 meter).

How long and how wide should I make the enclosure?

(A) l = 5/2 m, w = 5/2 m.

(B) l = 0 m, w = 5 m

(C) l = 1/2 m, w = 9/2 m

(D) l = 1/2 m, w = 19/2 m

Find absolute min of $A(w)=w(5-w)$ on $[1/2, 9/2]$.

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General structure of these problems

- There's an "objective function" (OF) that you want to maximize/minimize.
- The OF depends on more than one variable.
- There's a constraint relating the two variables.
- The constraint lets you simplify the OF to one variable.

$A(l,w)=lw$, $2l+2w=10$ --> $l=5-w$, $A(w)=(5-w)w$