Today

- Derivative of a^x.
- Converting between a^x and e^{kx}.
- Bacterial growth example
- Doubling time, half life, characteristic time

$f(x)=a^x$. $f'(x)=C_a a^x$. C_a=??

- Recall that we got stuck on this derivative.
- Time to get unstuck...

30s

$$f(x) = e^{\ln(2)x}.$$

(A) $f'(x) = e^{\ln(2)x}$.

(B) $f'(x) = \ln(2)e^{\ln(2)x}$.

(C) $f'(x) = \ln(2) \cdot 1/2 \cdot e^{\ln(2)x}$.

(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}$.

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(A) $f(x) = 2x.$

(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

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- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.
- $f(x) = e^{\ln(2)x}$ --> $f(x) = 2^x$.
- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.

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- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.
- $f(x) = e^{\ln(2)x}$ --> $f(x) = 2^x$.
- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.
- In general, $f(x) = a^x$ --> $f'(x) = a^x \ln(a)$.

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

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30s

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

Rough estimate:

(A) ~1 week.

(B) ~2 weeks.

(C) ~1 year.

(D) ~10^4 days ≈ 27 years.

(E) ~10^5 days ≈ 270 years.

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A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $p(t) = e^{t/24}$.

(B) $p(t) = 100{,}000\ 2^{t/24}$.

(C) $p(t) = e^{\ln(2)t}$.

(D) $p(t) = 2^{-t/24}$.

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A single cell is placed in dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $t=\ln(10^5)/\ln(2)$

(B) $t=10^5/\ln(2)$

(C) $t=\ln(10^5)/2$

(D) $t=100{,}000/24$ days

A single cell is placed in dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $t=\ln(10^5)/\ln(2)$ ≈ 16.6 days

(B) $t=10^5/\ln(2)$

(C) $t=\ln(10^5)/2$

(D) $t=100{,}000/24$ days

Doubling time

- Let $c(t) = c_0e^{kt}$. At $t=0$, $c(0) = c_0e^0 = c_0$.
  - If $k>0$, $c(t)$ is increasing and doubles when $c_0e^{kt} = 2c_0$.
  - That is when $t=\ln(2)/k$.
  - This is called the doubling time.

Half-life

- Let $c(t) = c_0e^{kt}$. At $t=0$, $c(0) = c_0e^0 = c_0$.
  - If $k<0$, $c(t)$ is decreasing and halves when $c_0e^{kt} = c_0/2$.
  - That is when $t=-\ln(2)/k$.
  - This is called the half-life.

Characteristic time

- Let $c(t) = c_0 e^{kt}$. At $t=0$, $c(0) = c_0 e^0 = c_0$.
  - If $k<0$, $c(t)$ is decreasing and reaches 1/e its original value when $c_0 e^{kt} = c_0/e$.
  - That is when $t=-1/k$.
  - This is called the **characteristic time** or **mean life**.

Log-log and semi-log plots

- A log-log plot is a plot on which you plot log(y) versus log(x) instead of y versus x.
- A semi-log plot is a plot on which you plot log(y) versus x instead of y versus x.

Log-log plot of power function

- Suppose $y = ax^p$.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ax^p) = \ln(a) + p \ln(x)$.
- $V = A + pU$ where $A=\ln(a)$, $U=\ln(x)$.
- On a log-log plot, $y=ax^p$ looks linear.

Semi-log plot of exponential function

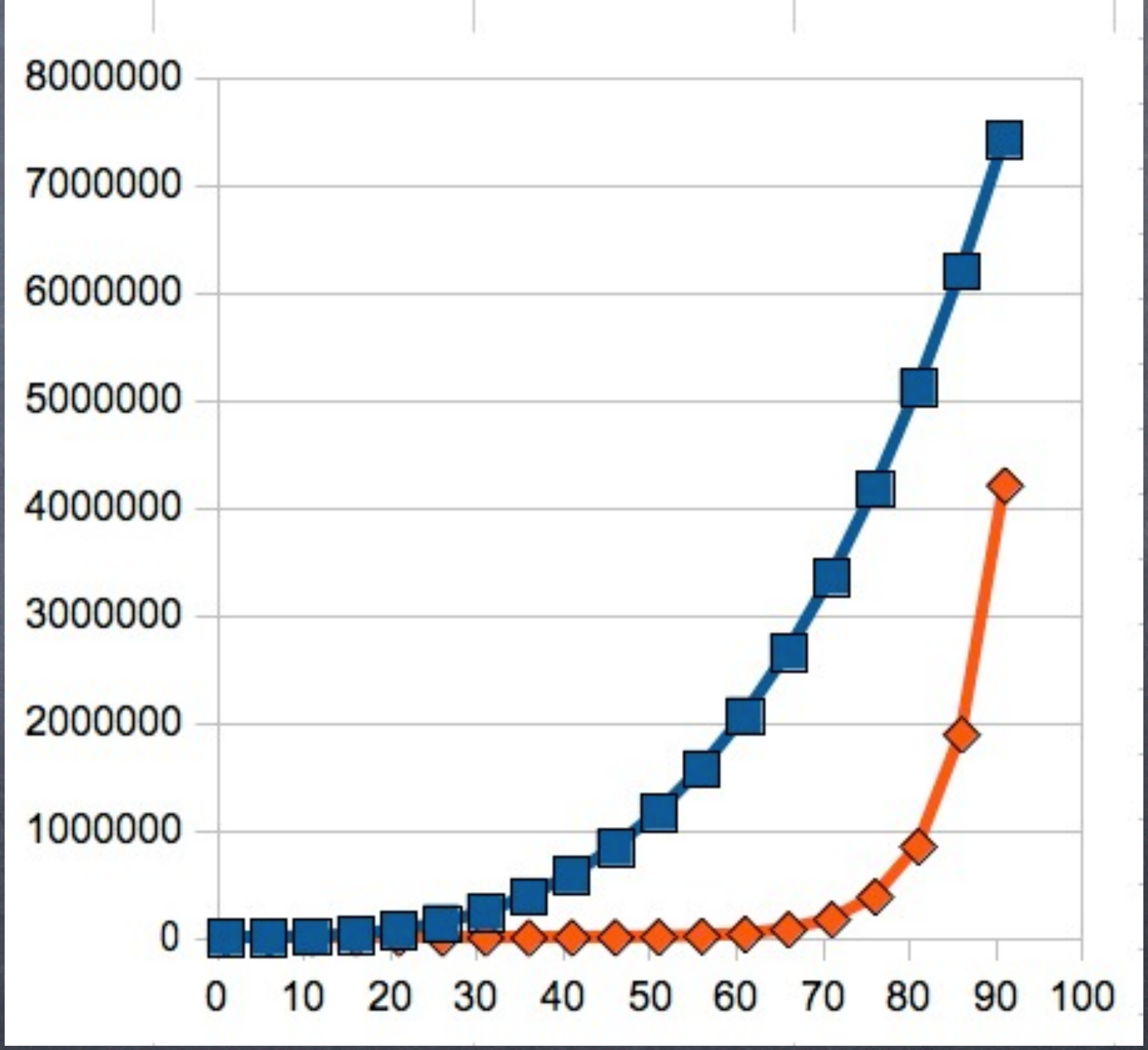
- Suppose $y = ae^{kx}$.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ae^{kx}) = \ln(a) + kx$.
- $V = A + kx$ where $A=\ln(a)$.
- On a semi-log plot, $y= ae^{kx}$ looks linear.

Regular, log-log and semi-log plots

Two data sets.

Power function?

Exponential function?



Regular x-y plot.

Plot $Y_i = \ln(y_i)$ versus x_i.

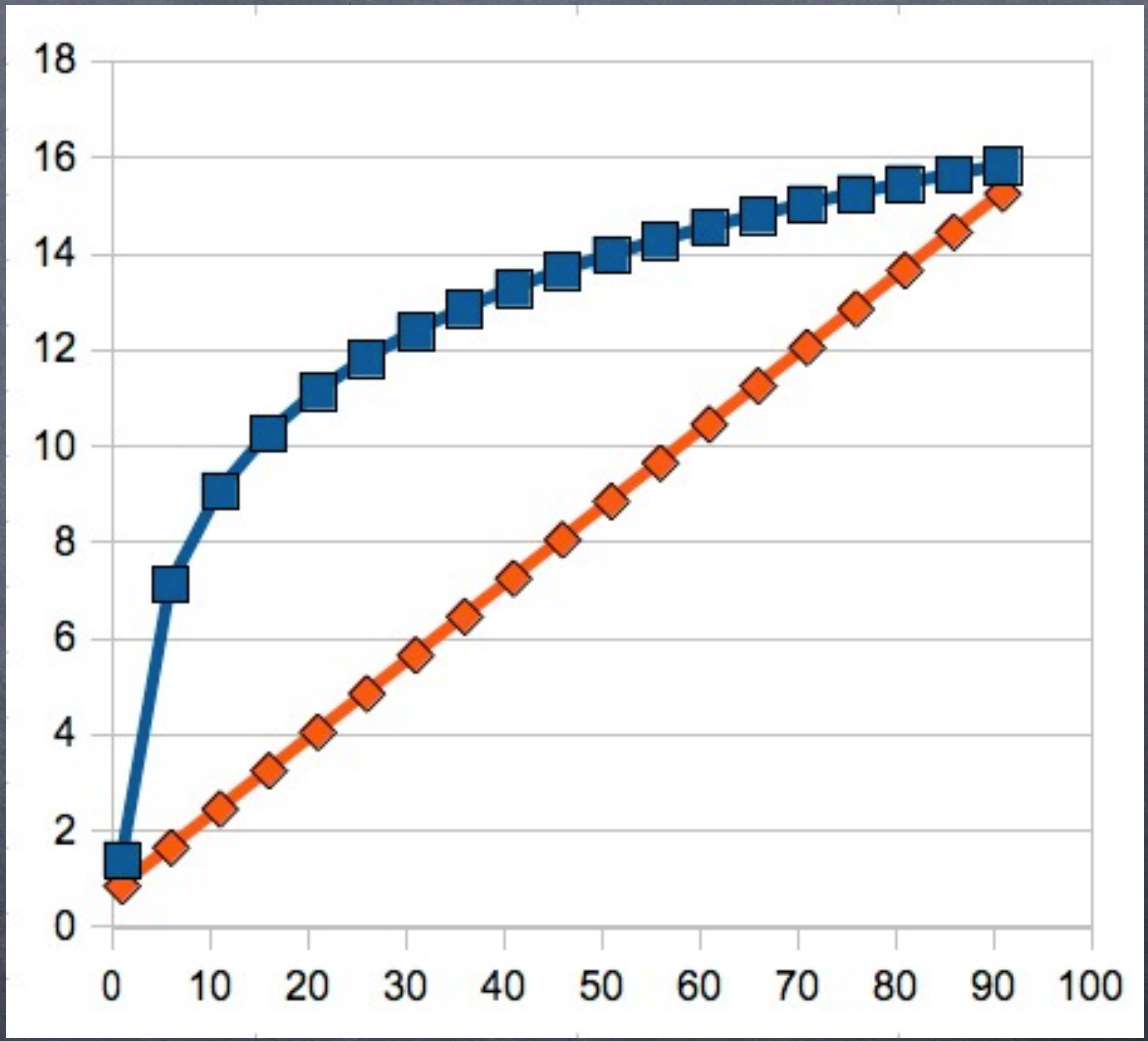
Conclude that:

(A) Blue is power function.

(B) Blue is exponential.

(C) Orange is power function.

(D) Orange is exponential.



Semi-log plot.

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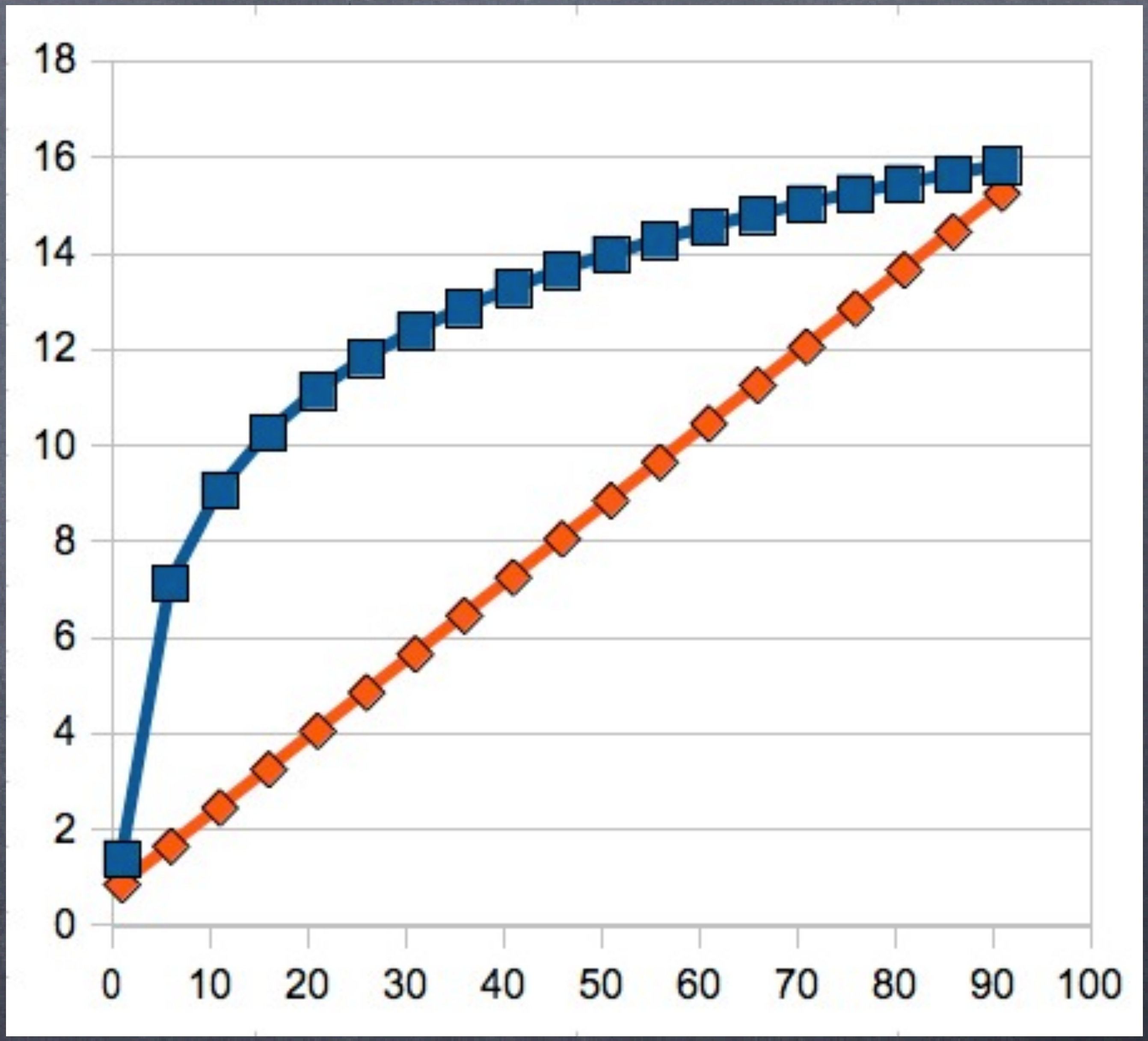

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Plot $Y_i=\ln(y_i)$ versus $X_i=\ln(x_i)$.

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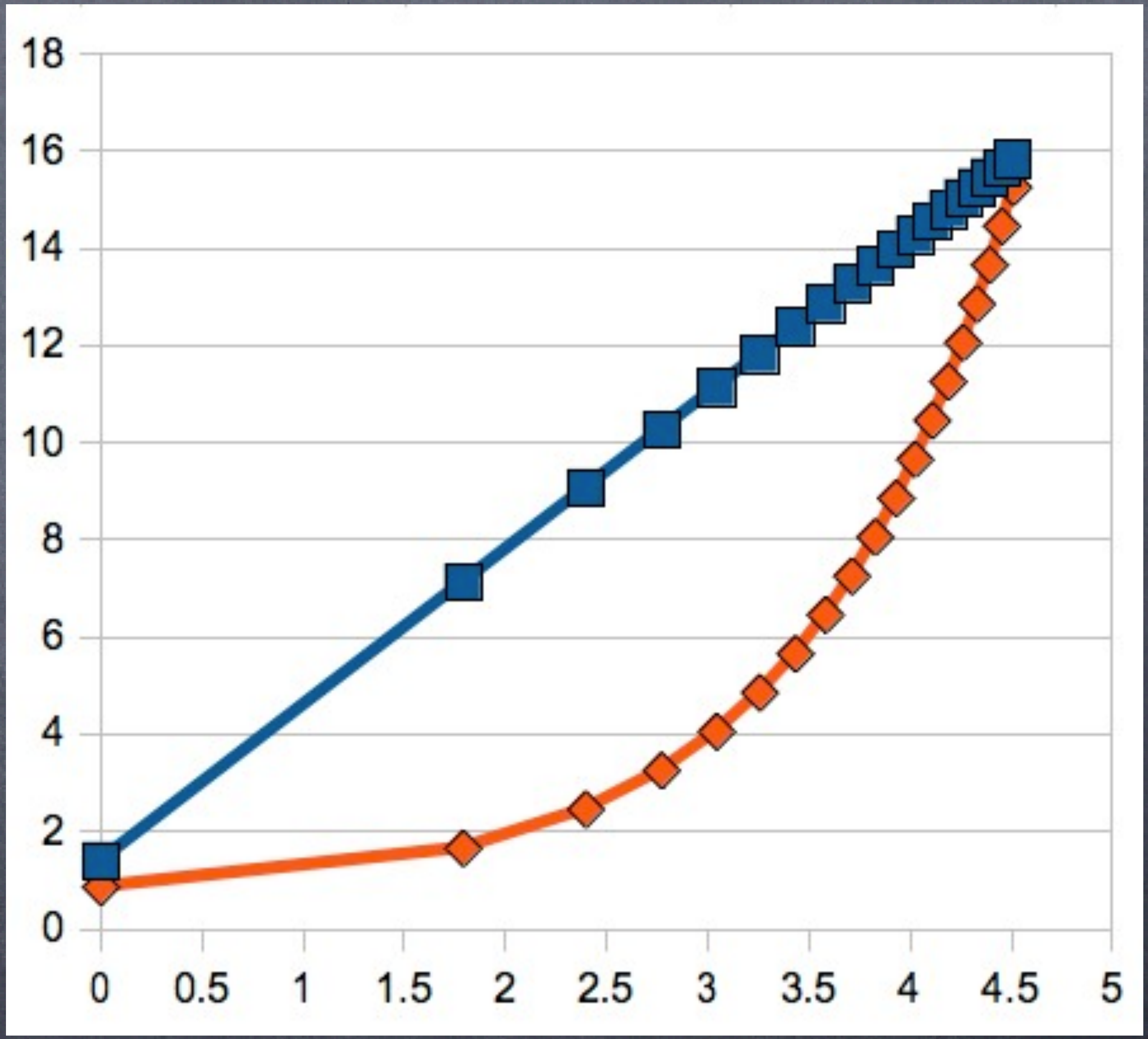

Log-log plot.

Plot $Y_i = \ln(y_i)$ versus $X_i = \ln(x_i)$.

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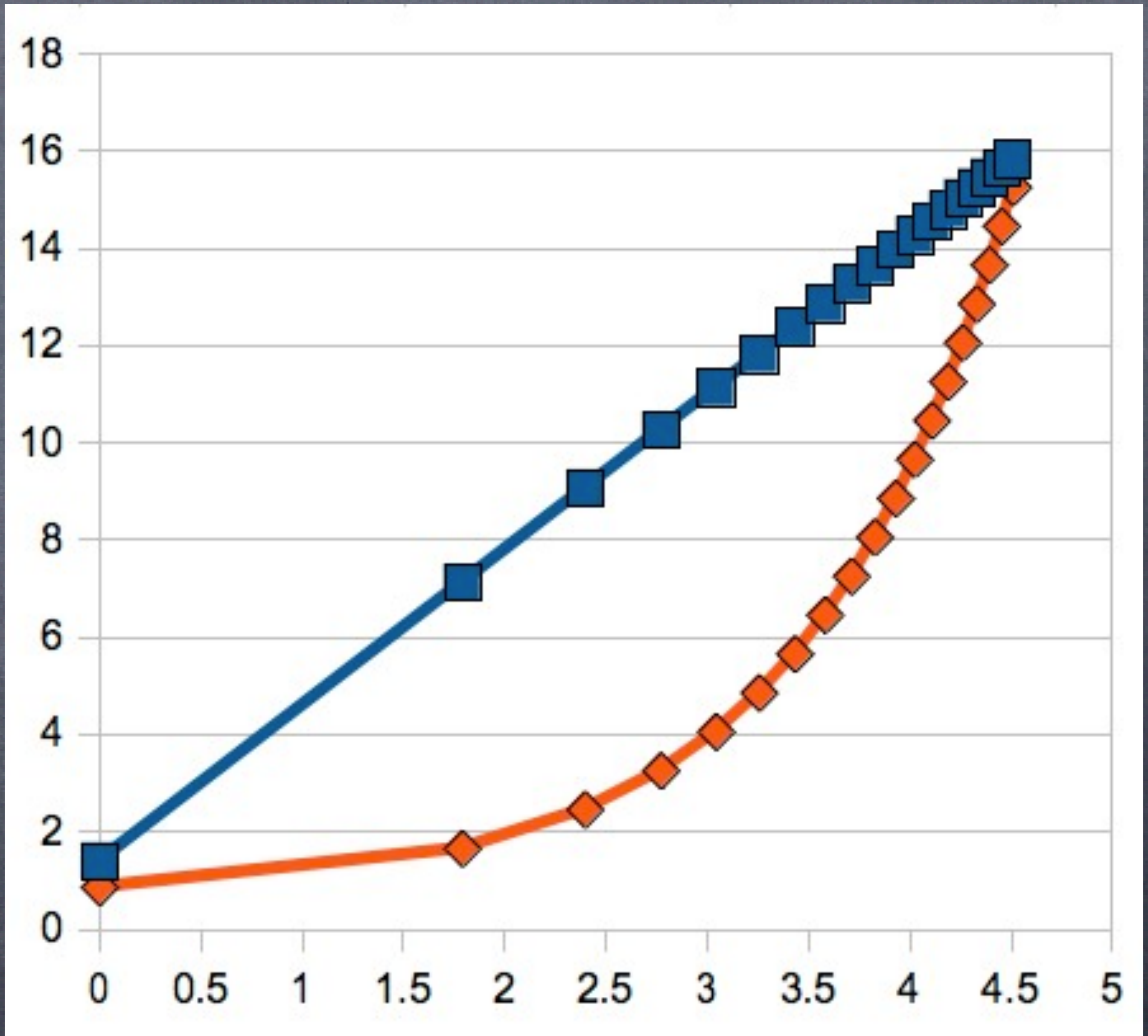

Log-log plot.