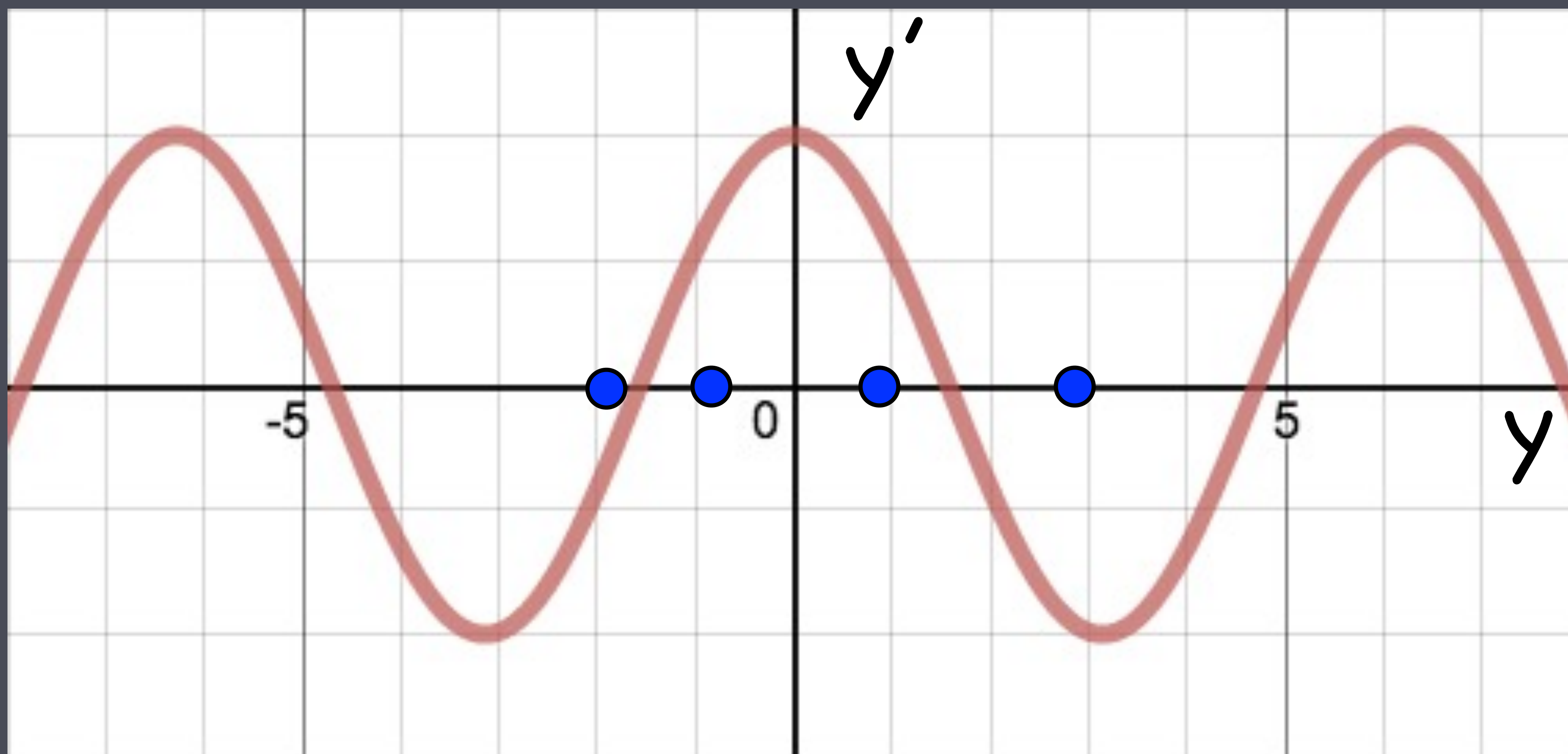
Today

- Phase-line to solution-sketching example (cont).
- Logistic equation in many contexts
  - Classic example of the power of mathematics - one unifying description for many apparently unrelated phenomena.

$y' = \cos(y)$

Sketch a few solutions y(t).



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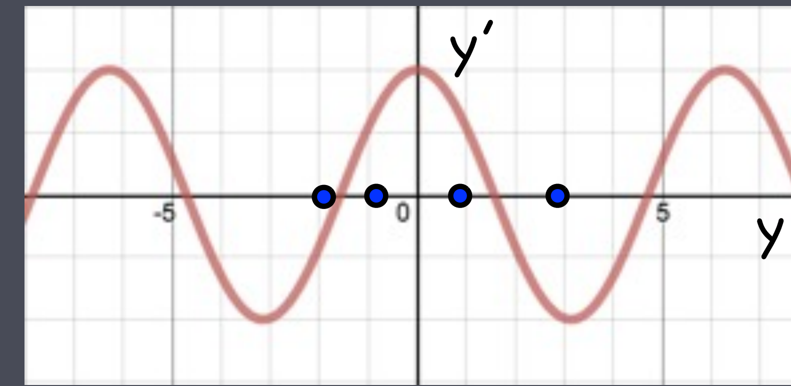

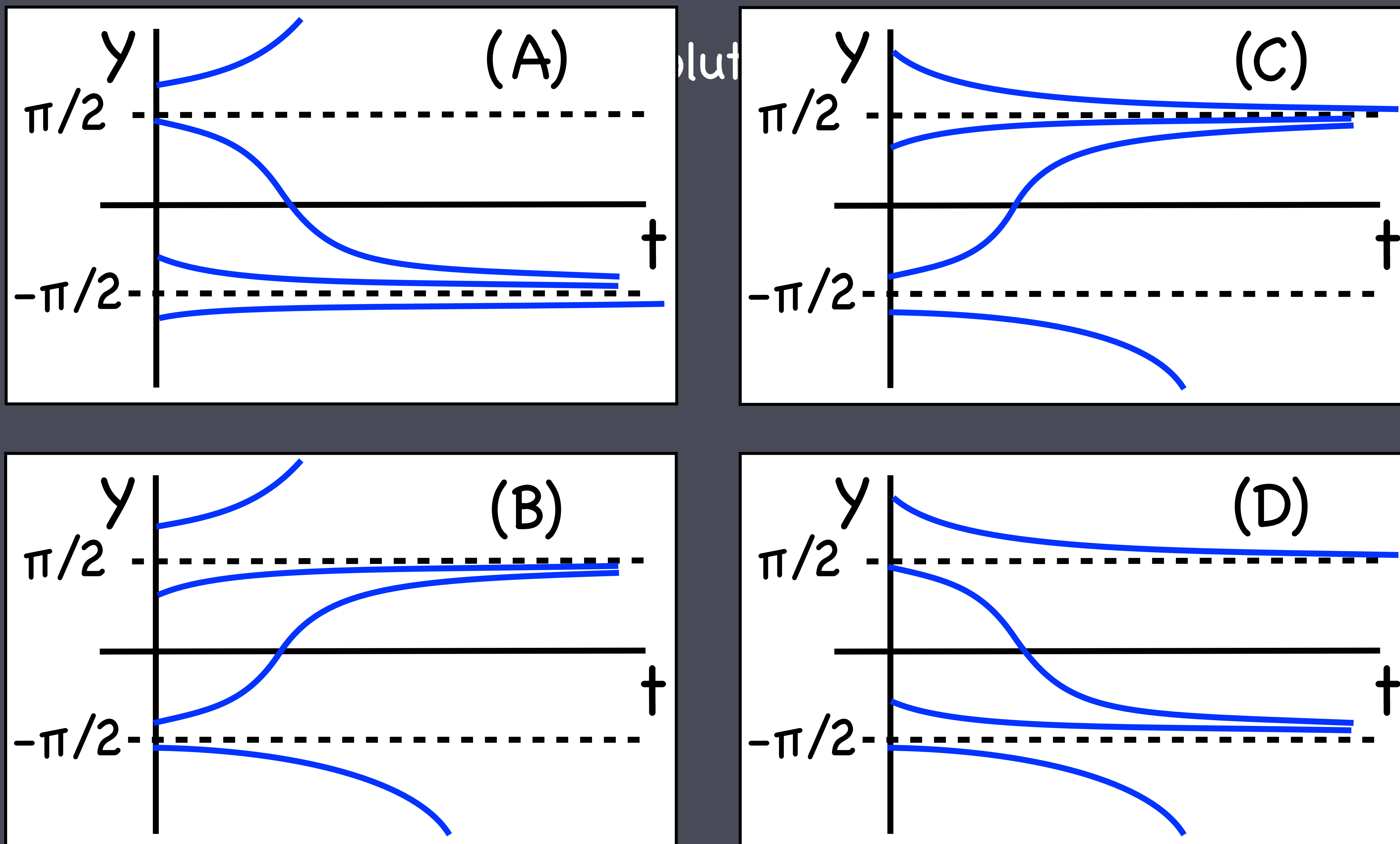

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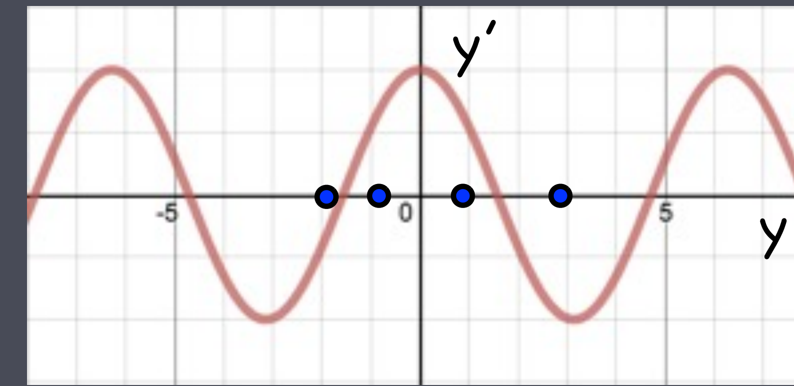

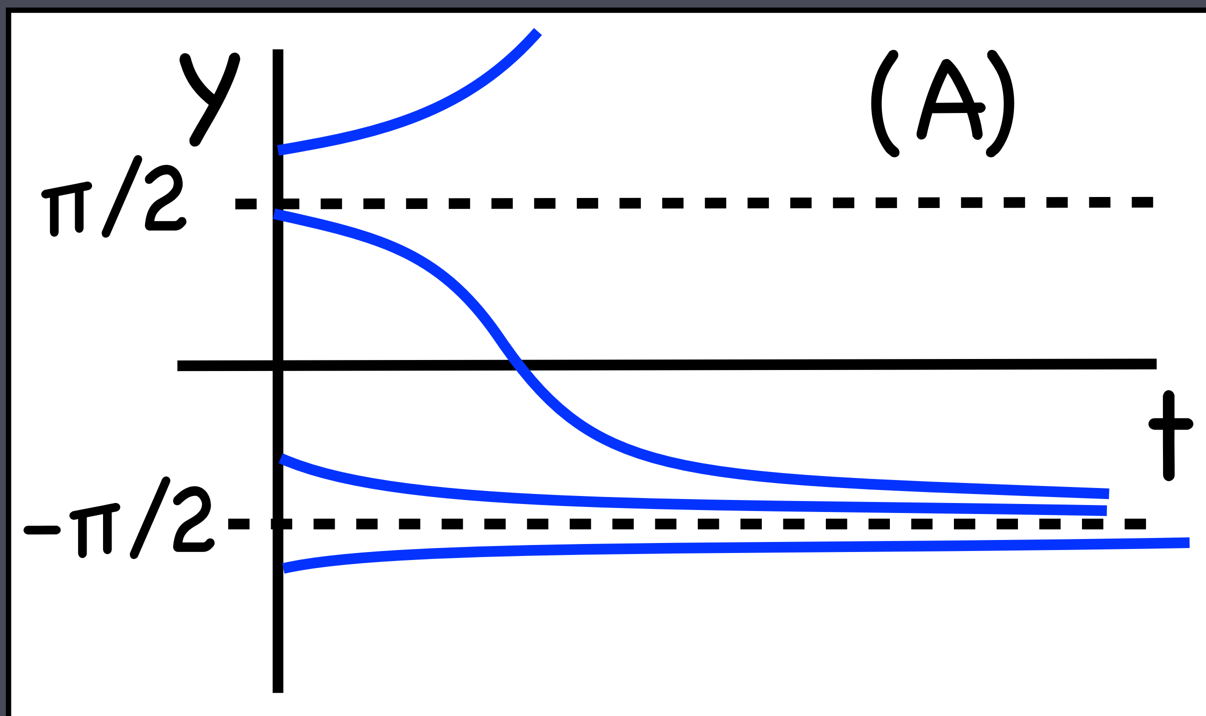

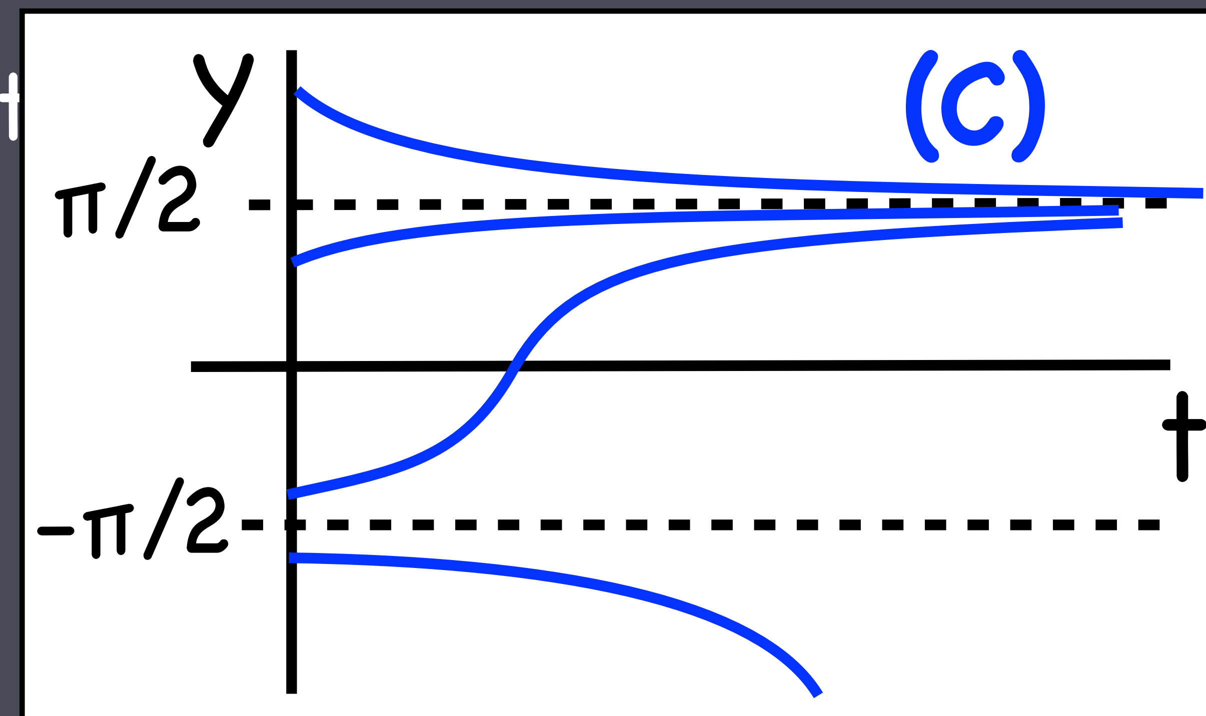

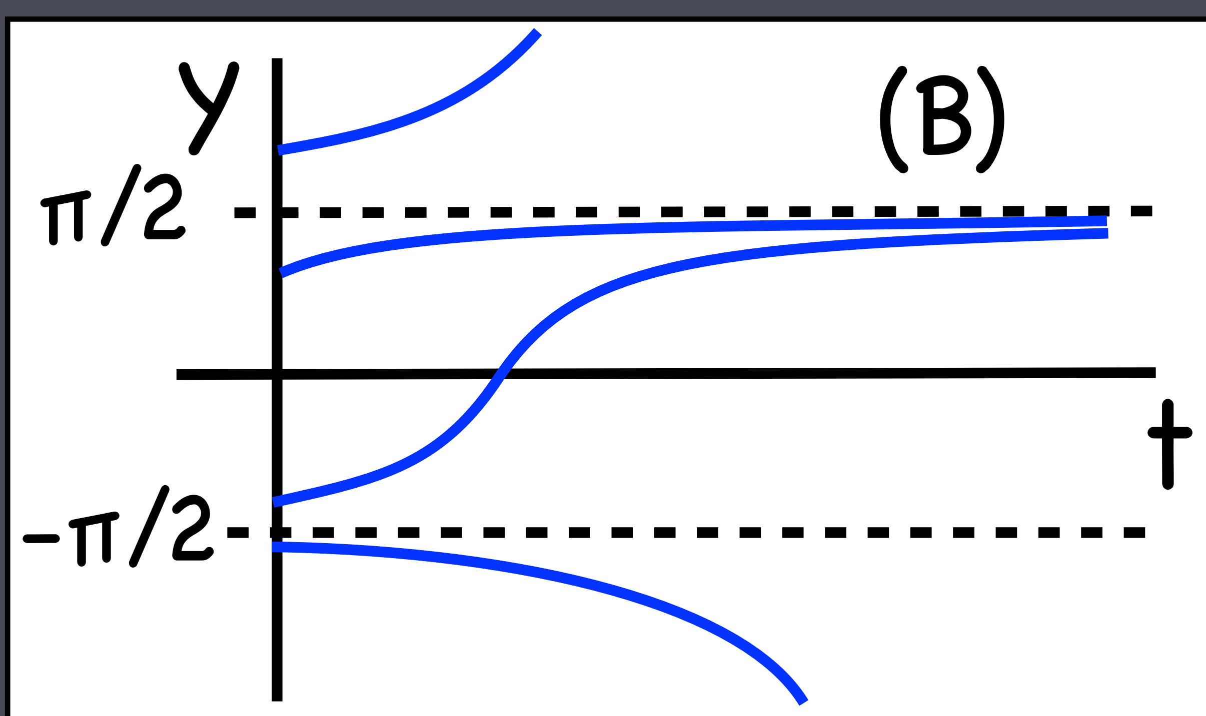

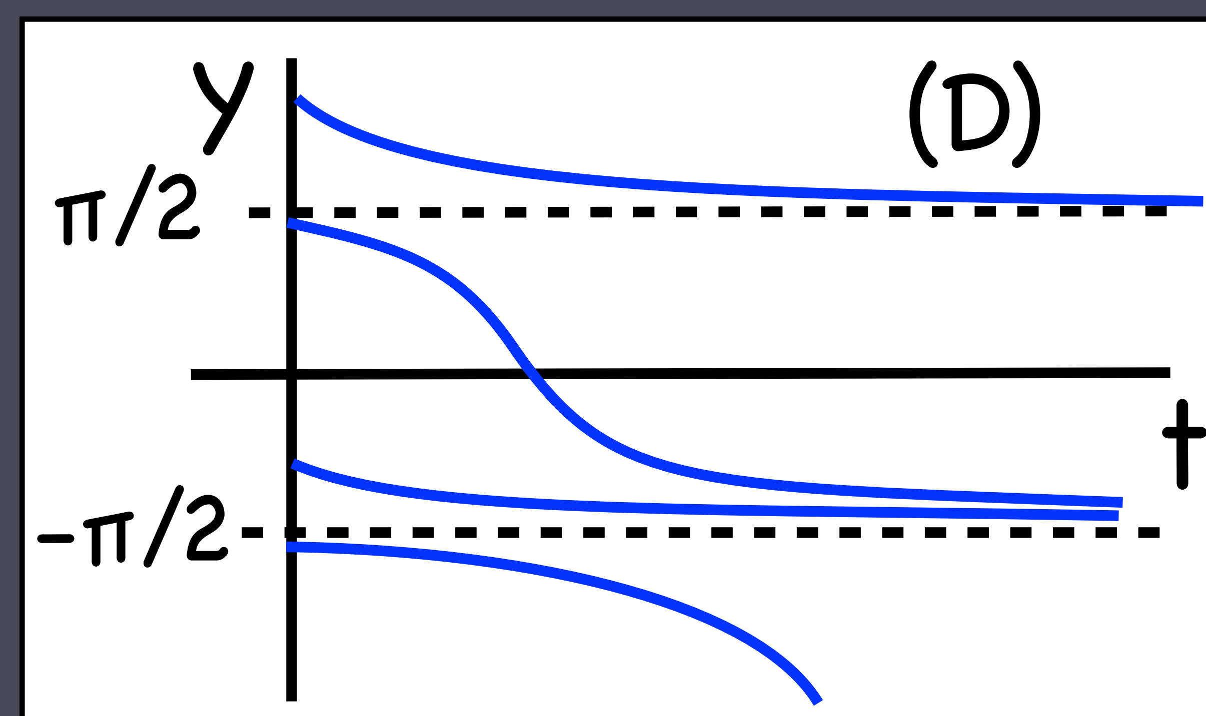

What you should be able to do:

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states.
- Determine long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, h-asymptotes).

Rates of change that are proportional to two things

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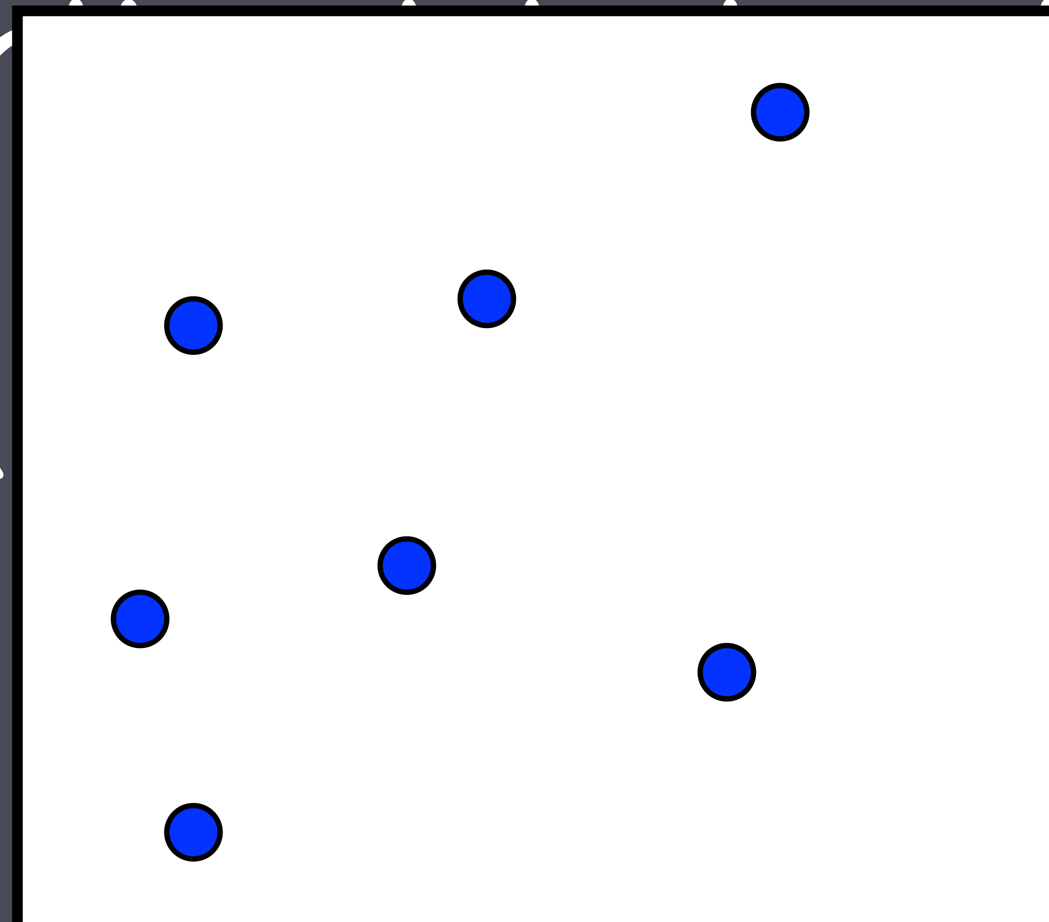

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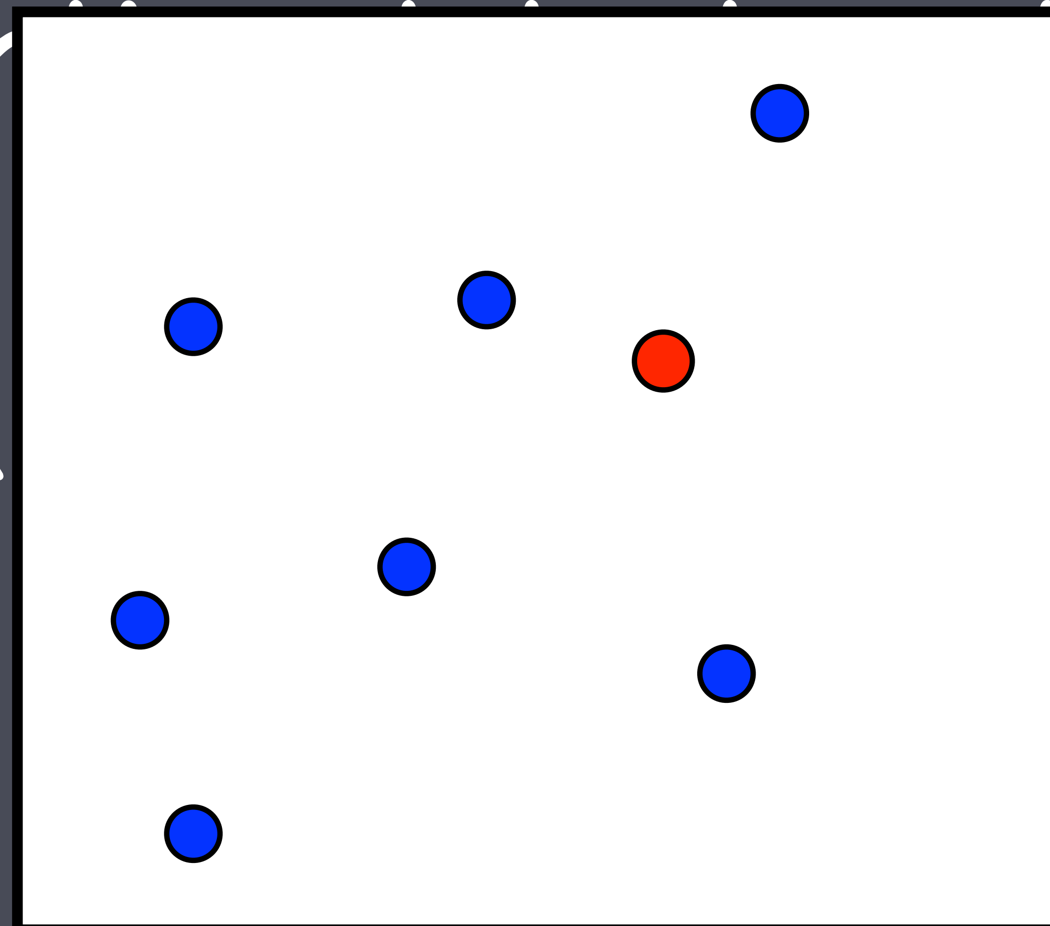

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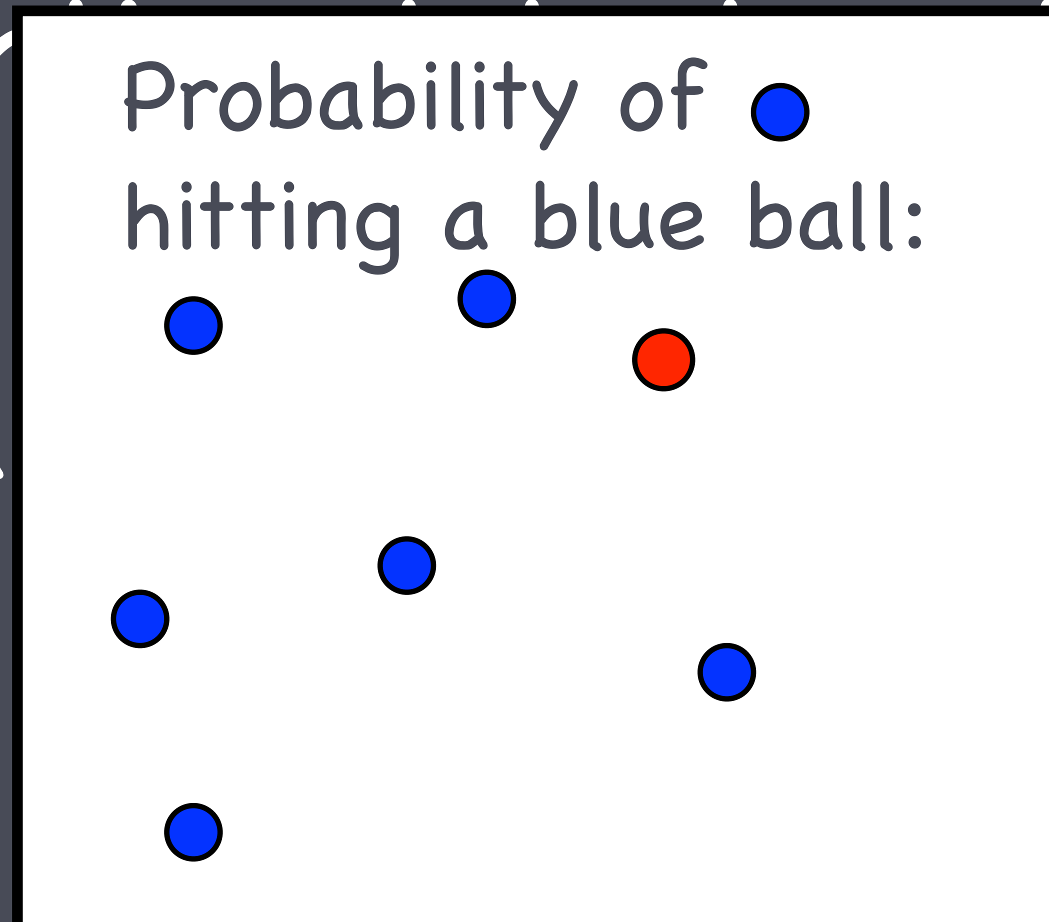

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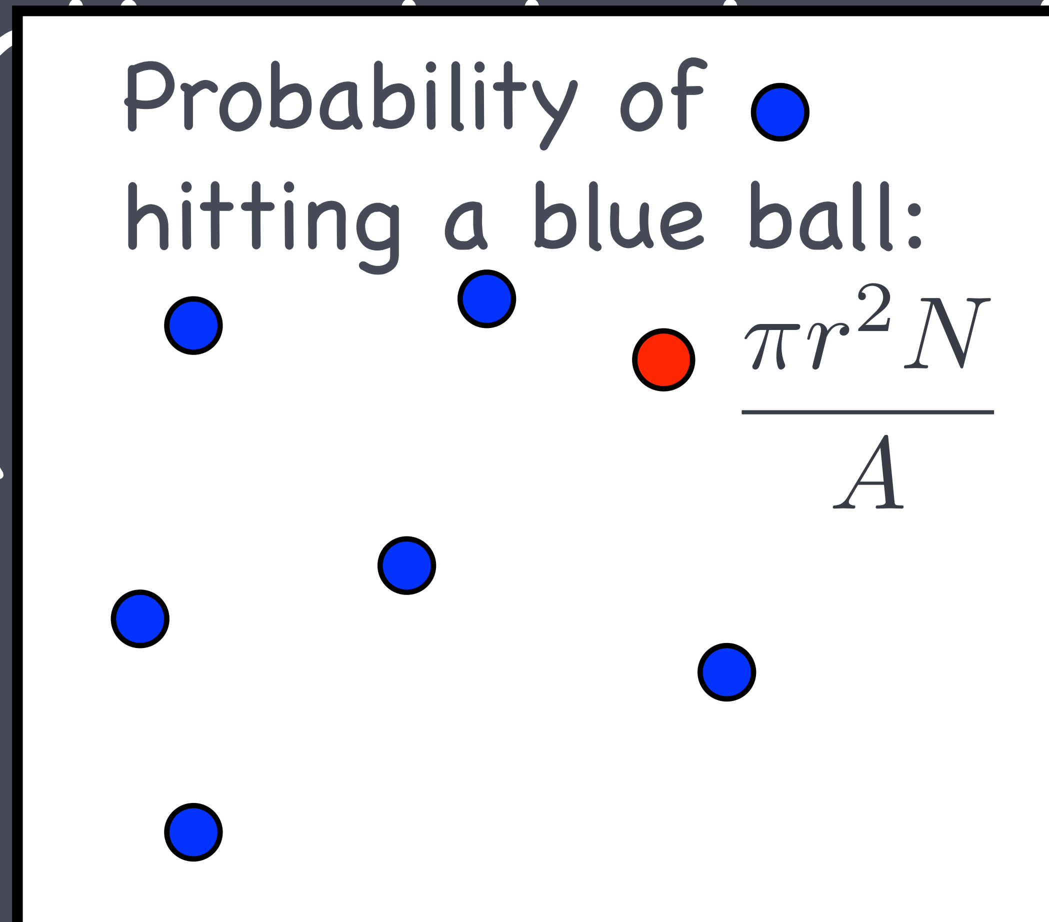

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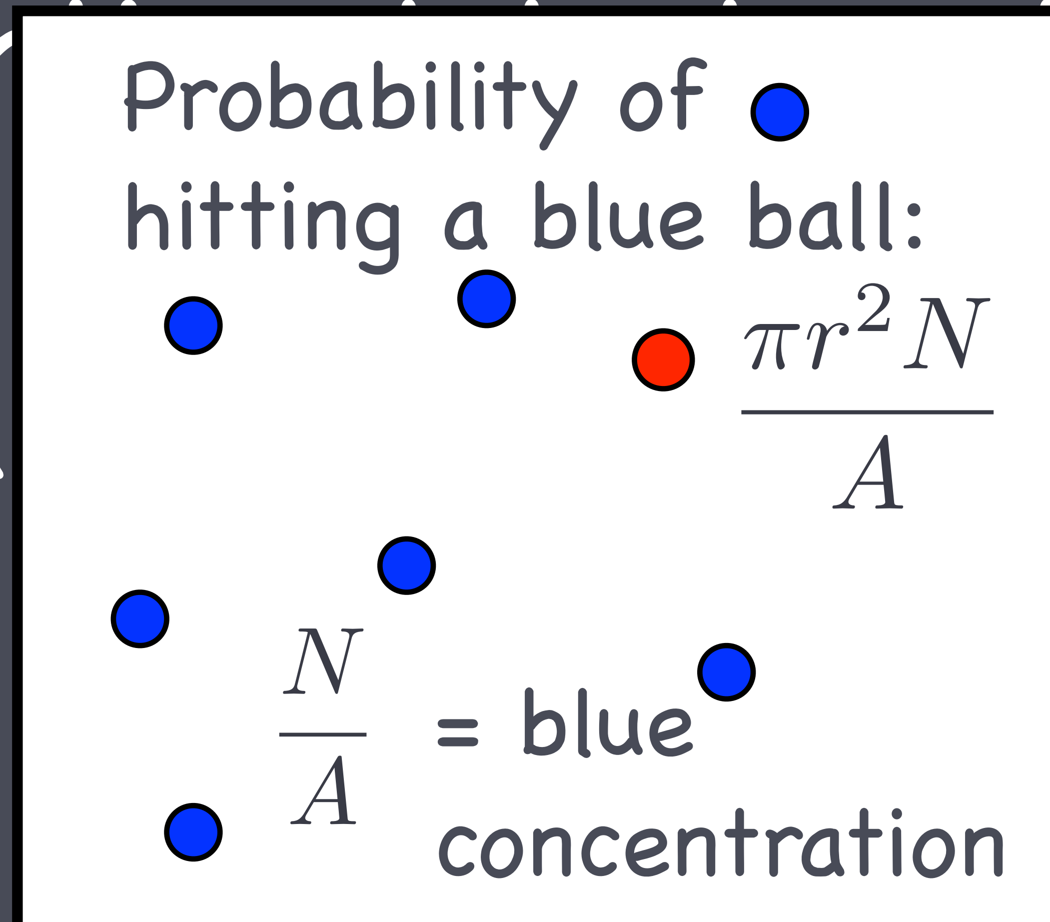

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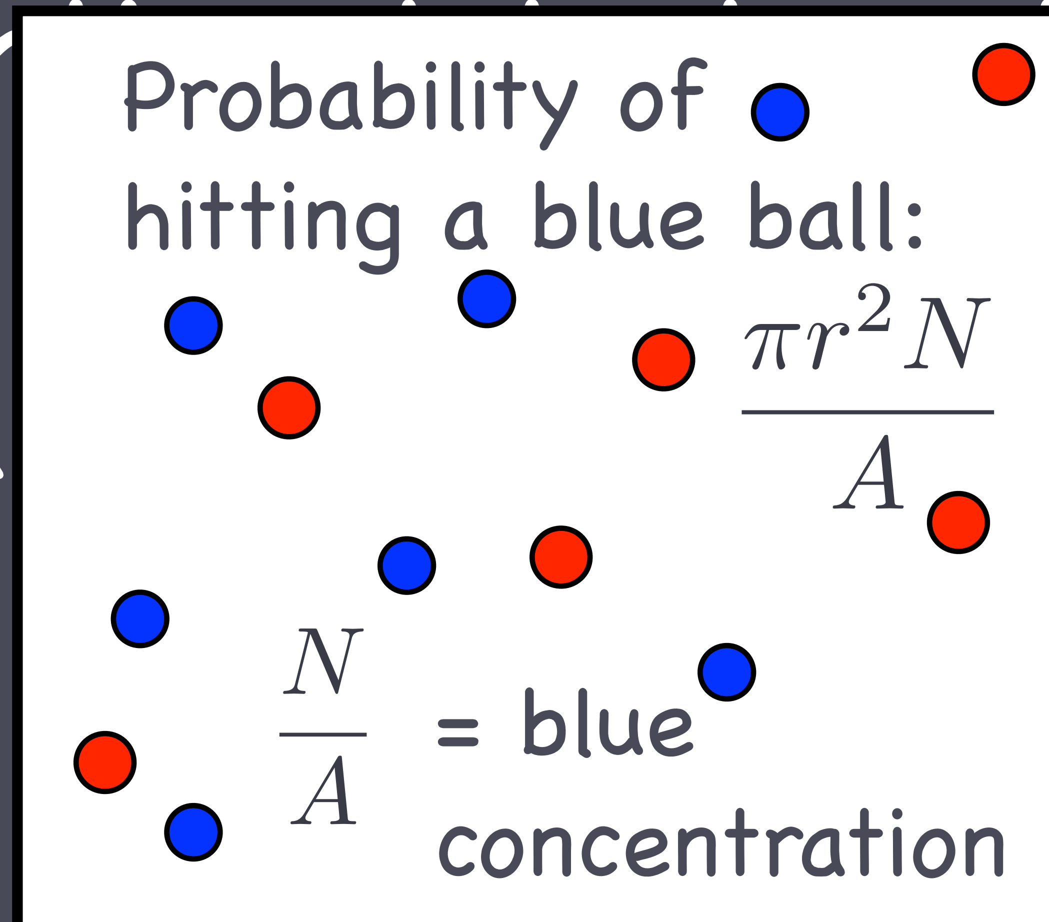

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Logistic equation in different contexts...

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- Active oil exploration sites: bUD (undiscovered and discovered).
- Waterlillies in a pond: bSW (waterlillies and space for waterwillies).

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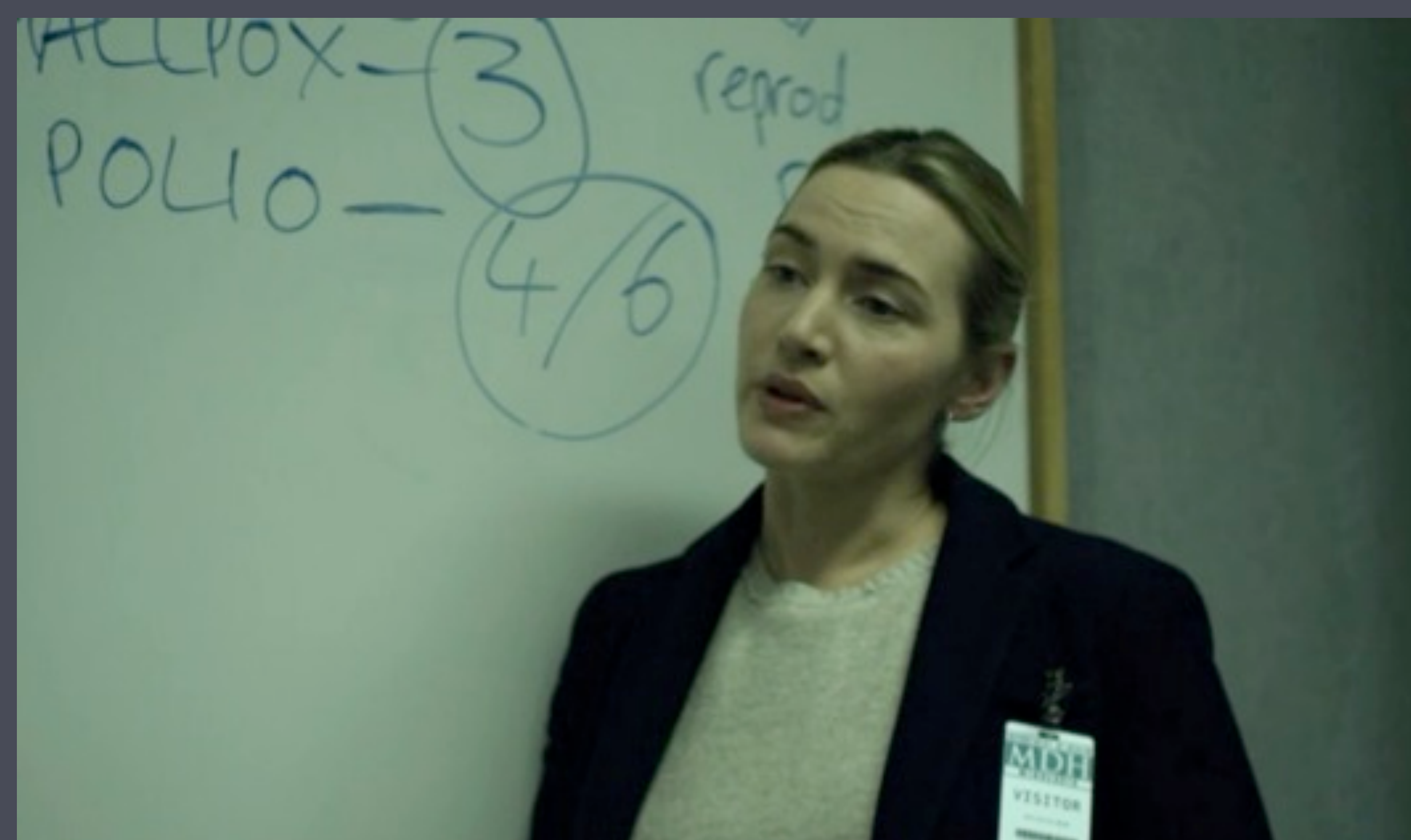
- X+Y= constant = C so Y=C-X.
- $\frac{dX}{dt} = bX(C - X)$

Infectious disease

Dr. Erin Mears: Once we know the R_0, we'll be able to get a handle on the scale of the epidemic.
Minnesota Health #4: So, it's an epidemic now. An epidemic of what?
Dave: We sent samples to the CDC.
Dr. Erin Mears: In seventy two hours, we'll know what it is, if we're lucky.
Minnesota Health #4: Clearly, we're not lucky.



Infectious disease

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(A) $$\frac{dI}{dt} = -bI(N-I)$$

(C) $$\frac{dS}{dt} = -bSI$$

(B) $$\frac{dI}{dt} = bI(N-I)$$

(D) $$\frac{dI}{dt} = bSI$$

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- Compare this with $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$.

Infectious disease

What is the carrying capacity?

$$\frac{dI}{dt} = bI(N - I)$$

(A) b/N

(B) N/b

(C) I

(D) N

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Infectious disease

What is the carrying capacity?

$$\frac{dI}{dt} = bI(N - I) = bNI\left(1 - \frac{I}{N}\right)$$

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$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

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(D) N

Everyone gets sick!

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

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- Suppose infected people recover at a rate proportional to how many there are.

Infectious disease

- Suppose infected people recover at a rate proportional to how many there are.
- The DE describing the spread of disease with recovery:

(A) $$\frac{dI}{dt} = bI(N-I) - \mu S$$

(B) $$\frac{dI}{dt} = bI(N-I) - \mu I$$

(C) $$\frac{dI}{dt} = -bI(N-I) + \mu I$$

(D) $$\frac{dI}{dt} = bI(N-I) + \mu I$$

Infectious disease

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$$\frac{dI}{dt} = bI(N - I) - \mu I$$

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$$= bI\left(N - \frac{\mu}{b} - I\right)$$

Infectious disease

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$$K = N - \frac{\mu}{b} < 0$$

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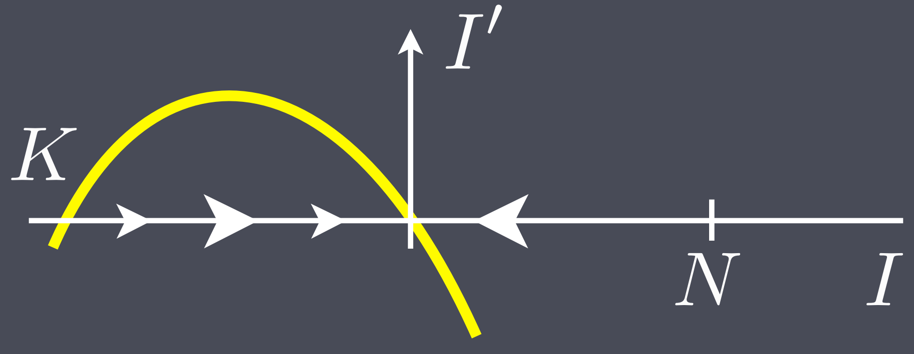

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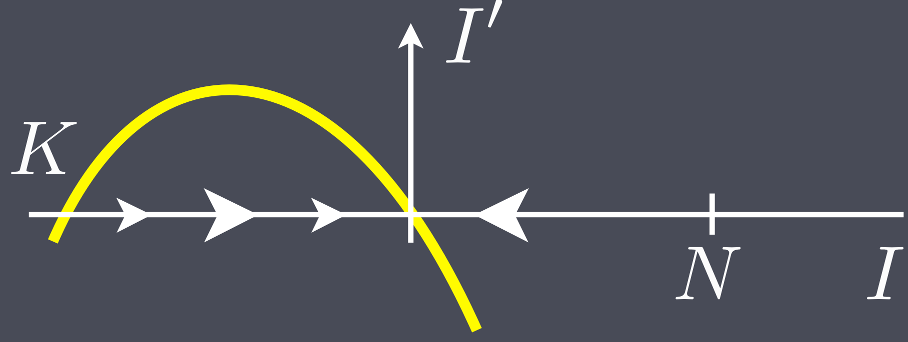

and the disease dies out.

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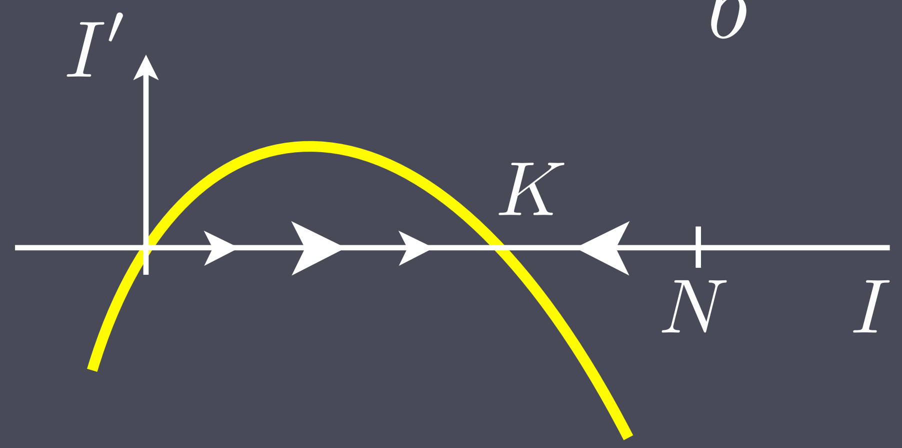

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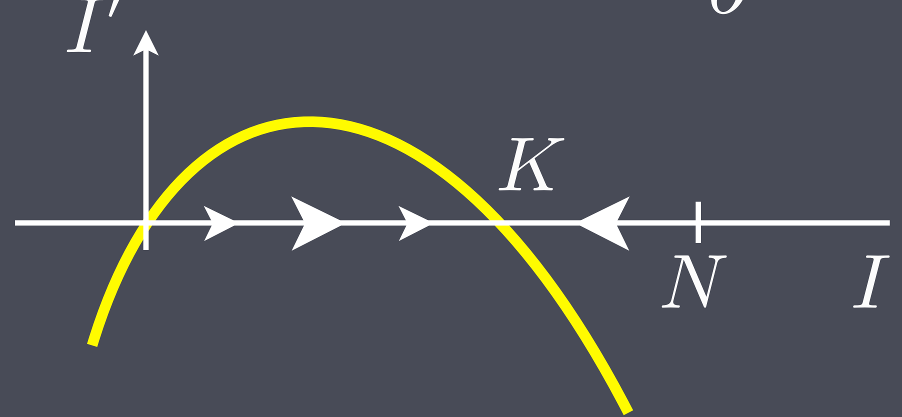

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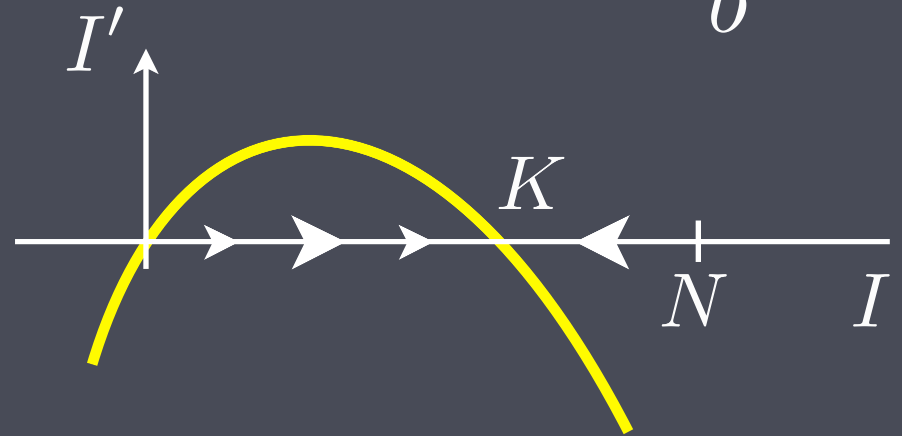

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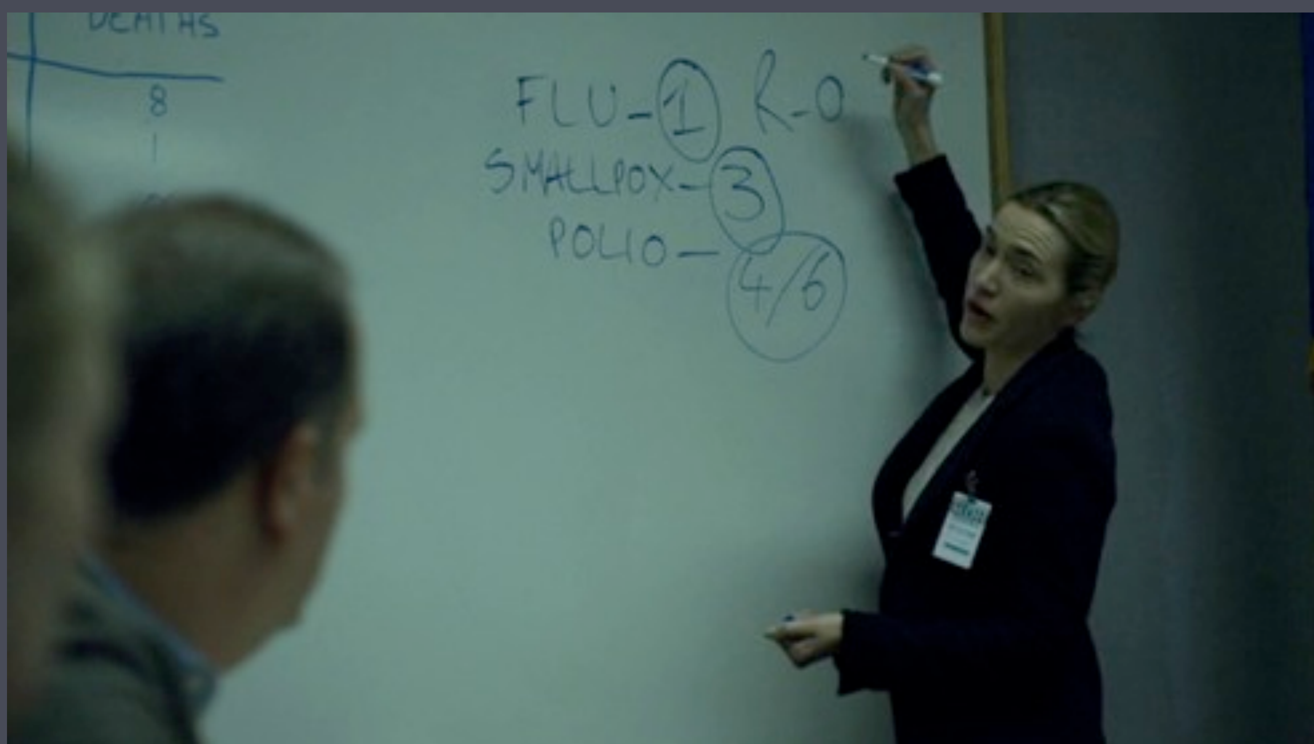

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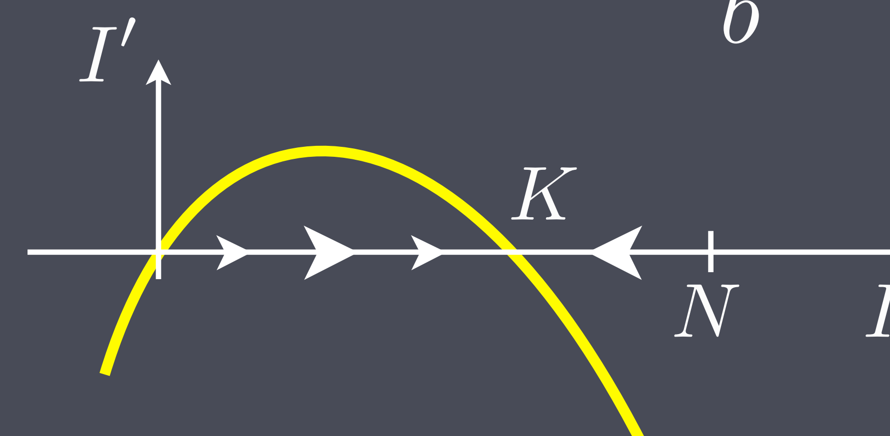

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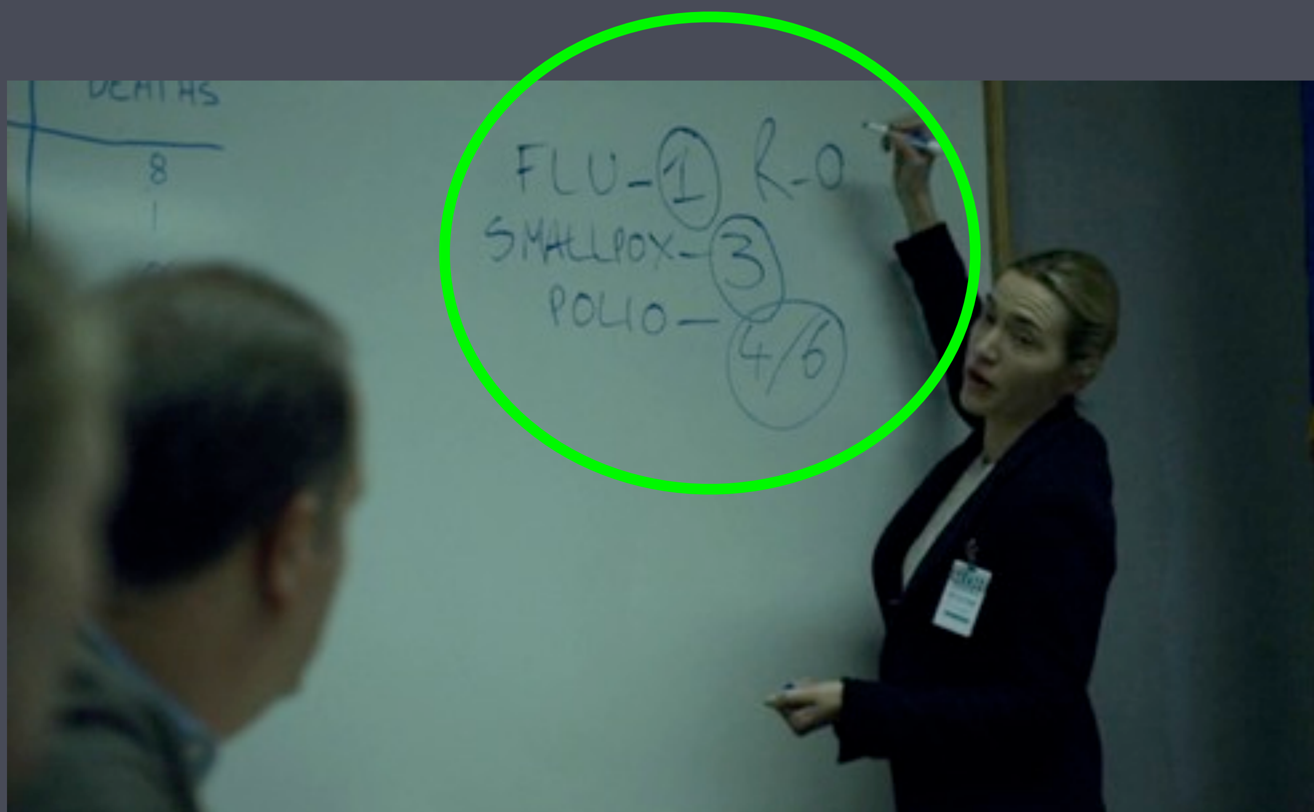

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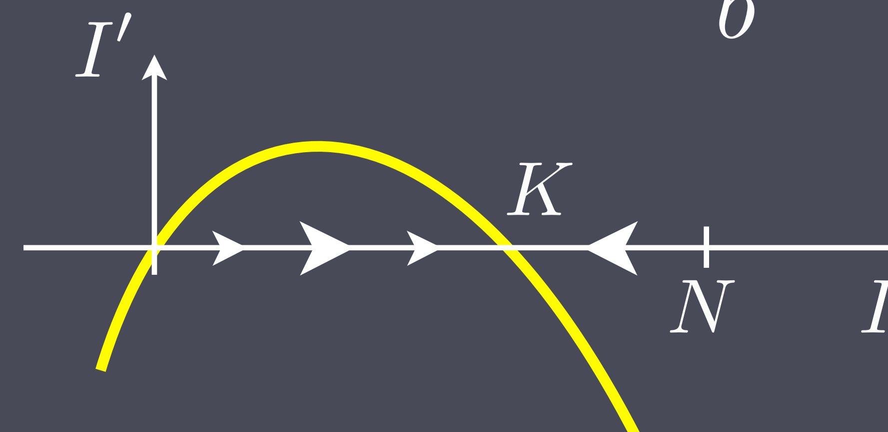

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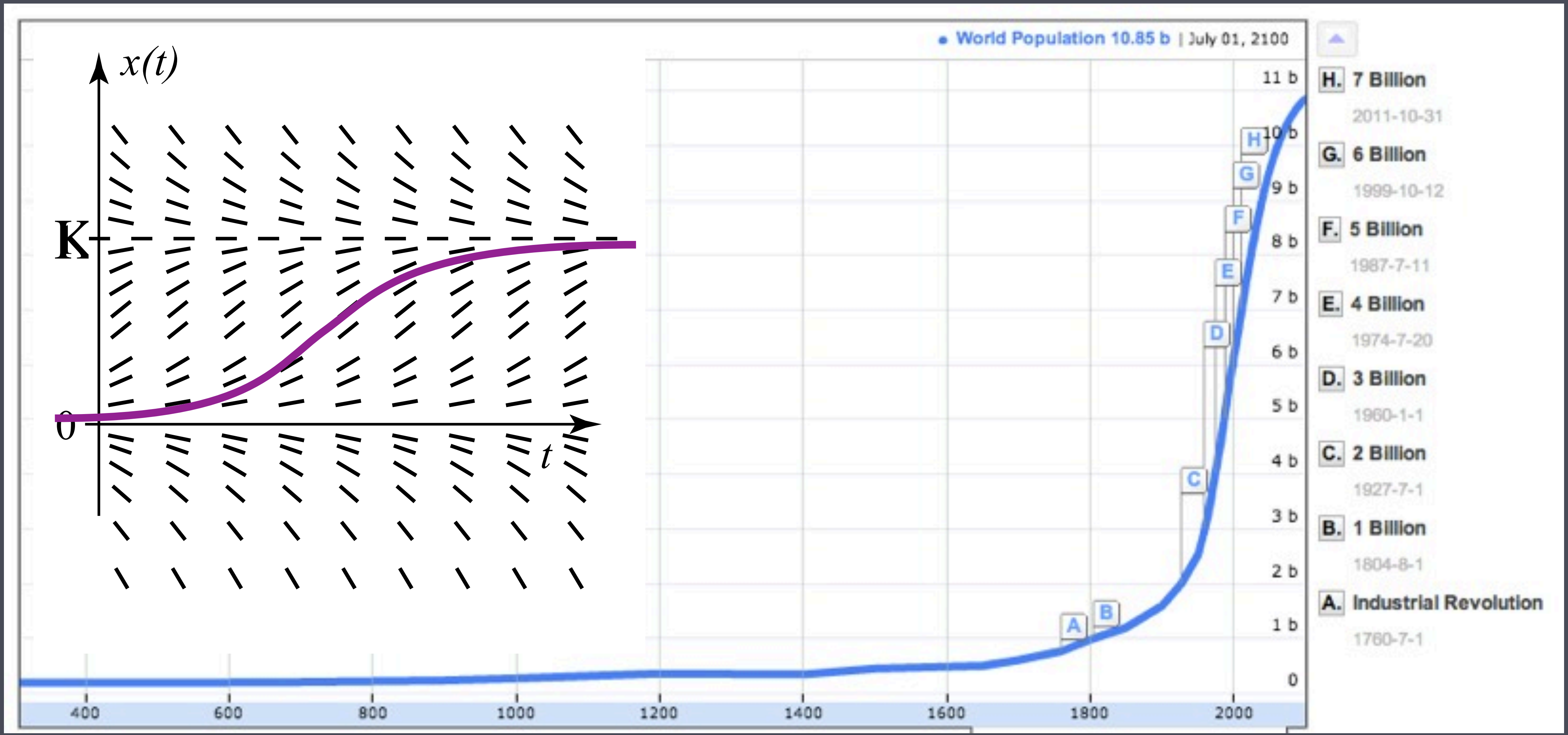
Some other logistic systems

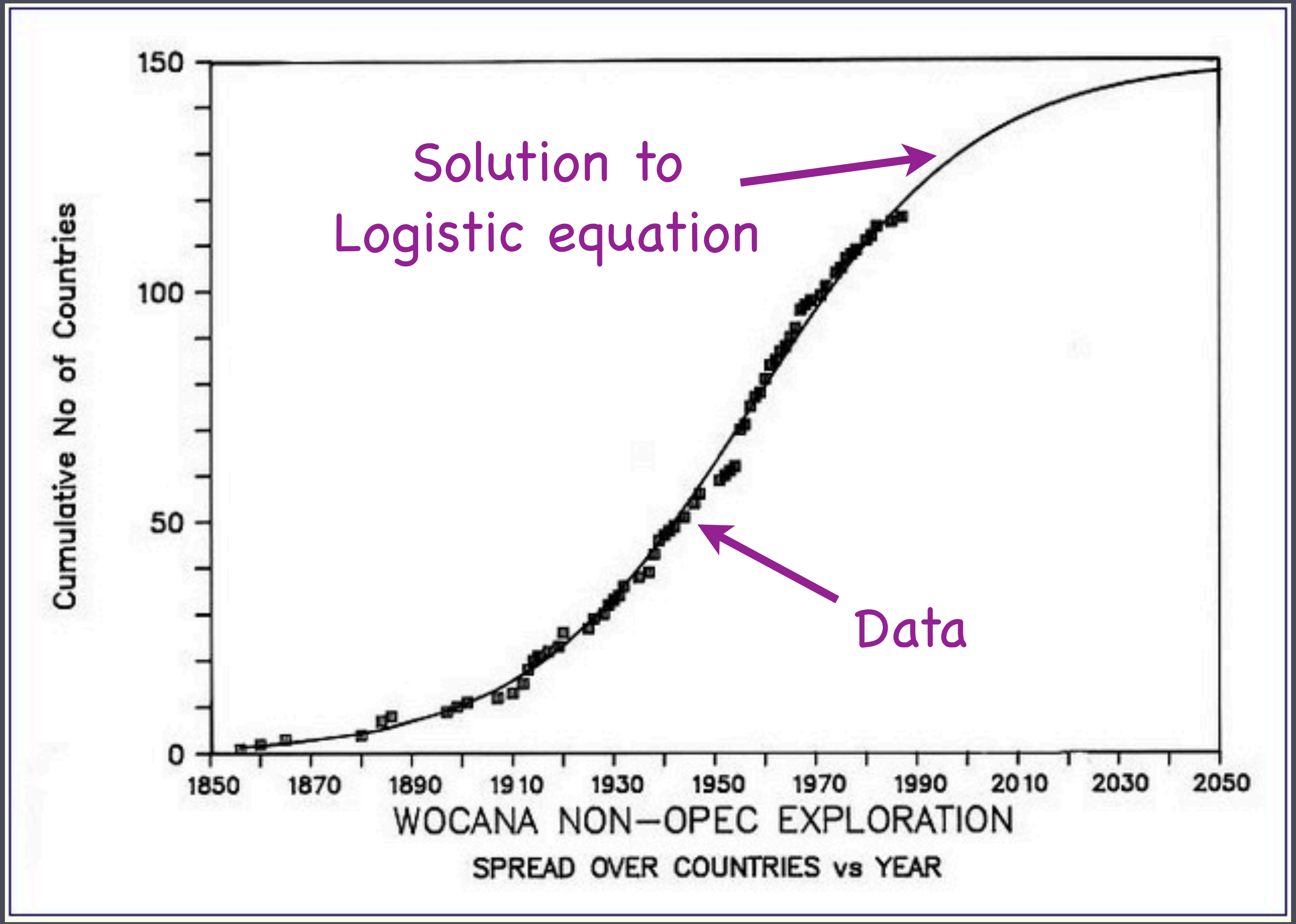
Human population



We're past the inflection point

- estimate K≈10 billion

Number of countries with active oil exploration



http://www.mhnederlof.nl/kinghubbert.html

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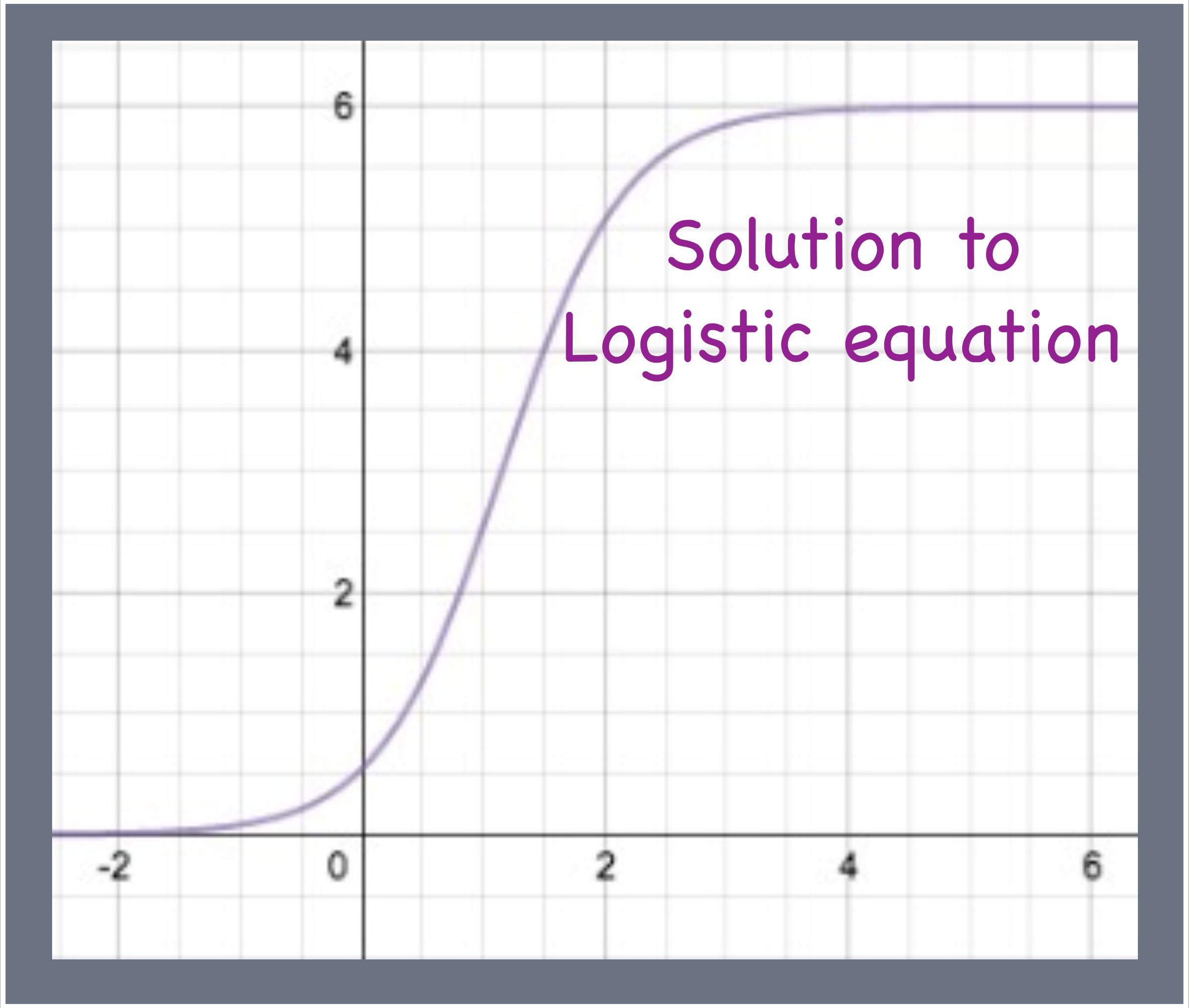

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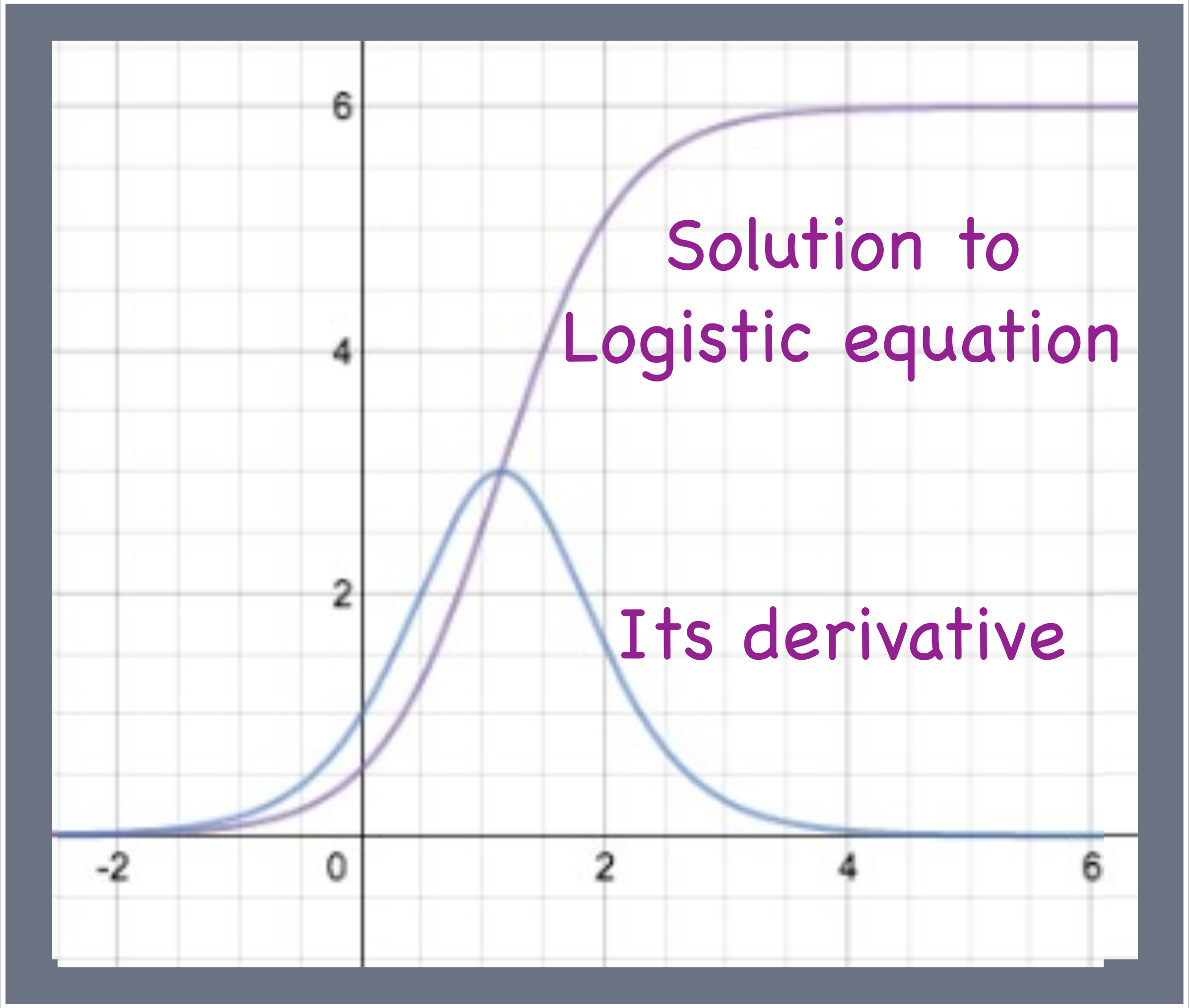

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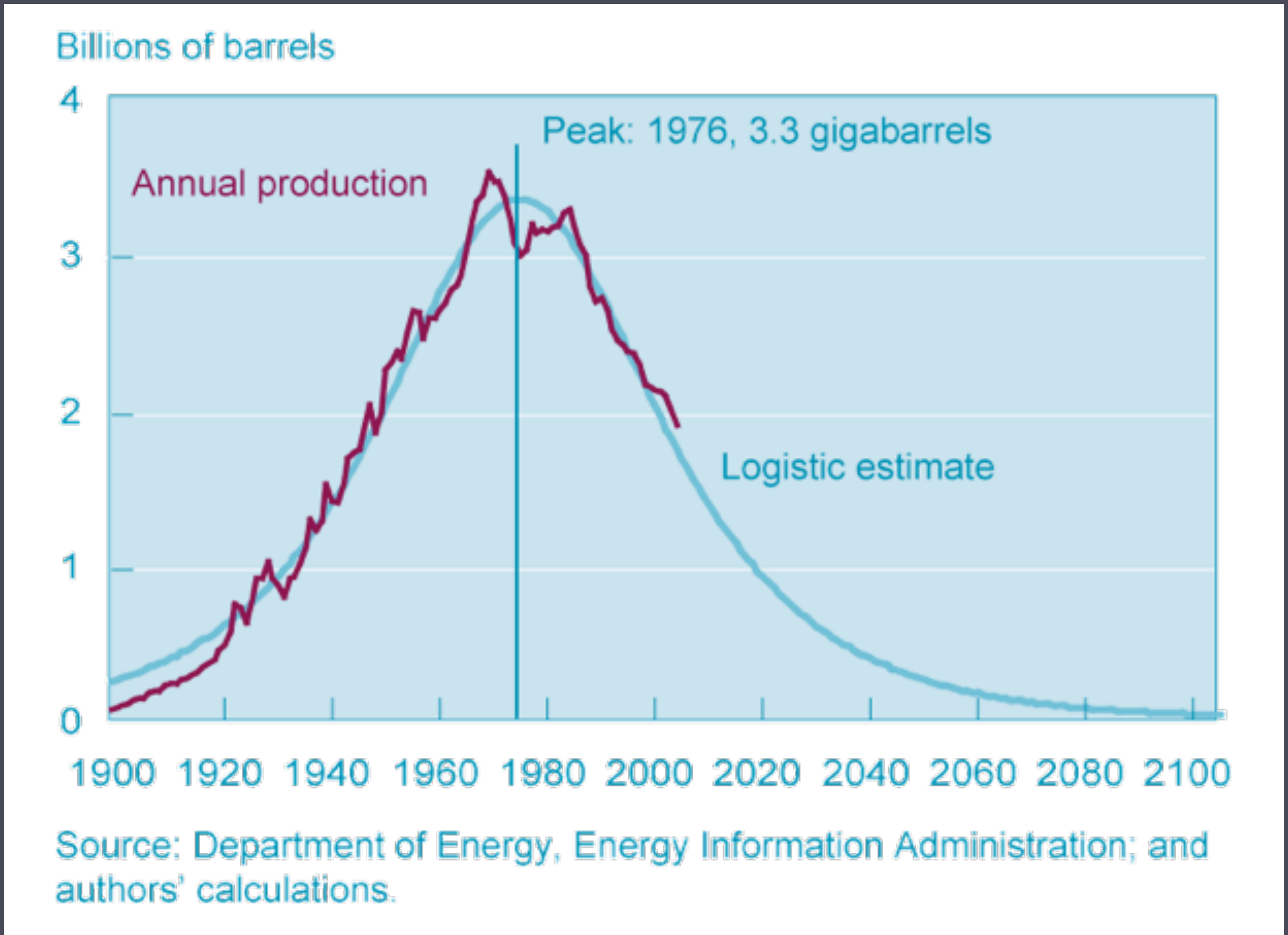

http://www.clevelandfed.org/research/commentary/2007/081507.cfm

World peak oil production