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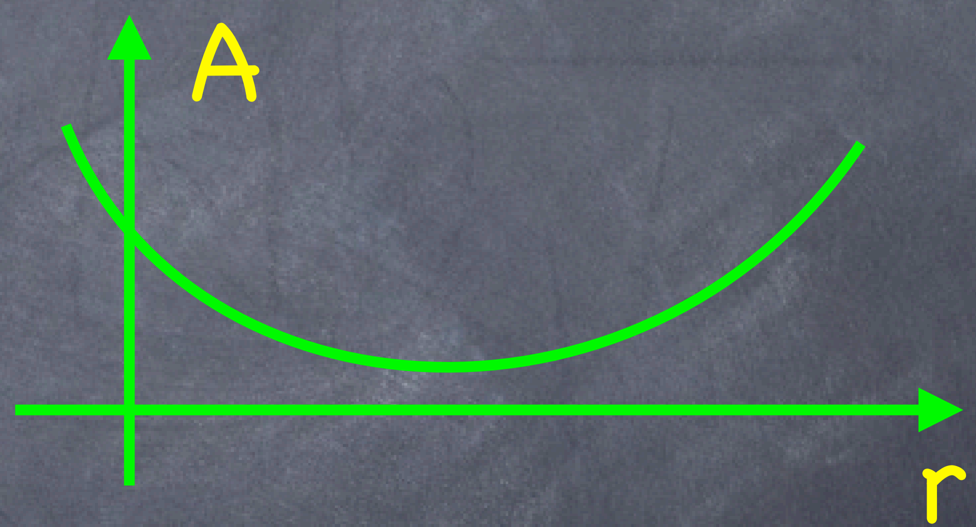

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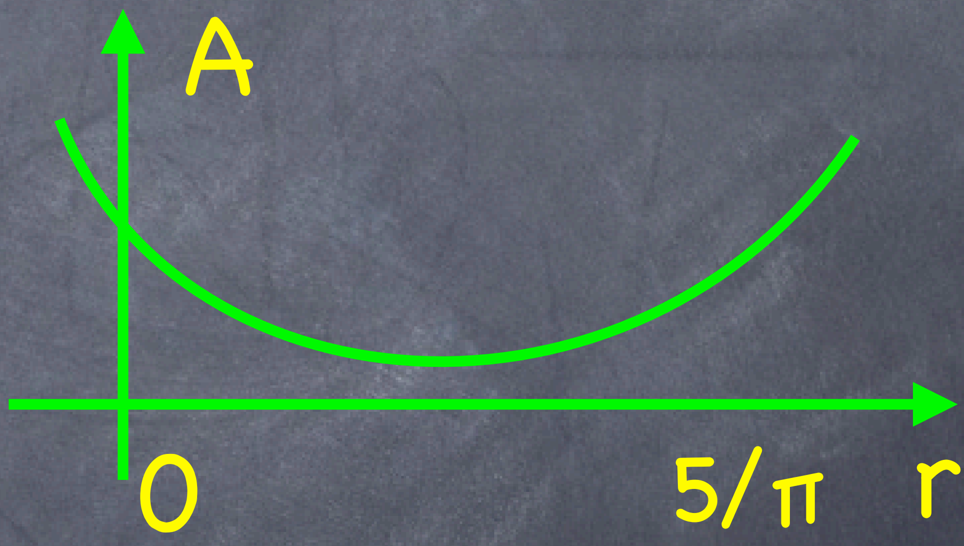
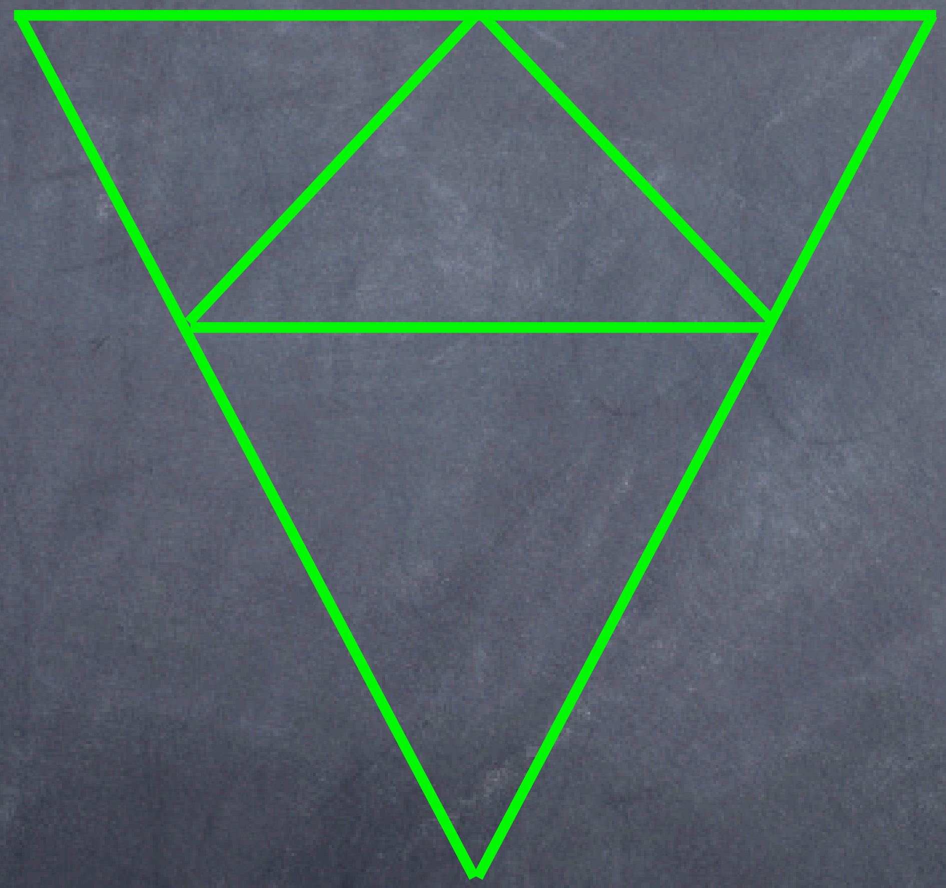
- $r=0$ and $w=0$ which means $r=5/\pi$...



Cone inside a cone



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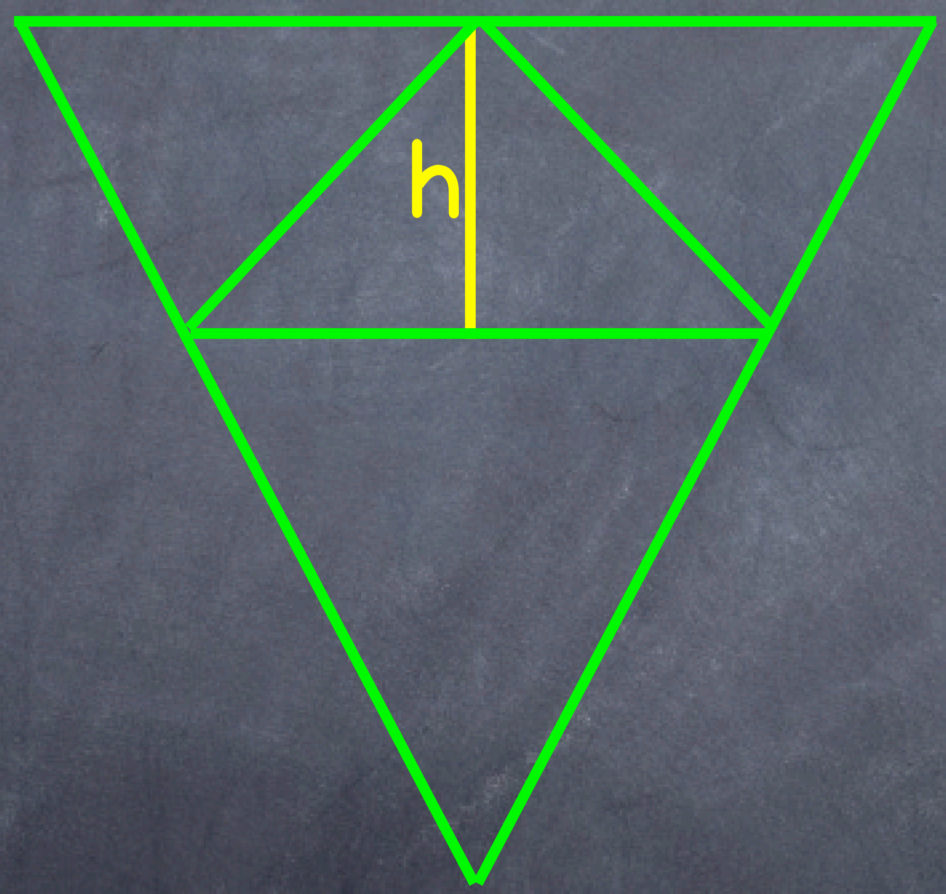

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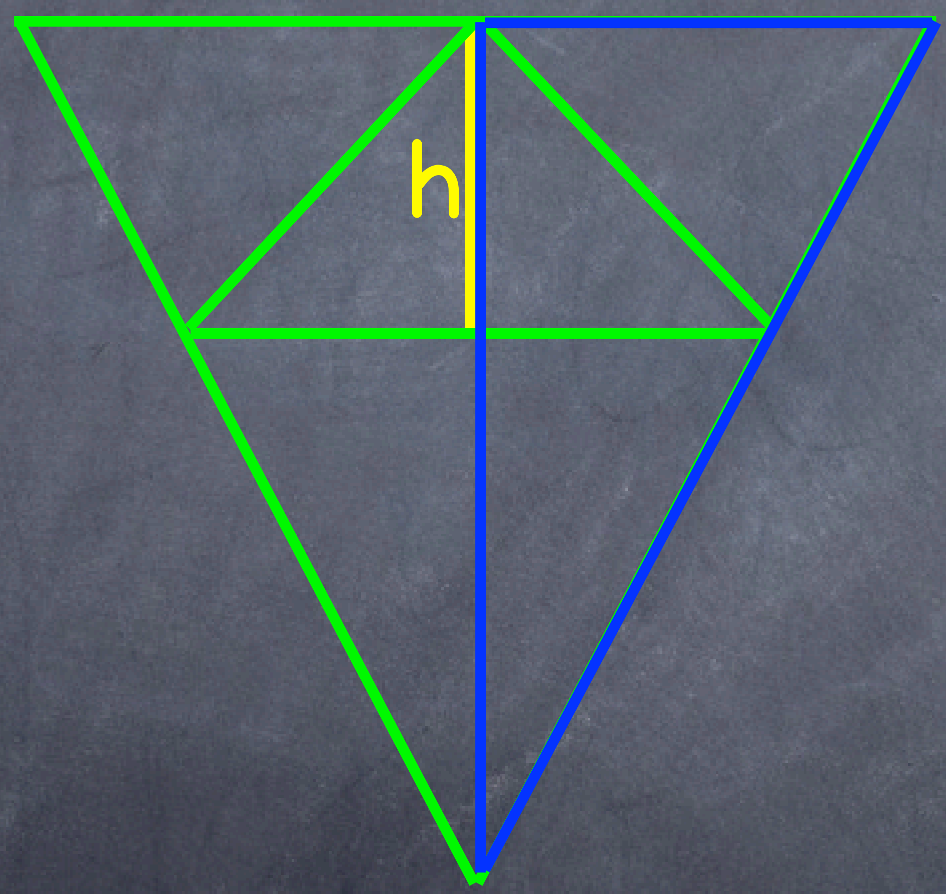

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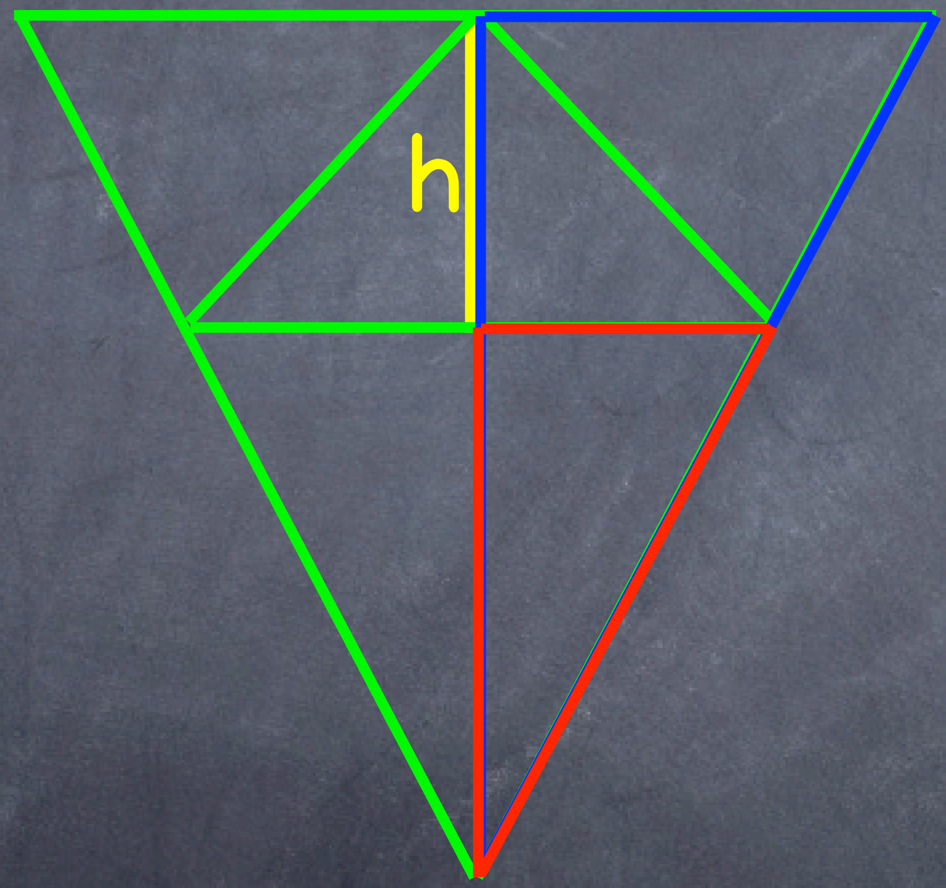

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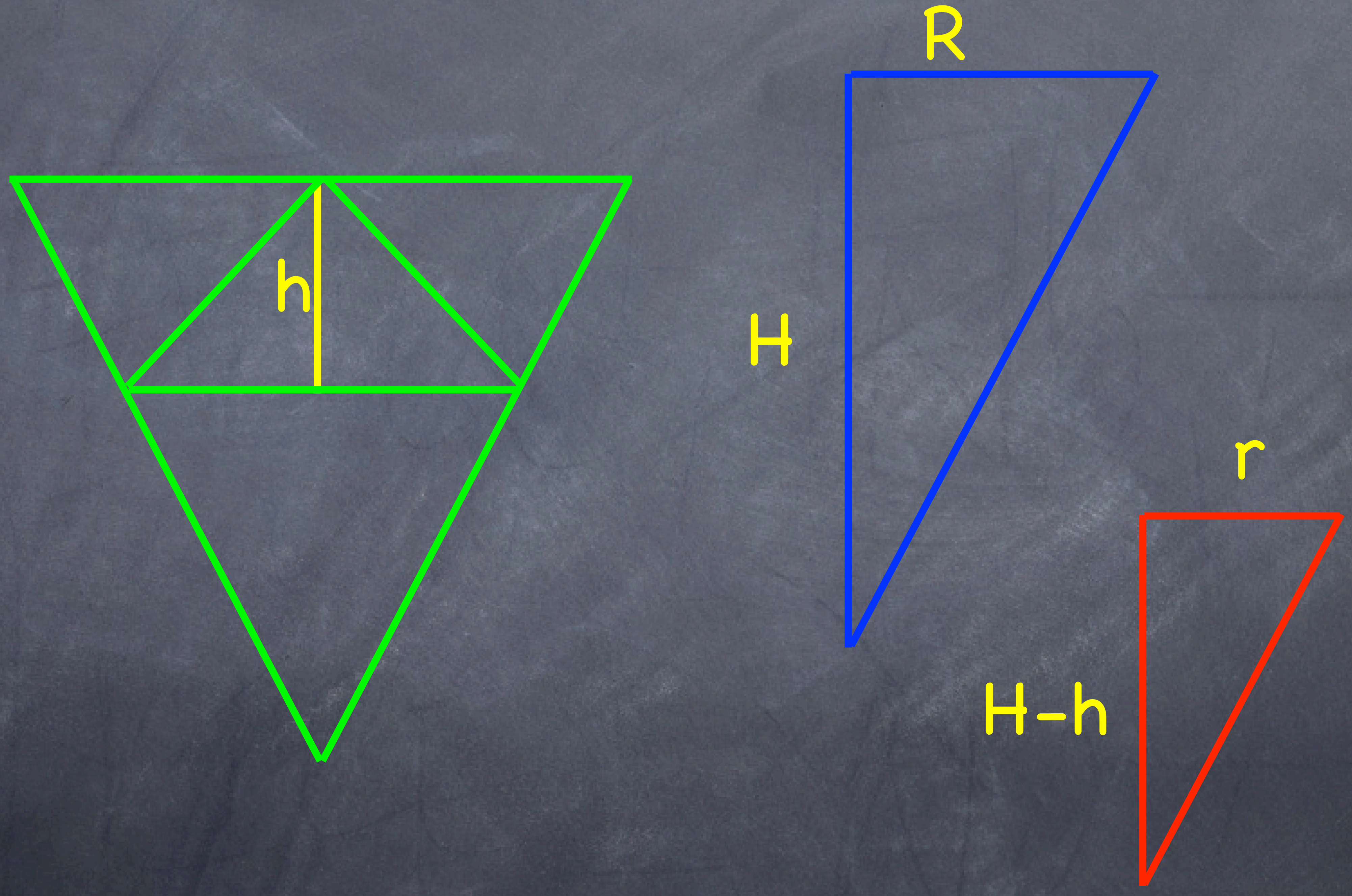

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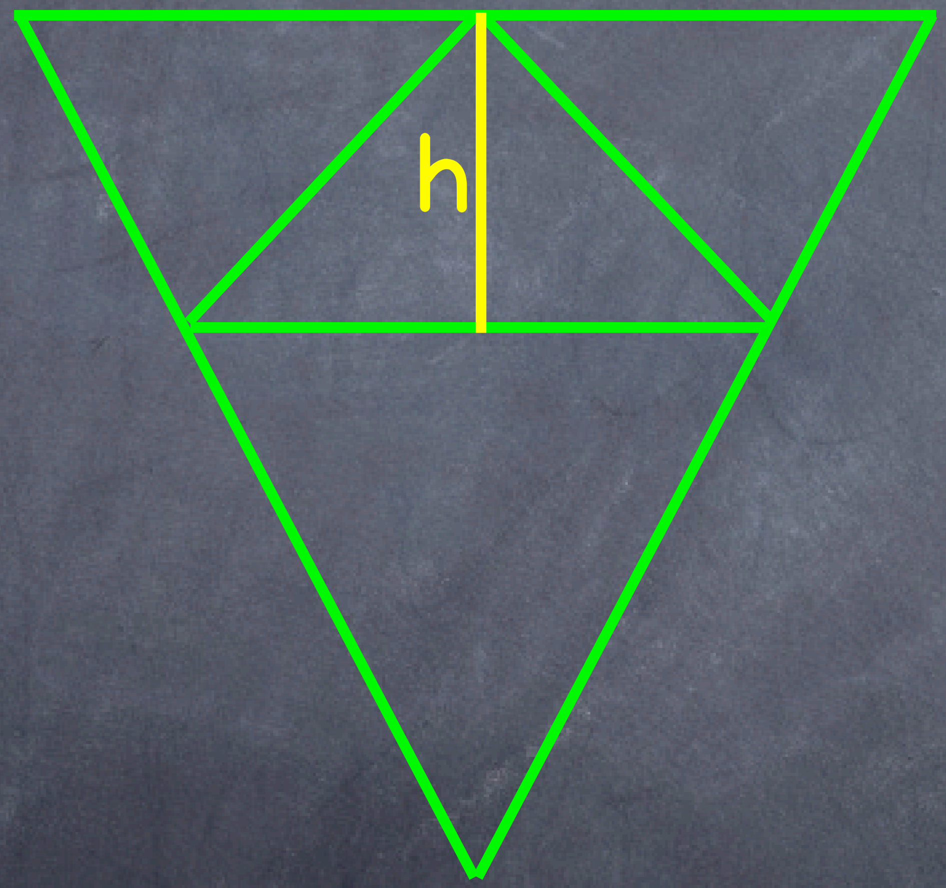

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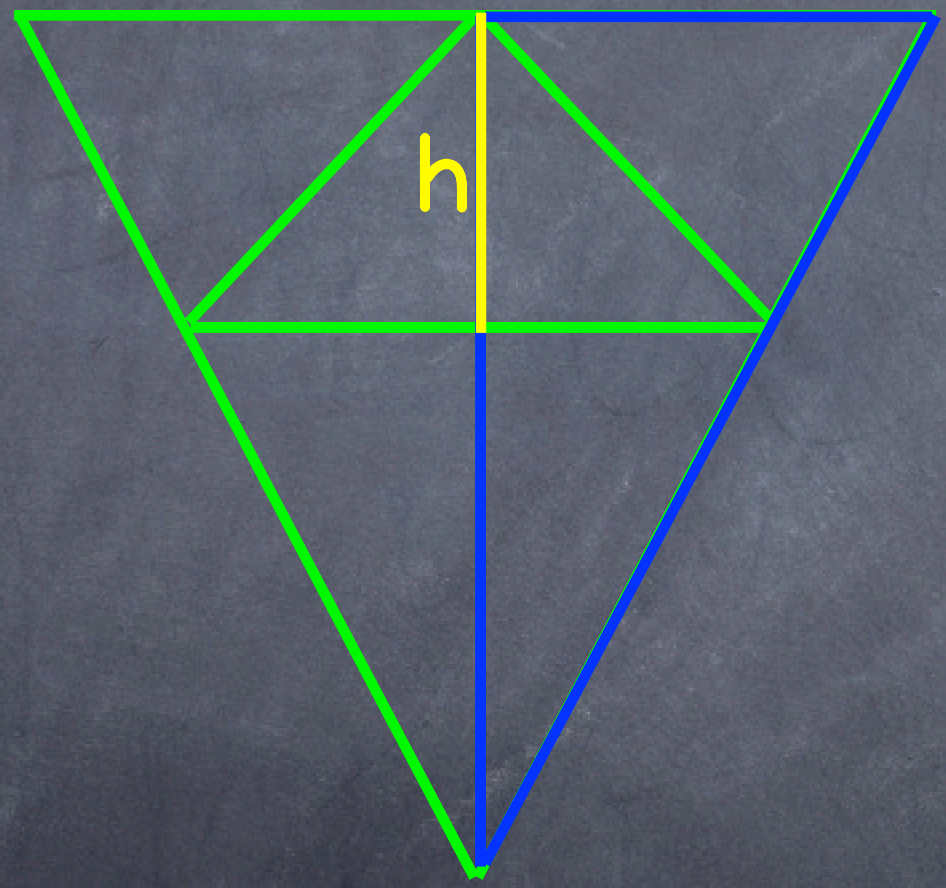

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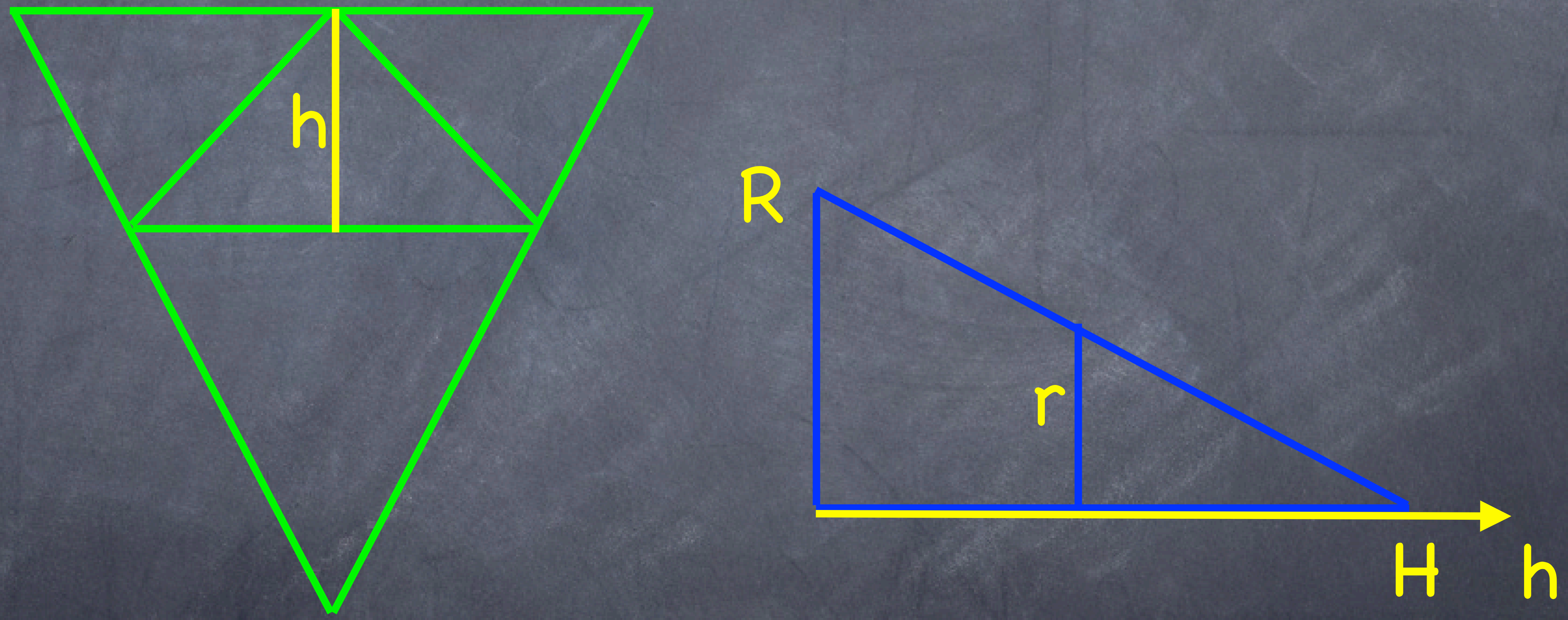

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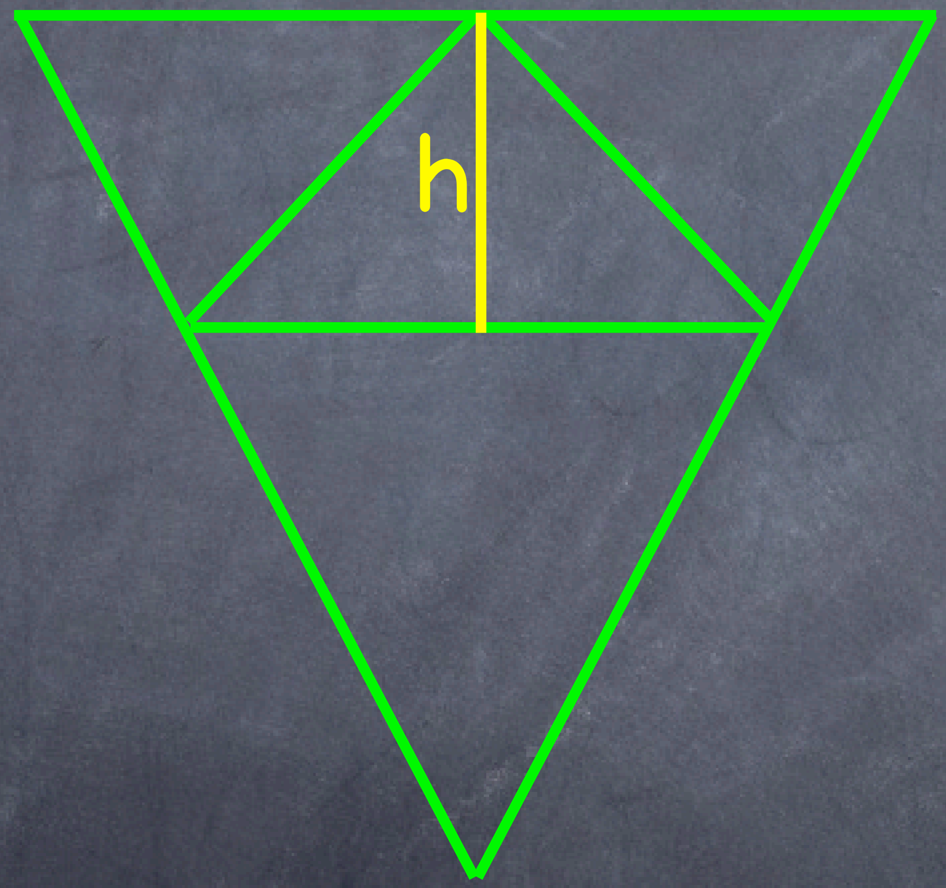

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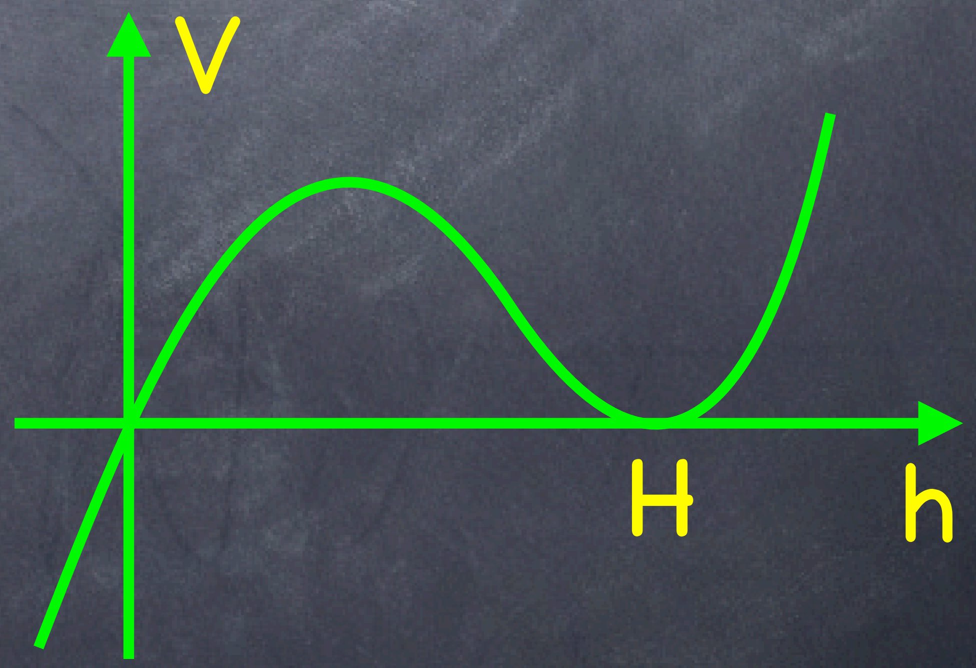

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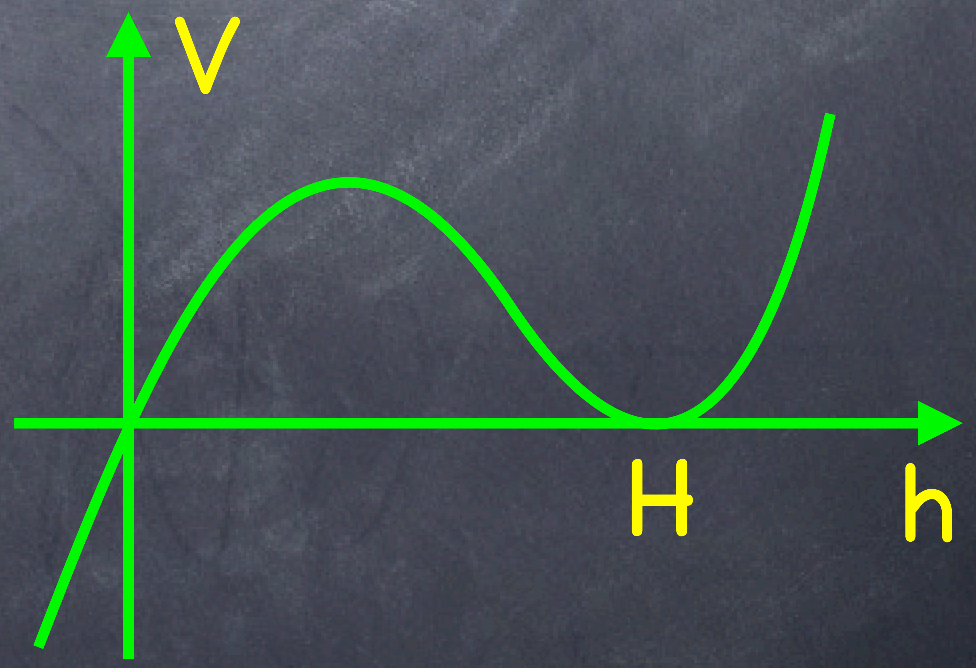
- Find max...



Long Answer Problems

5. During pregnancy, a fraction of twins are at risk of death due to entanglement of their umbilical cords. To prevent such deaths, the twins are removed early by surgery (Caesarean section). However, removing them earlier puts them at risk of death due to premature birth. In this problem, you will determine when to schedule delivery so as to maximize their chance of survival in the face of these opposing risk factors.

   (b) [**3 pt**] The overall probability of survival for a set of such twins, accounting for both survival to delivery and survival once delivered, is

   $$S(t) = A(t-7)\exp(-t/20)$$

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A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time t in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket $(h(0) = 4)$, what is the solution to this differential equation?

A. $h(t) = (2 - 3t)^2$
B. $h(t) = \sqrt{16 - 2t}$
C. $h(t) = (3 - 3t)^2$
D. $h(t) = 4 - 6t^2$

Related rates

A spotlight on the ground is shining on a wall 16m away. If a woman 2m tall walks from the spotlight toward the building at a speed of 0.6m/s, how fast is the length of her shadow on the building decreasing when she is 6m from the building?