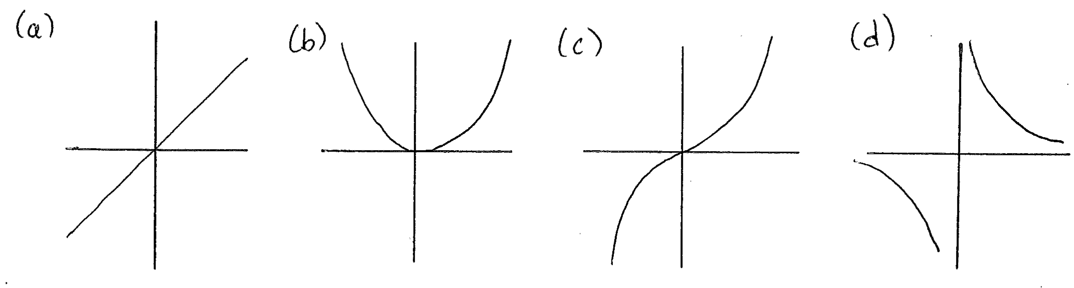
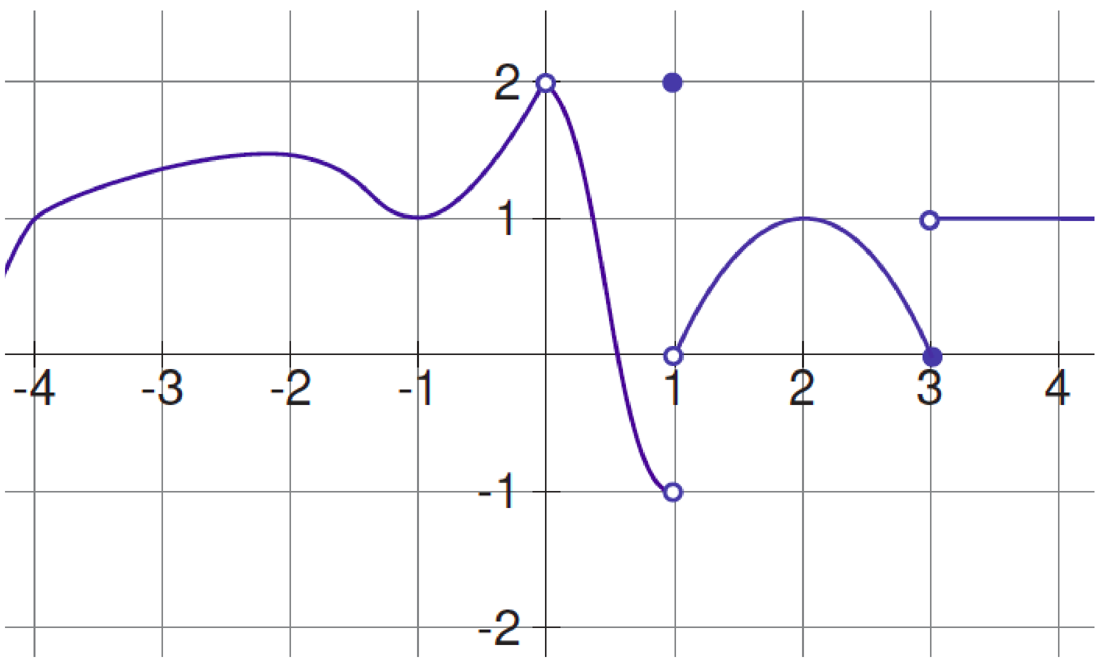
MATH 102 Practice problems

These problems appeared on various quizzes in various sections of the course this year (2016). For now, solutions are not available unless you can find solutions posted on the section pages.

1. Let $g(x) = \frac{x^3+3x^2}{9+2x^3}$. Which of the following represents the graph of $g(x)$ near the origin?



Using the graph of the function $f(x)$ below, answer the following three questions.



2. $\lim_{x\to 1} f(x)$

   (a) -1 (b) 0 (c) 2 (d) DNE

3. $\lim_{x\to 3^-} f(x)$

   (a) 0 (b) 1 (c) 3 (d) DNE

4. $\lim_{x\to -1} f(x)$

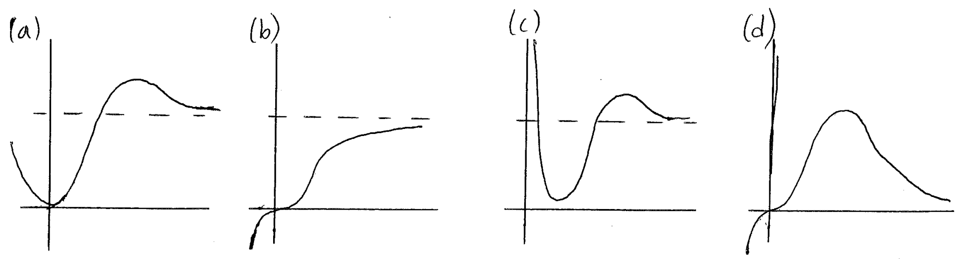
   (a) -1 (b) 1 (c) 2 (d) DNE

5. Using the following data for oxygen-hemoglobin binding, find the average change in hemoglobin saturation as oxygen pressure changes from 20 to 80.

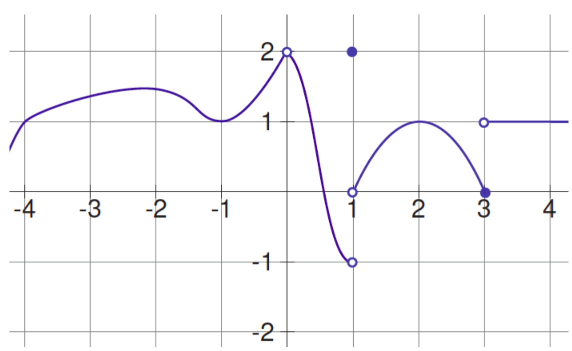
oxygen pressure (mm Hg)	0	20	40	60	80
hemoglobin saturation (%)	0	65	80	90	95

(a) 0.5 %/mm Hg (b) 0.75 %/mm Hg (c) 1.33 %/mm Hg (d) 1 %/mm Hg

6. Let $g(x) = \frac{x^3+3x^2}{9+2x^3}$. Which of the following represents the graph of $g(x)$?



7. Consider the function $f(x)$ shown below, which of the following limits do not exist?



(a) $\lim_{x\to 0} f(x)$ (b) $\lim_{x\to 1} f(x)$ (c) $\lim_{x\to 3} f(x)$ (d) a and b (e) b and c

8. Find the derivative of $f(x) = e^{kx}$ and $g(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$.

$f'(x) =$ ____________________ $g'(x) =$ ____________________

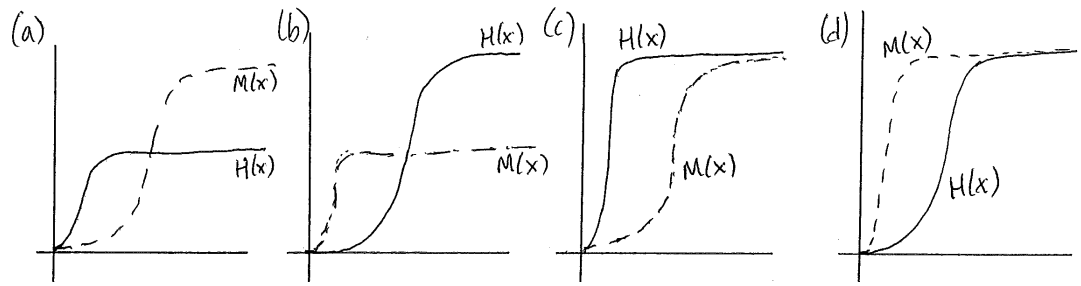
9. Use implicit differentiation to show that the derivative of $y = \ln x$ is $\frac{dy}{dx} = \frac{1}{x}$.

10. The percent saturation of hemoglobin by oxygen is given by the Hill function $H(x)$ and the percent saturation of hemoglobin by carbon monoxide is given by the Hill function $M(x)$ where x is the pressure of oxygen or carbon monoxide (mmHg).

$$H(x) = 100\frac{x^2}{h^2 + x^2} \quad \text{and} \quad M(x) = 100\frac{x^2}{m^2 + x^2}.$$

Carbon monoxide reaches half-maximal hemoglobin saturation (50%) at a lower pressure x than oxygen does.

Which of the following represents the graphs of $H(x)$ (solid) and $M(x)$ (dashed)?



11. A population of animals has a per capita growth rate of $b = 0.08$ per year and a per capital death rate of $m = 0.01$ per year. The population satisfies

$$\frac{dP}{dt} = bP - mP.$$

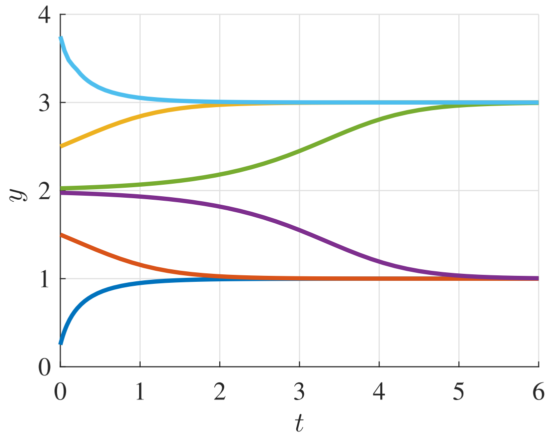
(a) If the population is initially $P(0) = 1000$, find $P(5)$.

$P(5) =$ ______________________

(b) At what time will the population double?

doubling time = ______________________

12. Suggest a *polynomial function*, $f(y)$, so that $\frac{dy}{dt} = f(y)$ has the solutions shown below.



$f(y) =$ __________________________

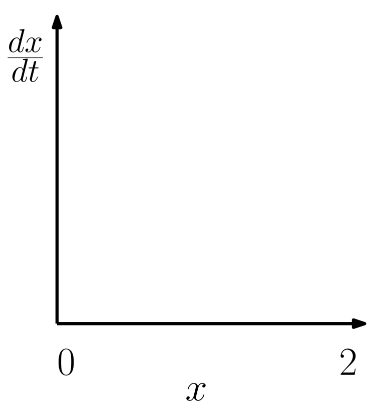
For the following three questions, consider the following differential equation:

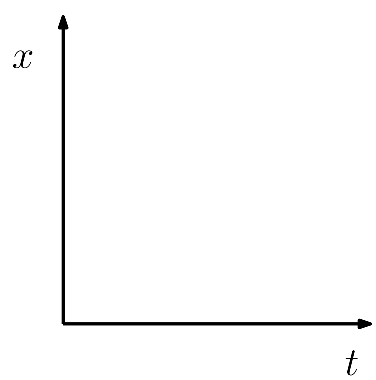
$$\frac{dx}{dt} = f(x) = 0.05 - \frac{x}{2} + \frac{x^2}{1+x^2}.$$

Suppose that $x \geq 0$**.**

13. Sketch the phase line for this differential equation on the appropriate axes below. Draw arrows on the x-axis to identify the direction of the flow. Identify steady-states and their stability on the x-axis.

14. Sketch the slope field for this differential equation on the appropriate axes below.





15. What is

$$\lim_{t\to\infty} x(t)$$

if

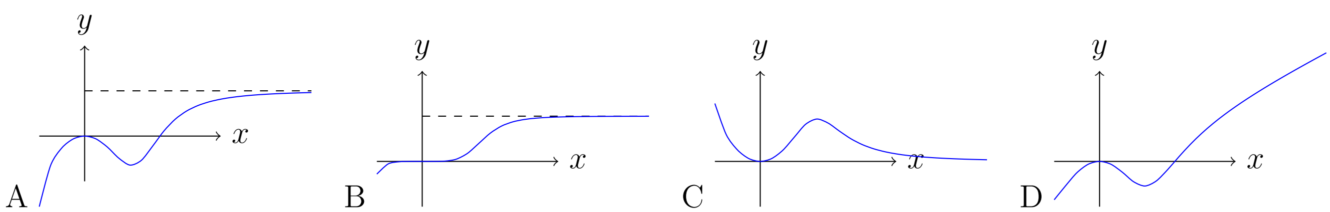
(a) $x(0) = 1$?

$$\lim_{t\to\infty} x(t) = \rule{4cm}{0.4pt}$$

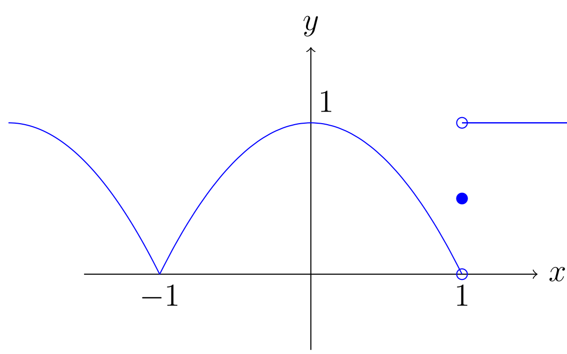
(b) $x(0) = \frac{1}{2}$?

$$\lim_{t\to\infty} x(t) = \rule{4cm}{0.4pt}$$

16. Let $f(x) = \frac{7x^5-4x^2}{2+14x^5}$. Which of the following represents the graph of $f(x)$?



17. Using the graph of the function below determine the following limits:



	A	B	C	D	E
a) $\lim_{x\to -1} f(x)$	1	0	-1	DNE	none of the above
b) $\lim_{x\to 1} f(x)$	0	0.5	1	DNE	none of the above
c) $\lim_{x\to 1+} f(x)$	0	0.5	1	DNE	none of the above
d) $\lim_{h\to 0} \frac{f(-1+h)-f(-1)}{h}$	0	-0.78	0.78	DNE	none of the above
e) $\lim_{h\to 0} \frac{f(h)-f(0)}{h}$	1	-1	0	DNE	none of the above

18. Let $f(t) = 3t^3 - t$. Calculate the slope of the secant line on the interval $1 \le t \le 3$. Show your work!

19. Let $g(x)$ be a function such that $g(0) = 0$ and $\lim_{h \to 0} \frac{g(h)}{h} = 1$. Calculate $f'(0)$ when

	A	B	C	D
(a) $f(x) = g(x) \cdot (x^3 + 2x)$	0	1	2	3
(b) $f(x) = g(x^3 + 2x)$	0	1	2	3

20. We are interested in the value $\sqrt[3]{9}$. Let $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$.

(a) Linear approximation at the point $x = 8$ gives

(A) $\frac{7}{12}$ (B) $\frac{31}{12}$ (C) $2\frac{1}{12}$ (D) $\frac{1}{4}$

(b) In this case the estimate of linear approximation is

(A) under (B) precise (C) over (D) other

(c) An appropriate function for calculating $\sqrt[3]{9}$ with Newton's method is

(A) $f(x) = \sqrt[3]{x}$ (B) $f(x) = x^3$ (C) $f(x) = 9$ (D) $f(x) = x^3 - 9$

(d) First iteration of Newton's method with initial value $x_0 = 1$ gives

(A) $\frac{-8}{3}$ (B) $\frac{11}{3}$ (C) 1 (D) $\frac{11}{8}$

21. Let $f(x) = x^3 - 3x^2 + 6x$.

(a) Find values of x where the tangent lines are parallel to the line $y = 6x + 1$.

(b) Calculate the equations of the tangent lines at above points.

22. Find the inverse function $f^{-1}(x)$ when

	A	B	C	D
(a) $f(x) = \sqrt[5]{x^5 - 1}$,	$\sqrt[5]{x^5 - 1}$	$\sqrt[5]{x^5 + 1}$	$\sqrt[5]{x^5}$	$\frac{1}{\sqrt[5]{x^5 - 1}}$
(b) $f(x) = \mathrm{e}^{x^3 + \ln 3}$.	$\frac{1}{\mathrm{e}^{x^3 + \ln 3}}$	$\sqrt[3]{\ln x}$	$\frac{1}{3} \ln \sqrt[3]{x}$	$\sqrt[3]{\ln \frac{x}{3}}$

23. Air is escaping from a spherical ballon ($V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$) at the rate of 2 cm^3 per minute. Units are suppressed below.

(a) The volume satisfies the differential equation

(A) $\frac{dV}{dt} = \frac{4}{3}\pi 3r^2$ (B) $\frac{dV}{dt} - 2 = 0$ (C) $\frac{dV}{dt} + 2 = 0$ (D) $\frac{dV}{dt} = \frac{4}{3}\pi$

(b) How fast the radius is *shrinking* when the radius is 1 cm?

(A) 0 (B) 1 (C) -2π (D) $\frac{1}{2\pi}$

(c) How fast the surface area is *shrinking* when the radius is 1 cm?

(A) 4 (B) -2π (C) -4π (D) $-\frac{1}{8}$

24. Consider the differential equation $y' = y^3 - 5y^2 + 6y$.

(a) Draw the slope-field and the state-space. Find the steady states and classify them with respect to their stability.

(b) Find the inflection points of the solution curves.

25. For x sufficiently large, $f(x) \approx$

$a)$ $-\frac{x}{a}$ $b)$ $-\frac{1}{x}$ $c)$ $\frac{K}{a+x^2}$ $d)$ K $e)$ $\frac{K}{a}x^2$ $f)$ Undefined

26. For $|x|$ sufficiently small, $f(x) \approx$

$a)$ $-\frac{x}{a}$ $b)$ $-\frac{1}{x}$ $c)$ $\frac{K}{a+x^2}$ $d)$ K $e)$ $\frac{K}{a}x^2$ $f)$ Undefined

27. Answer the following in your own words, possibly with the help of a graph: How does changing K and a change the asymptotic behavior of $f(x)$?

28. What is the average rate of change of $g(t) = t^2 - 3t^3$ from $t = 0$ to $t = 1$?

$a)\ -2$ $\qquad b)\ -1$ $\qquad c)\ 0$ $\qquad d)\ 1$ $\qquad e)\ 2$

29. The following tables give the total accumulation of rainfall at the UBC weather station starting from 6:00 AM last Sunday. What is the average rate of rainfall between 8:00 AM and 10:00 AM in mm/h?

$a)$ 2.2 $\qquad b)$ 2.3 $\qquad c)$ .9 $\qquad d)$ 1.6 $\qquad e)$ 5.4

Time, AM	mm
7:00	2.1
8:00	4.3
9:00	5.2
10:00	7.5

30. The volume, V, and surface area, S, of a cube whose sides have length a are given by the formulae

$$V = a^3, \qquad S = 6a^2.$$

Note that V and S are expressed as power functions of a, the length of a side of the cube.

(a) Ignoring units, take a to be very large. How do we expect the volume to relate to the surface area?

$a)\ V > S$ $\qquad b)\ V < S$ $\qquad c)\ V \approx S$

(b) Ignoring units, at what value of a is the surface area S the same as the volume V?

31. Evaluate the following limit:

$$\lim_{x \to 3} \frac{x^3 - 7x^2 + 15x - 9}{x^2 - x - 6}$$

32. Which of the following describes the derivative of a function $f(x)$?

(a) It is the average rate of change of $f(x)$ over an interval $0 < x < h$.

(b) It is the slope of the line we see when we zoom into a graph around a point.

(c) It is $\frac{f(x+h)-f(x)}{h}$.

(d) It is the value of $\lim_{x \to a} f(x)$ for some a

(e) More than one of the above.

33. To estimate the value of $\sqrt[3]{28}$ using Newtons Method, the best choice of $f(x)$ and x_0 would be:

$(a)\ f(x) = \sqrt[3]{x},$	$x_0 = 27.$	$(c)\ f(x) = x^3 - 28,$	$x_0 = 3.$
$(b)\ f(x) = \sqrt[3]{x} - 28,$	$x_0 = 27.$	$(d)\ f(x) = \sqrt[3]{x - 28},$	$x_0 = 0.$

34. Use linear approximation to estimate $\sqrt{63}$ from a reasonable starting point. Will this be an over estimate or an under estimate?

35. Using Newtons Method on $f(x) = 2x^2 - 5x + 2$, with an initial guess of $x_0 = 1$, what is x_1?

36. Let $g(x) = 2x + \frac{1}{2x}$.

   (a) Find the derivative of $g(x)$.

   (b) When is $g(x)$ increasing? Decreasing?

   (c) Where are the local maxima and minima of $g(x)$?

37. Using Newtons Method on $f(x) = x^3 - 2x + 2$ with $x_0 = 0$ yeilds a 2-cylce, $x_0 = 0, x_1 = 1, x_2 = 0, \ldots$ Can you construct another continous function f (that is not a scalar multiple of f) and an initial point x_0 such that Newtons Method yeilds a 2-cycle?

$f =$	$f' =$	$x_0 =$	$x_1 =$

38. Mark the following true or false:

   - **T or F** Let $f(x)$ have a unique inverse function $g(x)$. Assume $f(1) = 2$, $f(2) = 1$ and $f'(1) = 3$, $f'(2) = 1$. Then $g'(1) = 1$.
   - **T or F** Every solution to $y' = y^2 + 1$ diverges to infinity at large t.

39. Let $Y(t)$ be a solution to the differential equation $y' = -y(y-4)(5+y)$, with initial condition $Y(0) = 3$.

   (a) What is $Y'(0)$?

   (b) Draw a plot containing $Y(t)$, as well as the steady state solutions to the differential equation. Label the steady states as "stable" or "unstable," and CLEARLY indicate $Y(t)$.

40. The radius r of a tree grows logarithmically in relation to it's height h, and can be estimated by the formula $r = a\ln(1 + bh)$ where a and b are positive constants. If the radius is growing at a rate of 4 cm/year when the tree is 500 cm tall, how fast is the height changing? Note, your formula will be in terms of b and a.

41. Food Safety Agency requires that Chicken Nuggets taken out of a freezer be fried for long enough time so that the interior reaches at least 120^o C at the moment the frying process ends. An inspection at restaurant M, carried out at a room temperature of $E = 20^o$ C, reveals the following result. The temperature of the interior of a nugget is 84^o C after 1 min of cooling and 36^o C after 3 minutes of cooling. Use Newtons Law Of Cooling

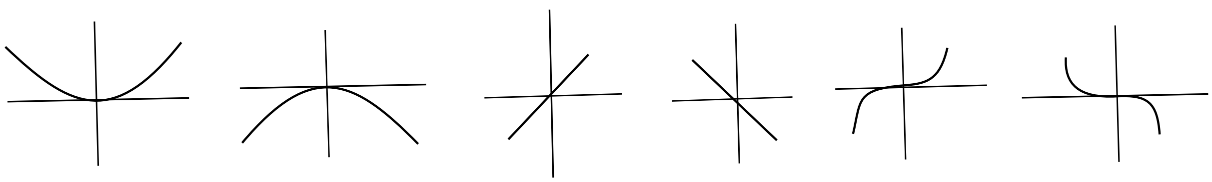
$$\frac{dT}{dt} = k(E - T)$$

to determine if restaurant M meet the safety standard.

42. Drug X is removed the blood stream at a rate proportional to the total amount present. The half life of Drug X in the blood stream is 15 minutes. At $t = 0$, 320 mg are injected into a patient, when will there be 20 mg left?

43. We want to fit a line $y = mx+b$ to a series of data point (x_i, y_i), $i = 1, \ldots n$. Minimize the sum of squares of residuals $S(a, b) = \sum_{i=1}^{n}(y_i - mx_i + b)^2$ with respect to both a and b to find a formula for best fit line. Ie, solve $\frac{dS}{da} = \frac{dS}{db} = 0$ simultaneously.

44. Which one is showing the graph of $f(x) = \frac{2x^2+x}{x^3-1}$ close to the origin?



45. Consider the Hill function $H(x) = \frac{ax^n}{b^n+x^n}$. Which of the following is **wrong** about $H(x)$:

a. The slope of the tangent line at $x = b$ is larger for higher n
b. If n is even, then $H(x)$ is even
c. If n is larger then $H(x)$ approaches faster to the max-response asymptote ($y = a$)
d. None of the above

46. Calculate the following limit:

$$\lim_{x \to -1^+} \sqrt{1 - x^2} =$$

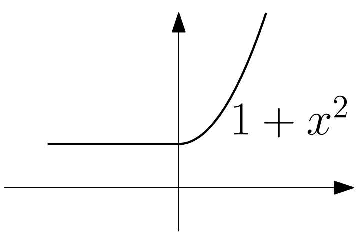
47. Consider a polynomial function $f(x) = ax^4 + bx^3 + cx^2 + 1$ and assume that $f'(1) = 1$. Which of the following is **wrong**:

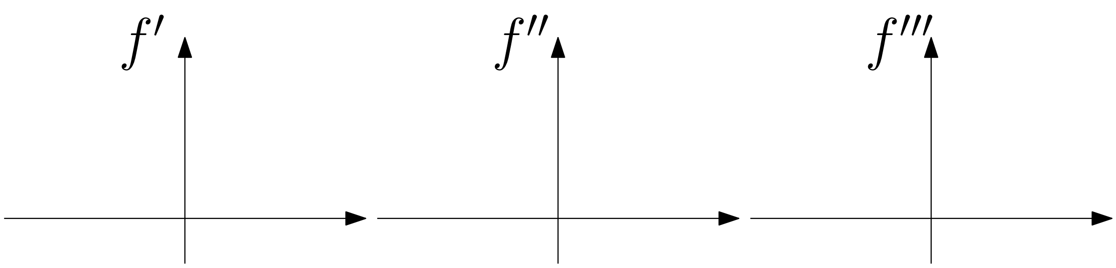
a. $\lim_{h\to 1} \frac{f(1+h)-f(h)}{h} = 1$
b. The function $g(x) = \frac{f(x)-f(1)}{1-x}$ has a hole-in-graph discontinuity
c. The $f(1)$ can be positive or negative
d. The $f(x)$ is continuous every where

48. Write the tangent line equation of $\frac{x^2+1}{x}$ at $x = 2$. Show your work

49. Suppose you want to approximate $\sqrt[3]{a}$ for some $a > 0$. Write the equation of Newton's method for $x_{n+1}(x_n)$ and then derive the formula for your specific case.

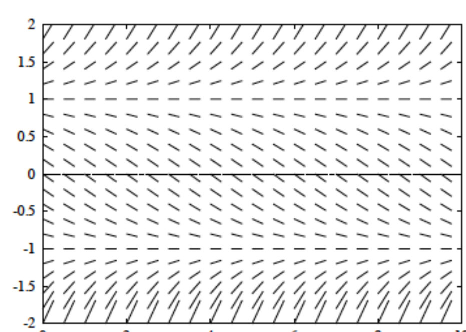
50. The graph of $f(x)$ is shown, draw f', f'', f''' ($1^{th}, 2^{nd}, 3^{rd}$ derivatives)





51. The following slope field corresponds to which differential equation?

a) $y' = y(y+1)$
b) $y' = (y-1)(y+1)$
c) $y' = -(y-1)(y+1)$
d) $y' = y(y-1)$
e) $y' = -y(y+1)$



What are the steady states and are they stable/unstable?

52. Let $R(t)$ be the rabbit population in a certain forest t months from now. Assume $R(t)$ satisfies the equation $\frac{dR}{dt} = kR$ for some $k > 0$. The rabbit population triples every 10 months and is currently 50 rabbits.
(Note: Leave the numbers in your answer without evaluating e.g. keep $5ln(5)$)

a) Write the solution to $\frac{dR}{dt} = kR$ with $R(0) = 50$:

b) Knowing the population triples every 10 months find k:

c) Write the final $R(t)$:

53. Find the limit

$$I = \lim_{x \to -4} \frac{x^2 + x - 20}{x^2 + 3x - 10} = ?$$

54. The distance of Alice falling from a helicopter to the ground is

$$y(t) = 1000 - 5t^2 \qquad \text{(ignore the unit).}$$

(a) Find her average velocity during time interval $[3, 5]$.

(b) Find her instantaneous velocity at time $t = 3$.

55. The function $f(x)$ defined by $f(x) = 1$ for $x \geq 0$ and $f(x) = x$ for $x < 0$ as $x \to 0$
(A) is continuous (B) has a hole (C) has a jump (D) is oscillating (E) blows up

56. Consider the Hill function

$$y(t) = \frac{100t^2}{144 + t^2}, \qquad (t \geq 0)$$

(a) What is its "maximal activation level"? (It is the "maximal rate" if $y(t)$ denotes the rate of a chemical reaction.)

(b) At which time does $y(t)$ achieve "half-maximal activation level"?

57. When $x = 1000$, the function $f(x) = \dfrac{6x^4 + 80x^2 + 12x + 64}{2x^3 - 6x + 1}$ is closest to
(A) 0.003 (B) 3000 (C) 1000000 (D) 6 (E) 3 (F) 64

58. Suppose the populations of crows, raccoons and squirrels in the UBC forest are given by

$$C(t) = 100t + \frac{1}{1000000}t^4, \quad R(t) = 200t + \frac{1}{10000}t^2, \quad S(t) = 300t + \frac{1}{10000}t^3.$$

Which species has more number at time $t = 10$?

59. The function $f(x) = \frac{x+x^3}{x^2+1}$ is

A) even (B) odd (C) both (D) neither.

60. From the point $P(1, 0)$ to the parabola $y = x^2$ there are two tangent lines. One is the x-axis with the tangent point the origin. Find the other tangent line L and its corresponding tangent point Q.

61. (a) Find the linear approximation $L(x)$ of the function $f(x) = x^{1/2}$ at $x = 4$.

(b). Approximate $\sqrt{3.8}$ using the result from (a).

62. We want to approximate the root of $f(x) = x^3 + x - 3$ using Newton's method with initial guess $x_0 = 0$. What is x_1, the value you get after one iteration of Newton's method?

63. To estimate $\sqrt[3]{9}$ using Newton's method, what are the best function and initial guess?

(A) $f(x) = \sqrt[3]{3x} - 3, \quad x_0 = 0,$ (B) $f(x) = x^3 - 9, \quad x_0 = 0,$

(C) $f(x) = \sqrt[3]{3x} - 3, \quad x_0 = 2,$ (D) $f(x) = x^3 - 9, \quad x_0 = 2$

64. Suppose $f'(x) = 6 + 6x$ and $f(0) = 2$. What is $f(x)$?

65. Find $h'(1)$, where $h(x) = f(x) \cdot g(x)$ with $f(x)$ and $g(x)$ satisfying

$f(1) = 2, \quad f'(1) = 3, \quad g(1) = 4, \quad g'(1) = 5.$

66. Find $\frac{d}{dx}\left(\frac{x^3}{x^2+1}\right)$. Do not simplify.

67. Suppose $f(x)$ satisfies $f(1) = 2.1$, $f(1.1) = 2.3$, and $f(1.2) = 2.4$. Approximate the derivative of $f(x)$ at $x = 1$ using finite difference (i.e., average rate of change) with step size $\Delta x = 0.1$.

68. Food Safety Agency requires that chicken nuggets taken out of a freezer be fried for long enough time so that the interior reaches at least 120 °C at the moment the frying process ends. An inspection at restaurant M, carried out at a room temperature of 20 °C, reveals the following result. The temperature of the interior of a nugget is 84 °C and 36 °C, respectively, when measured at 1 min and 3 min after the frying process is terminated. Newton's Law of Cooling applies. Does restaurant M meet the safety standard?

69. 10 mg of a radioactively labelled experimental drug is injected into the blood of a lab rat. Let $D(t)$ be the amount of this drug (in mg) inside the rat at time t. The rate of drug removal inside the rat is known to be proportional to the amount of drug that is present. A radioactive tracking device shows that 1 hour after the injection, the amount is reduced to 8 mg.

(a) Find the DE that describes the changes in $D(t)$ which contains no unknown constant.

(b) What is the half-life of the drug inside the rat?

70. Consider the DE $dy/dt = f(y)$ where the function $f(y)$ is graphed below. How many of the 4 fixed points of the DE are stable?

71. Find $h'(1)$, where $h(x) = \ln \frac{x^9}{(x+1)^4}$

72. Find the derivative of $f(x) = x^{2x}$.

73. Find $f^{-1}(x)$ for $f(x) = \sqrt[3]{x^3 - 5}$.

74. Find the derivative of $f(x) = e^{x^2+x}$.