Lecture 27 (Nov. 07, 2013)

Learning Goals: ① $\sin(t)$, $\cos(t)$ and their properties

② $y = f(t) = A\sin(\omega t - \phi)$

③ use trig. function to describe rhythmic processes

• Euler's method: how to tell the approximation underestimates or overestimates the actural function value?

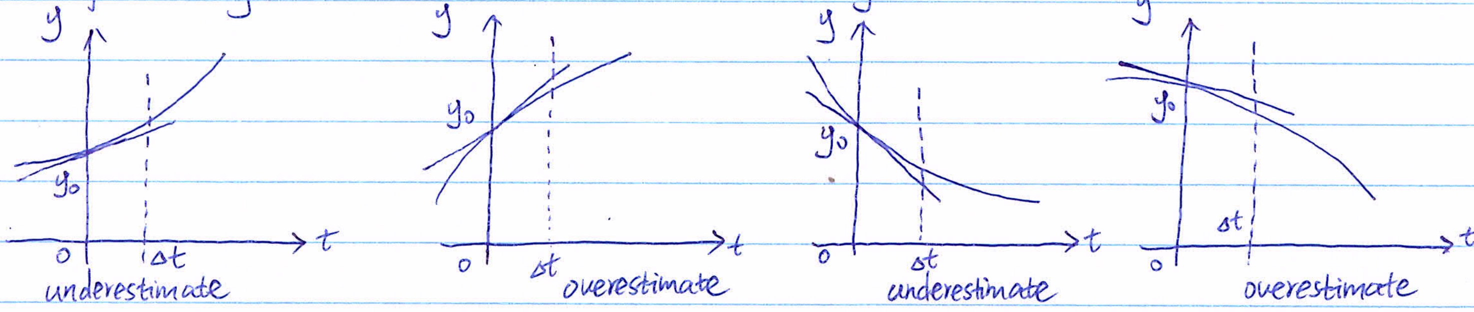
The tangent line that passes through $(0, y_0)$ on the curve $y(t)$ given by $\begin{cases} \frac{dy}{dt} = f(y) \\ y(t=0) = y_0 \end{cases}$

is $y = y_0 + f(y_0)\cdot t$

Recall the one step Euler's method, approximation at $t = \Delta t$ is $y_1 = y_0 + f(y_0)\cdot \Delta t$

$\Rightarrow (\Delta t, y_1)$ is on the tangent line.

To tell if the approximation underestimates or overestimates the actural function value, we need to know if the tangent line is above or below the curve $y(t)$



$\Rightarrow$ the approximation underestimates the actural value if $y(t)$ is concave up

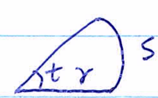
the approximation overestimates the actural value if $y(t)$ is concave down.

$\Rightarrow$ Concavity is given by $\frac{d^2y}{dt^2} = f'(y)\cdot\frac{dy}{dt} = f'(y)\cdot f(y)$

take derivative with respect to y

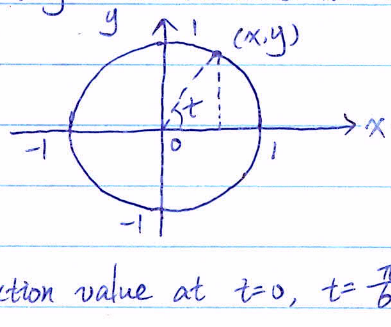
• Trigonometric function $\sin(t)$, $\cos(t)$

① t - angles in radians not degrees



$s = r\cdot t$, $\frac{\theta}{180} = \frac{t}{\pi}$ (in degrees)

②



a particle moves along the curve $x^2 + y^2 = 1$

angle t satisfies $\begin{cases} \sin(t) = y \\ \cos(t) = x \end{cases}$ range $[-1, 1]$

③ function value at $t=0$, $t=\frac{\pi}{6}$, $t=\frac{\pi}{4}$, $t=\frac{\pi}{3}$, $t=\frac{\pi}{2}$, ...

④ $\sin^2(t) + \cos^2(t) = 1$, $\cos(t) = \sin(t + \frac{\pi}{2})$

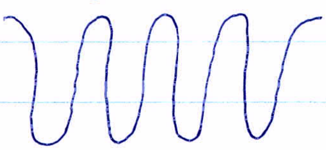
⑤ Period $T = 2\pi$. T is called a period if $f(t) = f(t+T)$ for any t in the domain.

⑥ Other trig. functions: $\tan(t) = \frac{\sin(t)}{\cos(t)}$, $\cot(t) = \frac{1}{\tan(t)}$, $\sec(t) = \frac{1}{\cos(t)}$, $\csc(t) = \frac{1}{\sin(t)}$

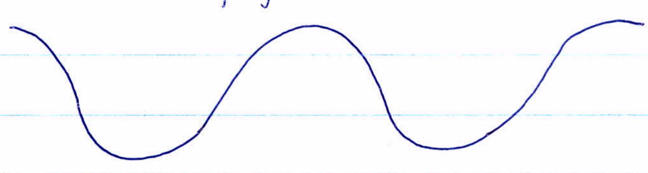
- $y = f(t) = A\sin(\omega t - \phi)$, A, ω, ϕ - constants.

  A - amplitude, reflects the peak (maximum) and the valley (minimum) of the function.

  ω - frequency, how fast the oscillation is. Period of $f(t)$ is $T = \frac{2\pi}{\omega}$



large ω, small T



small ω, large T

  ϕ - phase shift. Recall the geometric transformation that change the curve of $A\sin(\omega t)$ to $A\sin(\omega t - \phi) = A\sin(\omega(t - \frac{\phi}{\omega}))$: move the curve to the right by $\frac{\phi}{\omega}$

- Rhythmic process: use $y = c + A\sin(\omega t - \phi)$ to describe some periodic phenomena

  Example 1: The level of a certain hormone in the bloodstream fluctuates between undetectable amount at 7:00 and 100 ng/ml at 19:00. Use a trig. function to describe the variation of hormone level. Let t be time in hours from 0:00.

  * use given information to determine values of c, A, ω, ϕ

  valley is 0 and peak is 100 $\Rightarrow \begin{cases} c + A = 100 \\ c - A = 0 \end{cases} \Rightarrow A = c = 50$

  hormone level is 0 at 7:00 and back to 0 at 7:00 after one day $\Rightarrow T = 24 \Rightarrow \omega = \frac{2\pi}{T} = \frac{\pi}{12}$

  hormone level is 100 at 19:00 $\Rightarrow y = f(t=19) = 50 + 50\sin(\frac{\pi}{12} \cdot 19 - \phi) = 100$

  $\Rightarrow \sin(\frac{\pi}{12} \cdot 19 - \phi) = 1$

  $\Rightarrow \frac{\pi}{12} \cdot 19 - \phi = \frac{\pi}{2} \Rightarrow \phi = \frac{13}{12}\pi$

  $\Rightarrow f(t) = 50 + 50\sin(\frac{\pi}{12}t - \frac{13\pi}{12})$

  * $f(t)$ can be written in the form of cosine function