MATH 102 - MIDTERM TEST
University of British Columbia

Last name: First name:

Student number:

Section number:

Date: *October 20, 2015.*

Number of pages: *10 (including both cover pages and blank page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/record-ers/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For marking purposes only		
Page	Mark	Possible marks
3		6
4		6
5		7
6		8
7		6
8		7
9		8
Total		48

Multiple choice

Each multiple choice question is worth **2 pts**. No partial points will be given for work shown. **Enter your answers using the bubbles on the front page.**

1. When $x = 100$ the function $g(x) = \dfrac{x^5 - 2x^3 - 3x^2}{5x^7 + x^4 + x^2}$ is closest to

   (A) -3 (B) 0 (C) 1/50000 (D) π (E) 1000000

   C

   $$g(100) \approx \frac{100^5}{5 \cdot 100^7} = \frac{1}{50000}$$

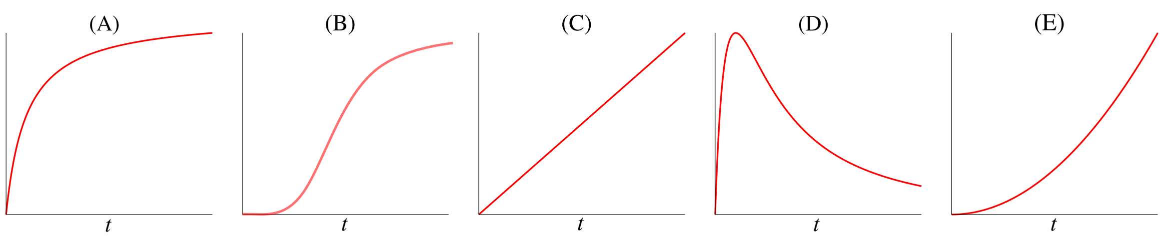
2. When $x = 0.1$ the function $g(x) = \dfrac{x^5 - 2x^3 - 3x^2}{5x^7 + x^4 + x^2}$ is closest to

   (A) -100000 (B) -3 (C) 0.01 (D) π (E) 20

   B

   $$g(0.1) \approx \frac{-3(0.1)^2}{(0.1)^2} = -3$$

3. Each of the graphs represents $f(t)$, the amount of food gained during time t spent in a food patch. Which of these graphs represents the type of patch where food can be collected rapidly at first, but there is only some maximal amount that can be obtained no matter how long the search continues?



   A

Enter your answers using the bubbles on the front page.

Multiple choice (continued)

Enter your answers using the bubbles on the front page.

4. Consider a differentiable function $y = f(x)$. Which of the following statements best describes a tangent line to the graph of this function?

   (a) The slope of the tangent line is given by $\dfrac{f(x+h) - f(x)}{h}$.

   (b) The tangent line at x_0 is close to any secant line through x_0.

   (c) The tangent line can only intersect the graph of $f(x)$ at one point.

   (d) The tangent line locally approximates the function.

   (e) More than one of the above is correct.

   d

5. The absolute maximum of the function $y = f(x) = x + \frac{1}{x}$ on the interval $0.1 \leq x \leq 2$ occurs at

   (a) $x = -1$

   (b) $x = 0.1$

   (c) $x = 1$

   (d) $x = 2$

   (e) $x = \pm 1$

   b

6. Let $y = f(x)$ be a smooth function (derivatives of all orders exist) at x_0. Which of the following statements is correct?

   (a) If $f''(x_0) = 0$, then the function has an inflection point at x_0.

   (b) If the function has an inflection point at x_0, then $f''(x_0) = 0$.

   (c) Both (a) and (b) are correct.

   (d) If $f''(x_0) = 0$, then the function has a critical point at x_0.

   (e) If $f''(x_0) = 0$, then the function never has a critical point at x_0.

   b

Enter your answers using the bubbles on the front page.

Short-answer problems

Show your work. Enter your answer in the box provided.

7. **[2 pt]** Using any method, determine the value of the following limit

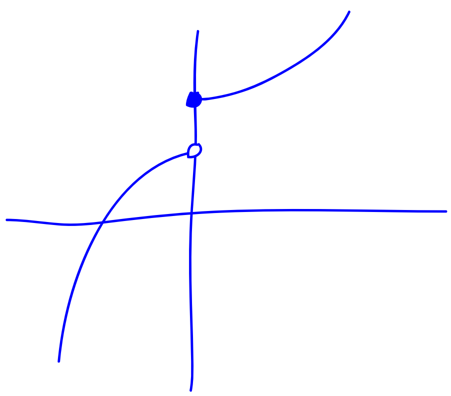
$$\text{Limit} = \lim_{h \to 0} \frac{(x+h)^{10} - x^{10}}{h}.$$

$f(x) = x^{10}$ $\quad$ Limit $= f'(x) = 10x^9$

Limit = $\boxed{10x^9}$

8. **[5 pt]** Consider the function $f(x) = \begin{cases} 2x^3 + 1 & x < 0, \\ x^2 + 2 & x \geq 0. \end{cases}$

Calculate the following. Enter DNE if a limit does not exist.



$\lim_{x \to 0^-} f(x) =$ $\boxed{1}$

$\lim_{x \to 0^+} f(x) =$ $\boxed{2}$

$\lim_{x \to 0} f(x) =$ $\boxed{\text{DNE}}$

Is $f(x)$ continuous at $x = 0$ (Yes/No): $\boxed{\text{No}}$

Is $f(x)$ differentiable at $x = 0$ (Yes/No): $\boxed{\text{No}}$

9. **[5 pt]** Consider a differentiable function $f(x)$ for which we want to estimate $f(2)$.

(a) Suppose that we found out that $f(1) = 1$, $f'(1) = 1/2$. Estimate $f(2)$ using this information.

$f(x) \approx f(1) + f'(1)(x-1)$ 1

$f(2) \approx 1 + \frac{1}{2}(2-1) = 3/2$ 1

$f(2) \approx$ $\boxed{3/2}$

(b) We subsequently discovered that $f(3) = 3$ and $f'(3) = 2$, and that f is concave up. Use this new information to get another estimate for $f(2)$. Would that estimate be better or worse than the one in the previous part?

$f(3) \approx 3 + 2(2-3) = 1$ 1

$f(2) \approx$ $\boxed{1}$

Circle one of these: **better** or **worse** and explain:

f is concave up 1 so linear approximations underestimate values of f. The second approximation is less 1 than the first one so must be further from f(2).

10. **[3 pt]** Using Newton's method, with iterations given by $x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$, to approximate the value of x at which $\sqrt{x} = 5/x$, what **polynomial** would you choose as your function $g(x)$? For what **integer** value for x_0 would the iterations get close to the solution the fastest?

$\sqrt{x} = 5/x$

$x = 25/x^2$

$x^3 = 25$

$x^3 - 25 = 0$

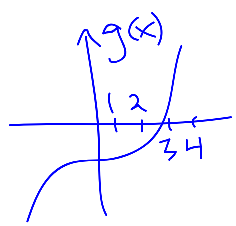
$g(x) = x^3 - 25$ 2

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$x_0 = 3$ 1



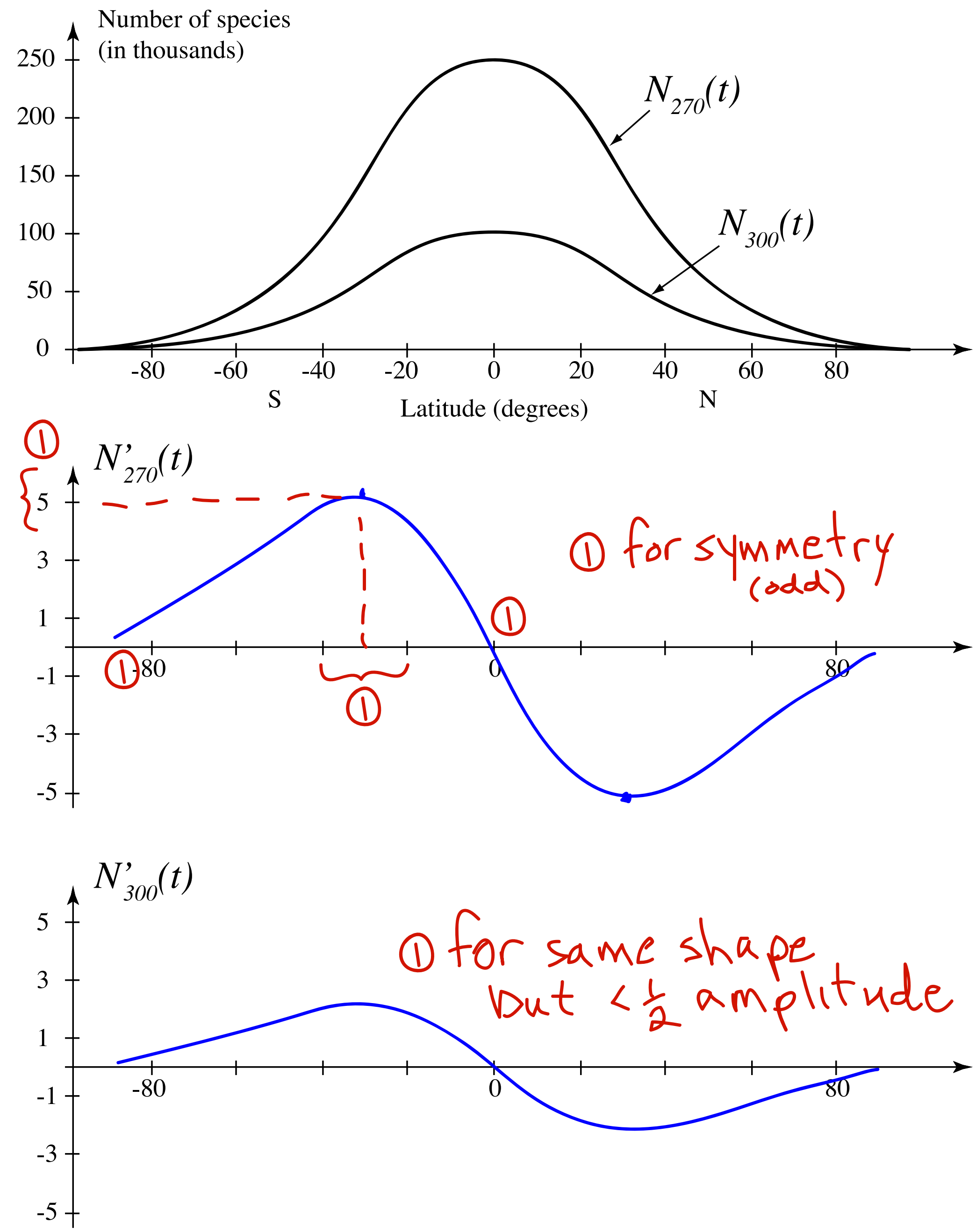
$g(x) =$ $\boxed{x^3 - 25}$

$x_0 =$ $\boxed{3}$

Sketching problems

Show your work.

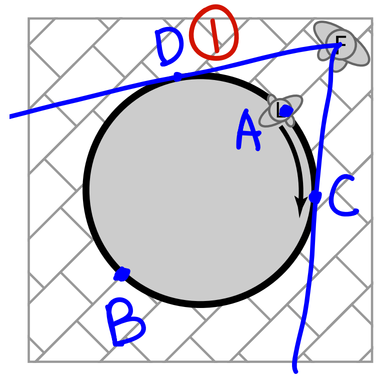
11. **[6 pts]** It is well known to ecologists that diversity of species is greatest close to the equator (latitude 0) and much lower near the North and South poles (latitudes ±90 degrees). Shown below are two graphs of the number of species ($N(x)$) versus latitude (x) at two geological ages, 300 and 270 million years ago. On the axes provided, sketch the derivatives of each of these two functions.

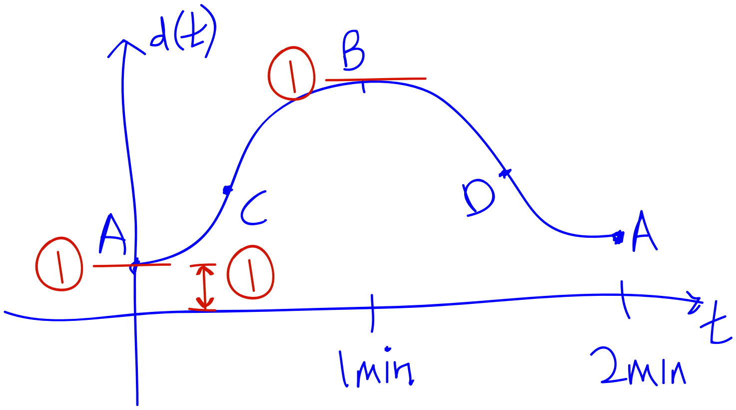


12. There is a circular pool in the centre of a square courtyard. Lucas' father (F) stands in the corner of the courtyard. Lucas (L) starts on the edge of the pool at the point closest to his father and walks once around the edge of the pool at a constant speed. It takes Lucas two minutes to go around the pool.

(a) **[3 pts]** Sketch the graph of the distance between Lucas and his father as a function of time. Do not try to write down an equation for the distance - just make an approximate sketch based on the diagram with a focus on getting key features (minima, maxima) correct.

(b) **[4 pts]** If your graph has any minima, maxima and/or inflection points, explain which points on the edge of the pool correspond to each of these special points and label them on your graph and on the diagram with the letters A, B, C...





A: min when Lucas is closest to his father 1

B: max when Lucas is farthest from his father 1

C: inflection point when Lucas is walking directly away from his father.

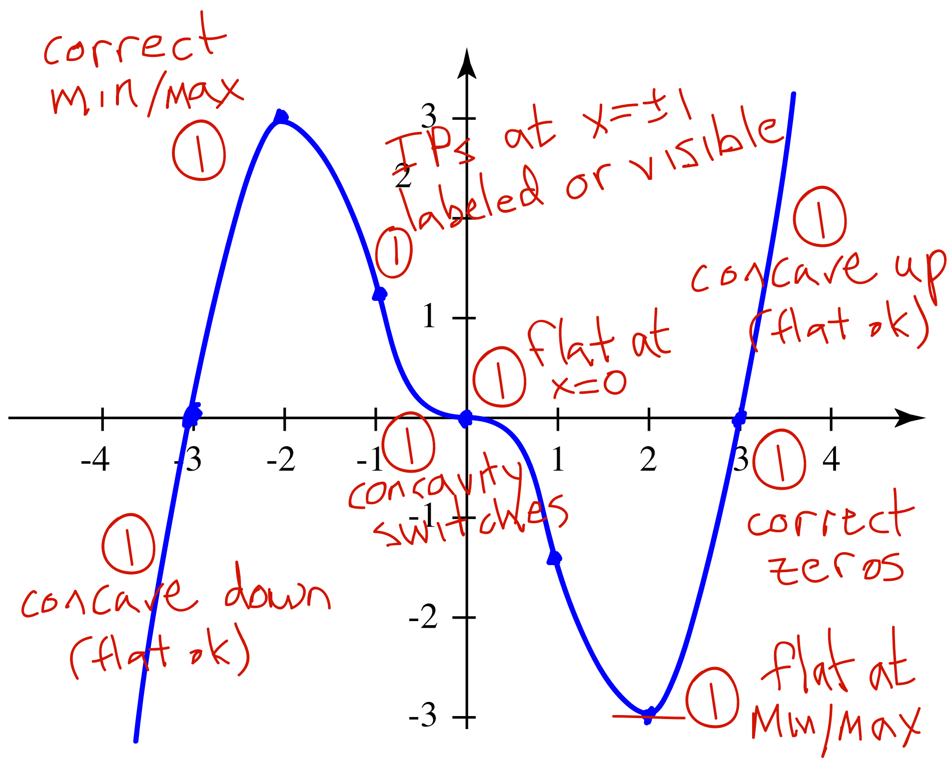
D: inflection point when Lucas is walking directly toward his father.

1

13. **[8 pt]** Consider the function $f(x)$ which is twice differentiable ($f''(x)$ exists) for $-4 \leq x \leq 4$. The table below summarizes ALL important information about $f(x)$, $f'(x)$, and $f''(x)$. Use this information to graph the function, **clearly indicating zeros, local minima, local maxima and inflection points** of the function on the interval [-4,4].

The columns indicate either the value or sign of $f(x), f'(x), f''(x)$ at specific points or in the intervals in between. For example, at $x = -3.5$, the function and its second derivatives are both negative and the derivative is positive.

x	[-4,-3)	-3	(-3,-2)	-2	(-2,-1)	-1	(-1,0)	0	(0,1)	1	(1,2)	2	(2,3)	3	(3,4]
$f(x)$	-	0	+	3	+	+	+	0	-	-	-	-3	-	0	+
$f'(x)$	+	+	+	0	-	-	-	0	-	-	-	0	+	+	+
$f''(x)$	-	-	-	-	-	0	+	0	-	0	+	+	+	+	+



Recommended: Draw a practice sketch below before using the axes above. Only the sketch on the axes above will be marked.

This page may be used for rough work. It will not be marked.