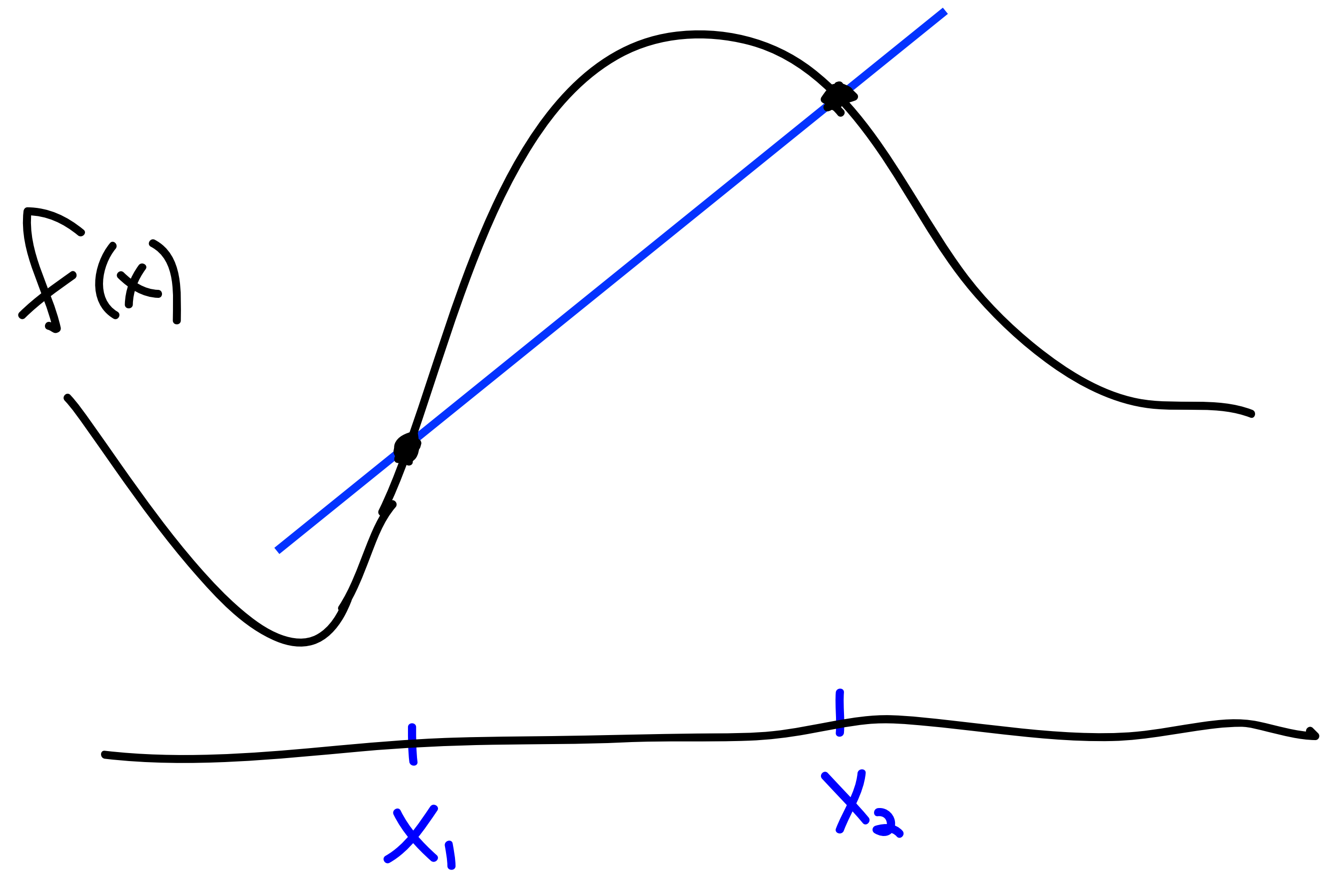
Today...

- From secant line to tangent line.
- Definition of the derivative.
- Limits, left limit, right limit.
- Continuity.
- Types of limits we'll see this semester.

What if you want the rate of change AT x_1?

(instantaneous instead of average)



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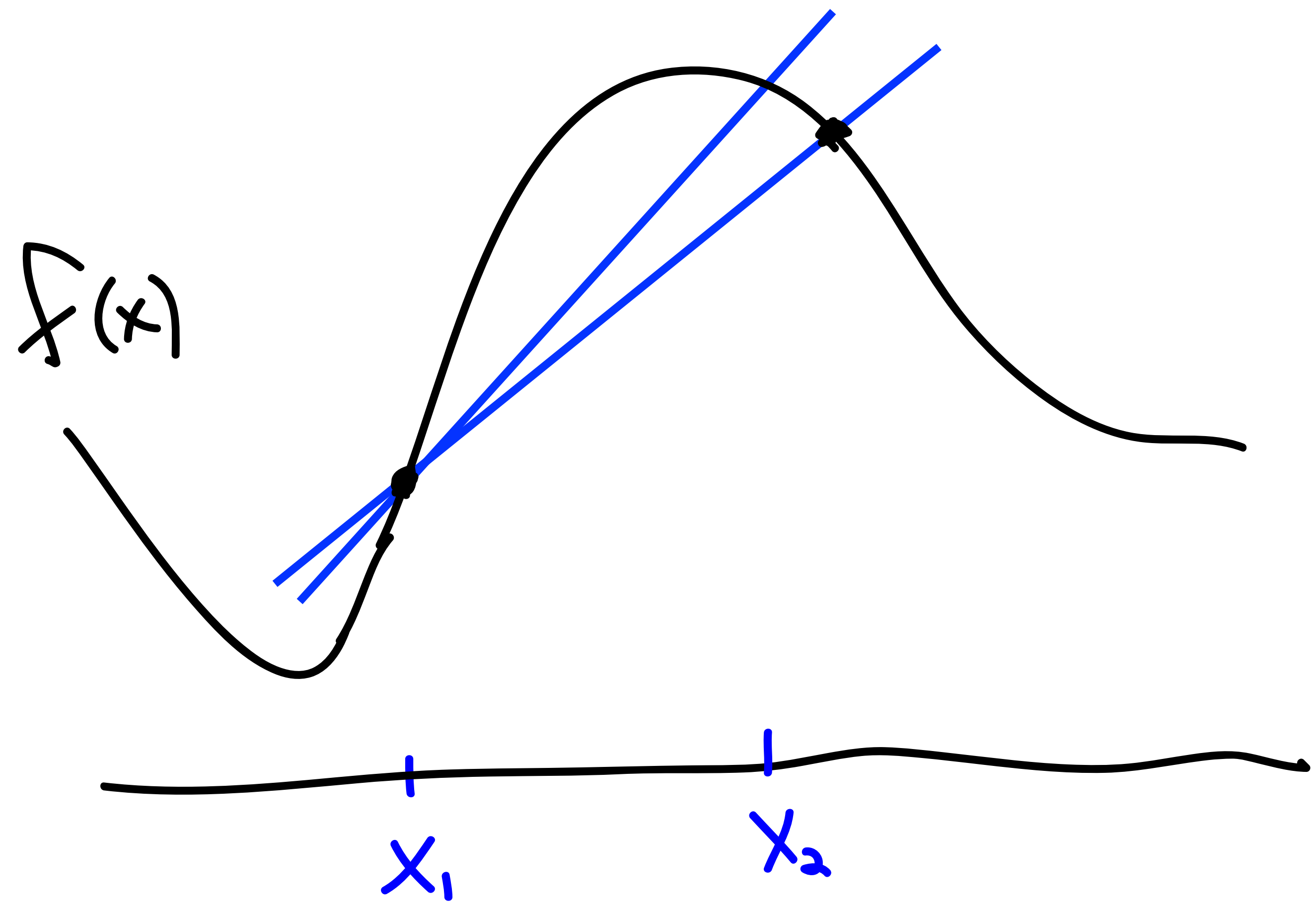

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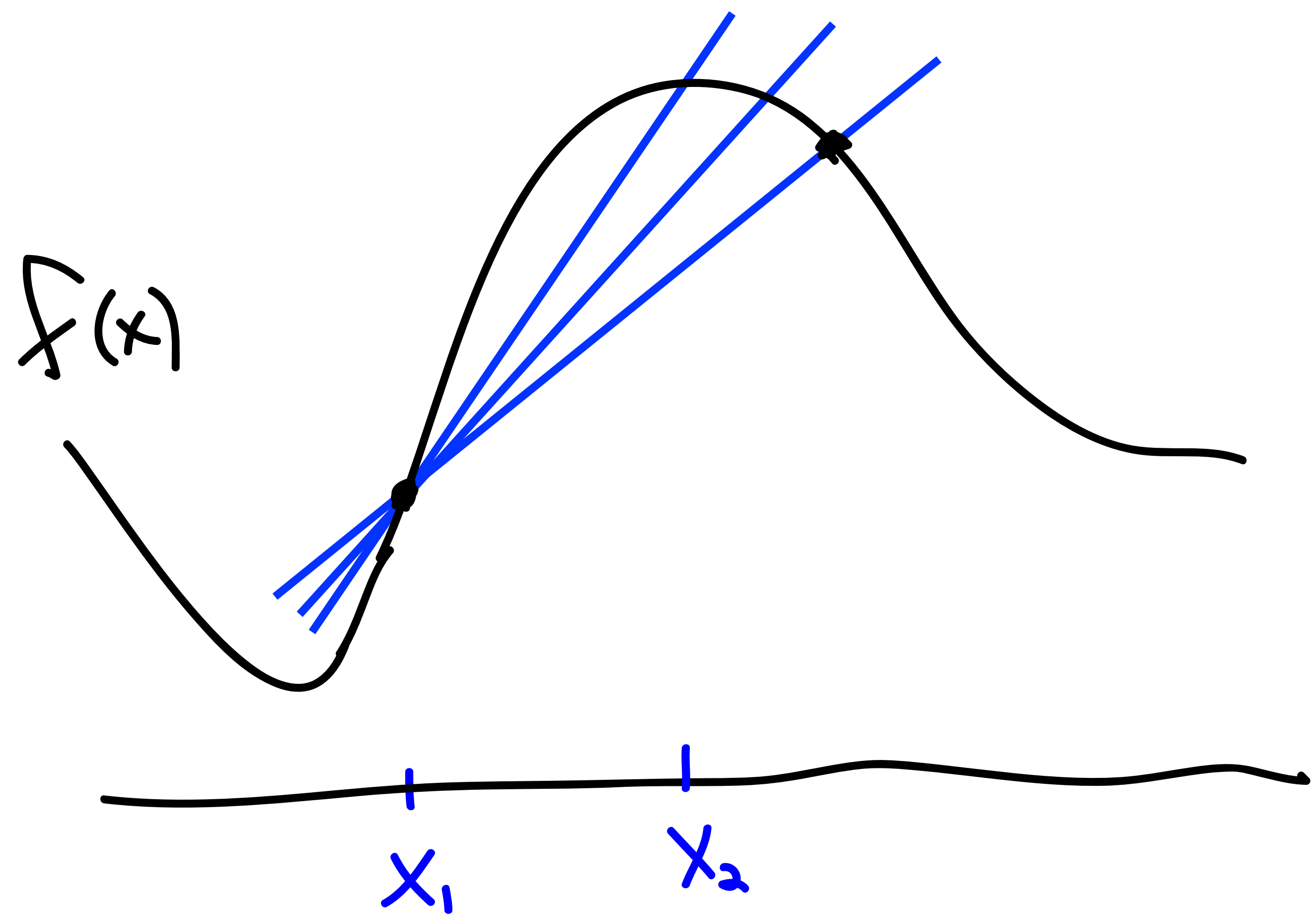

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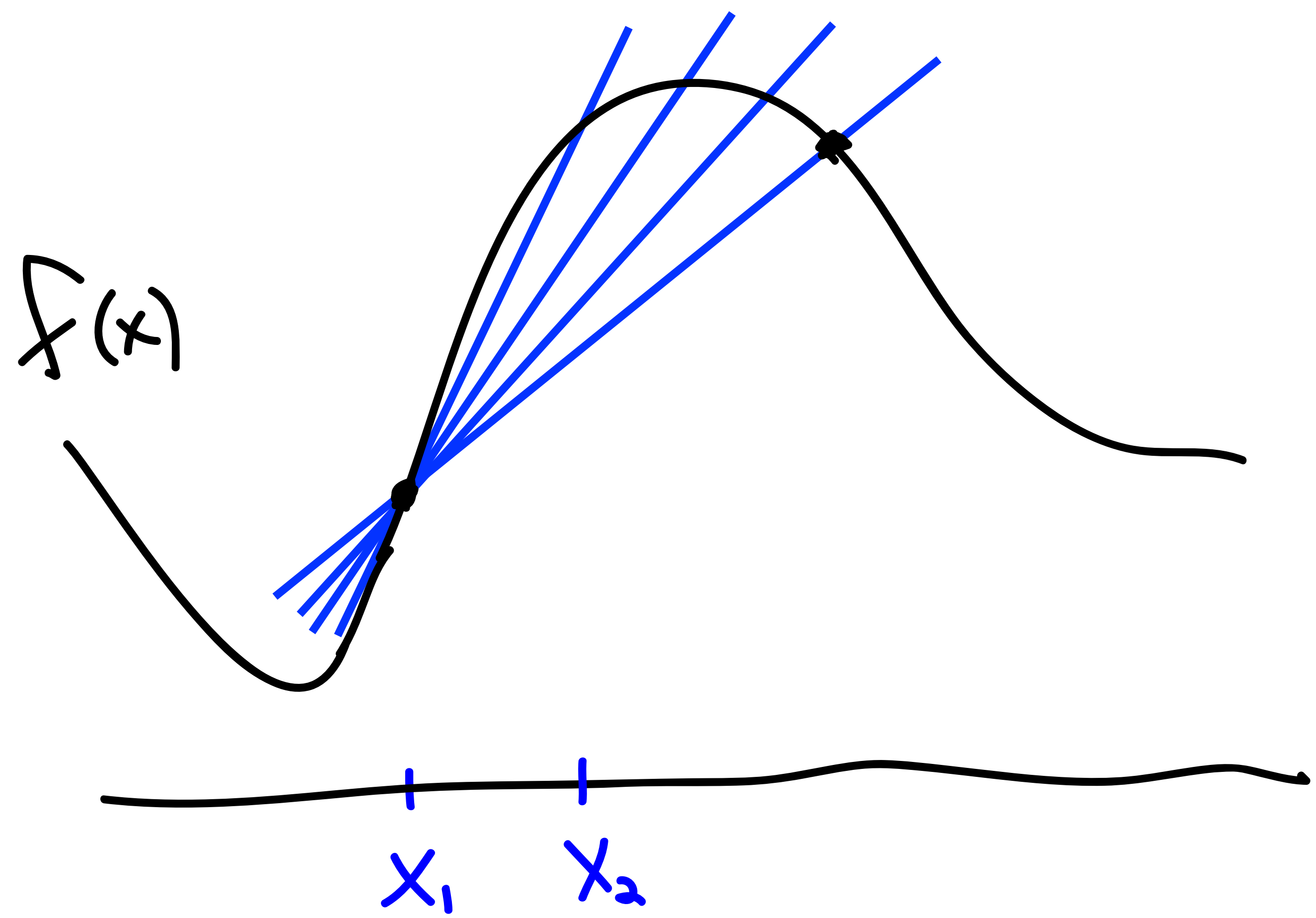

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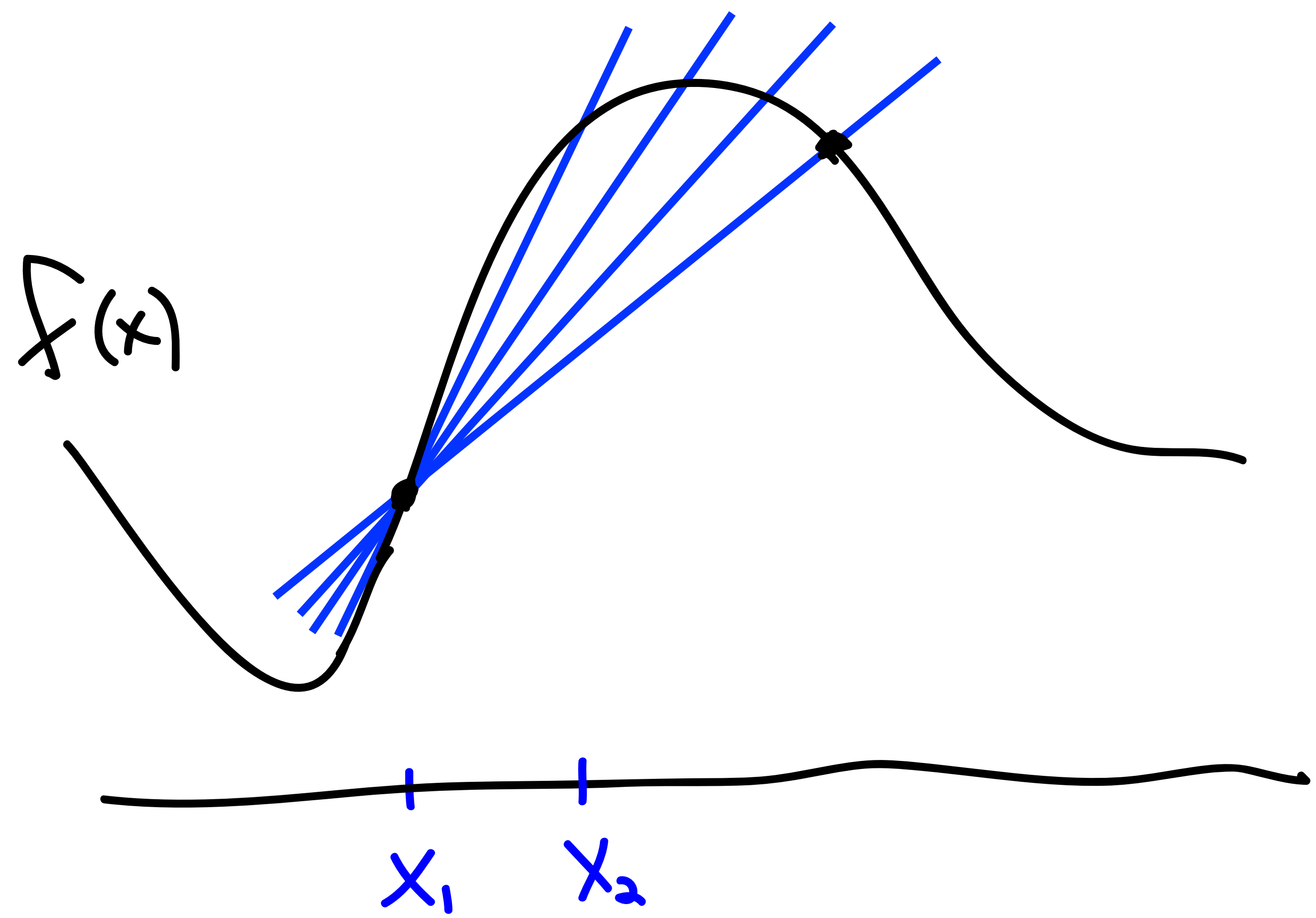

Alternate notation: let $x_2=x_1+h$ so that

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

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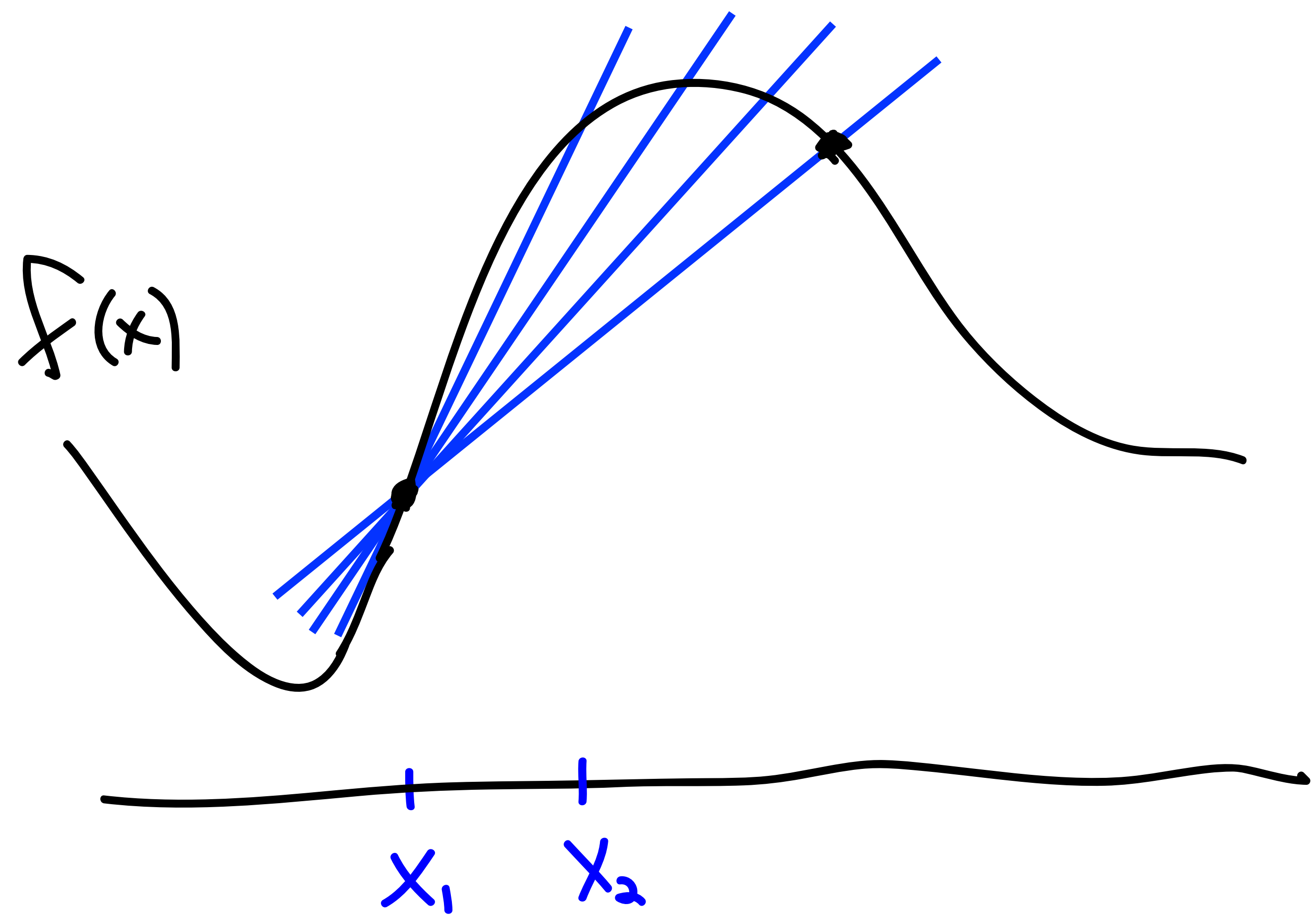

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$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1+h) - f(x_1)}{x_1 + h - x_1}$$

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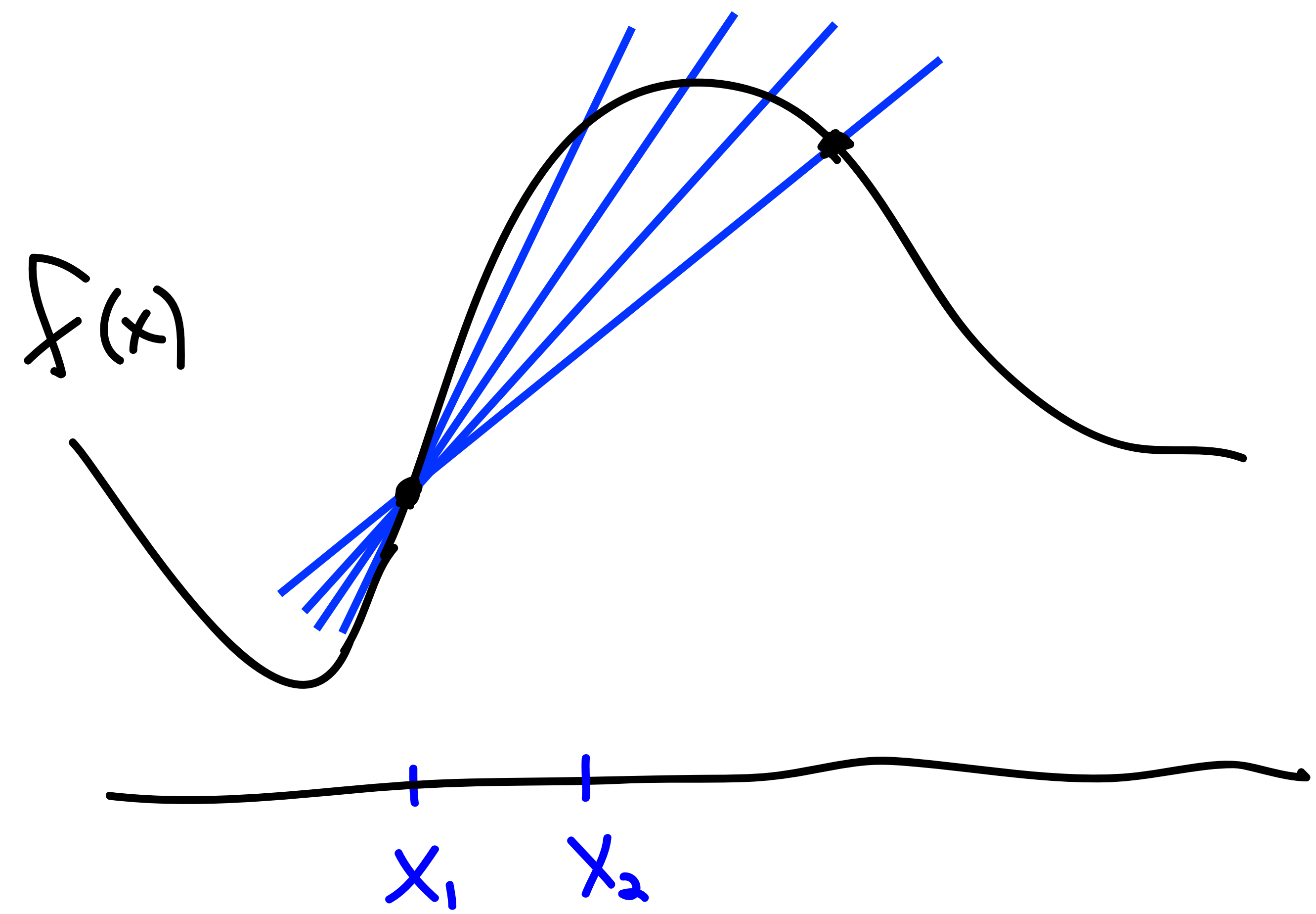

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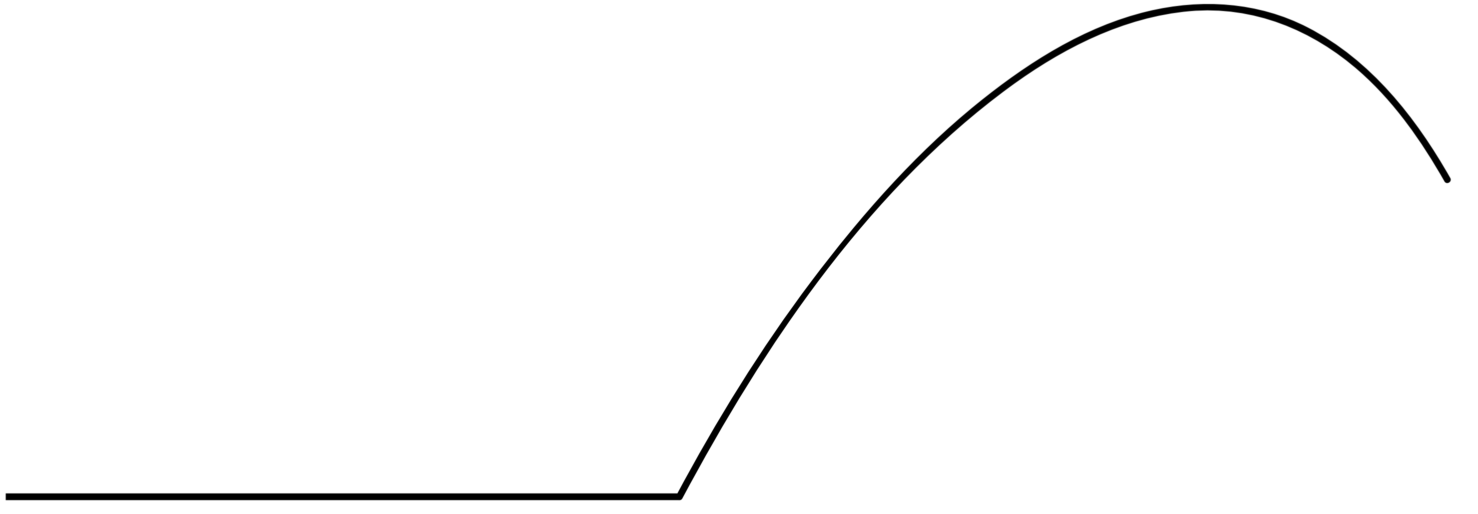
If we take h values closer and closer to 0...

- The secant line approaches the tangent line.
- The slope of the secant line approaches the slope of the tangent line.
- We call the resulting slope the derivative at x_1.
- We now have to learn how to take limits!

$$\text{slope at } x_1 = f'(x_1) = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$

Another example



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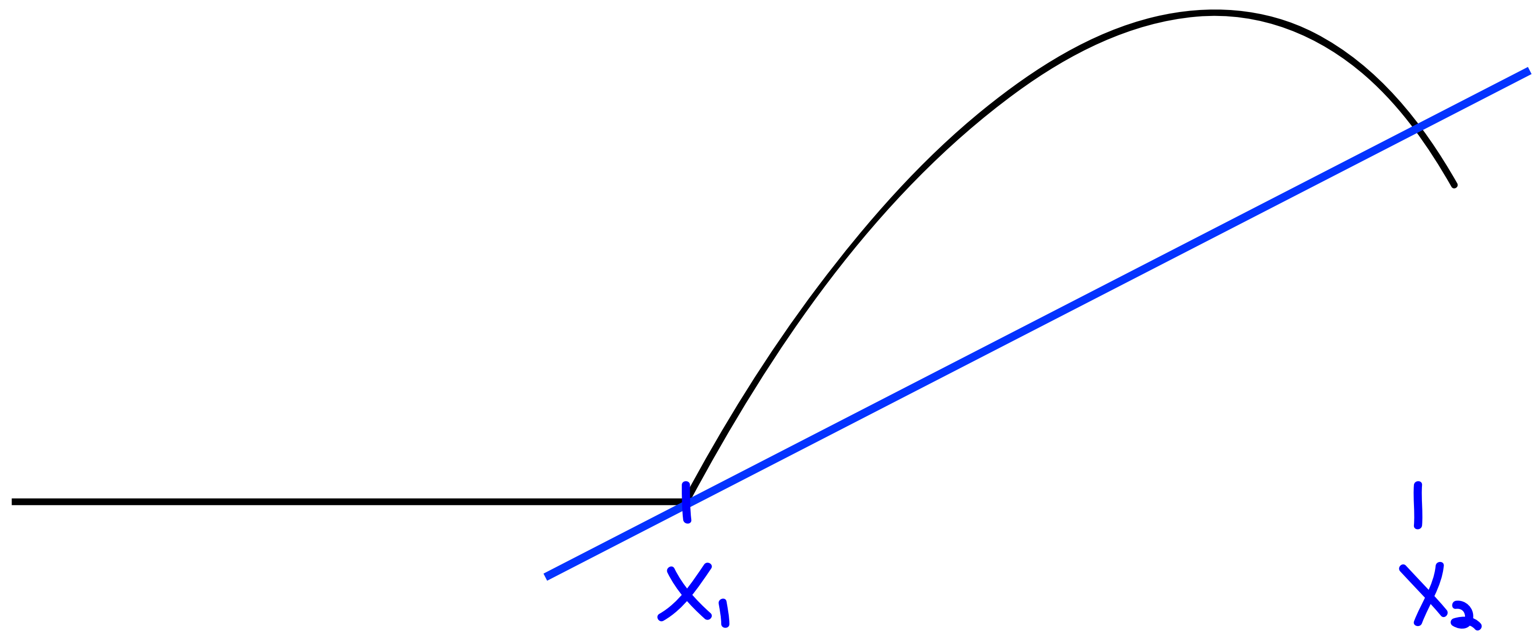

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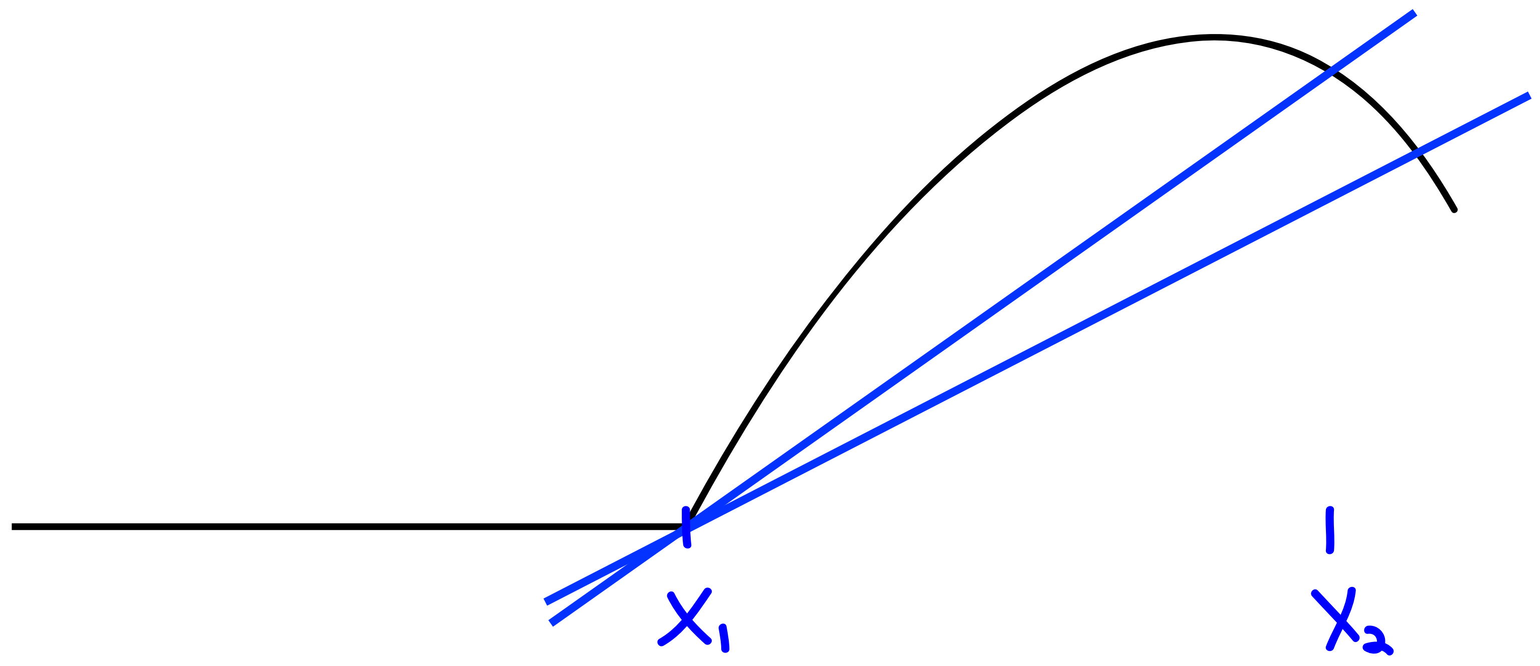

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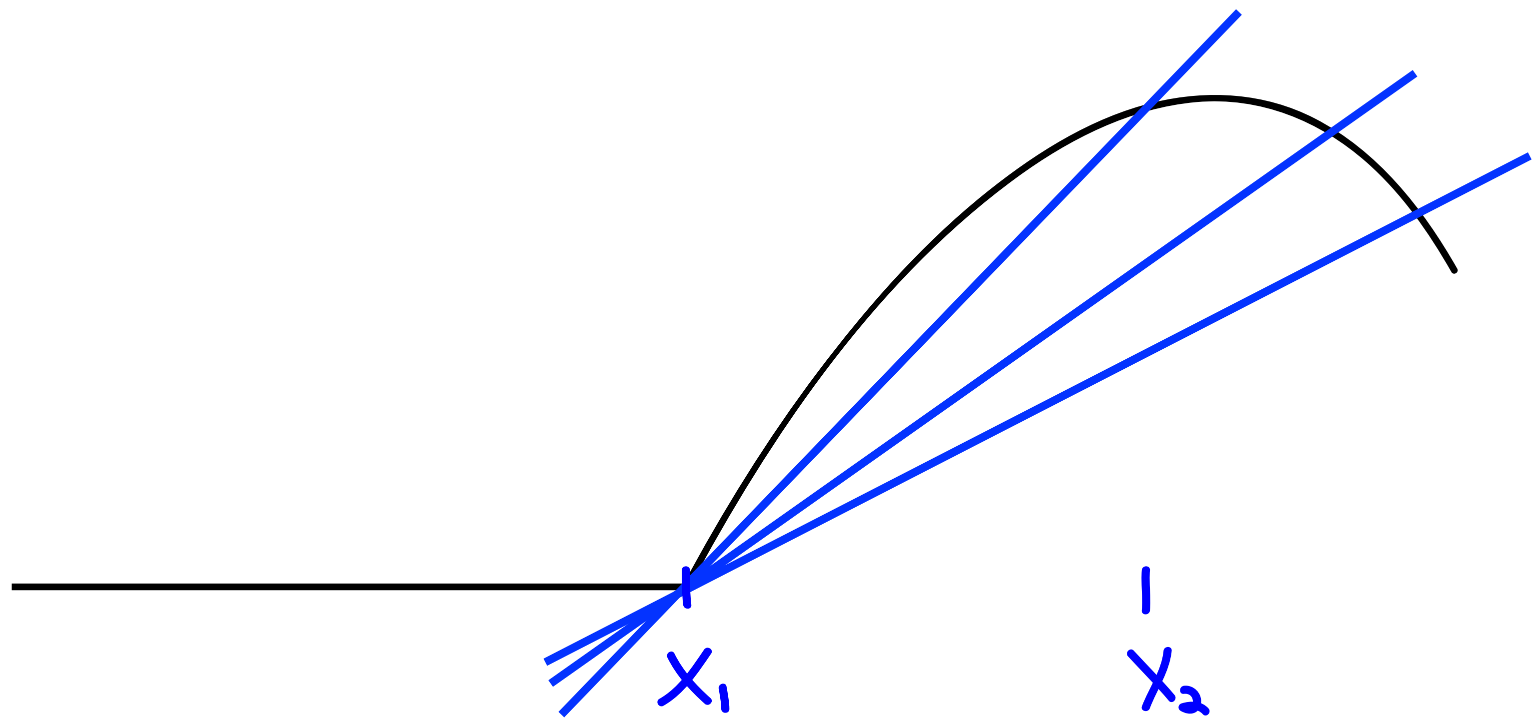

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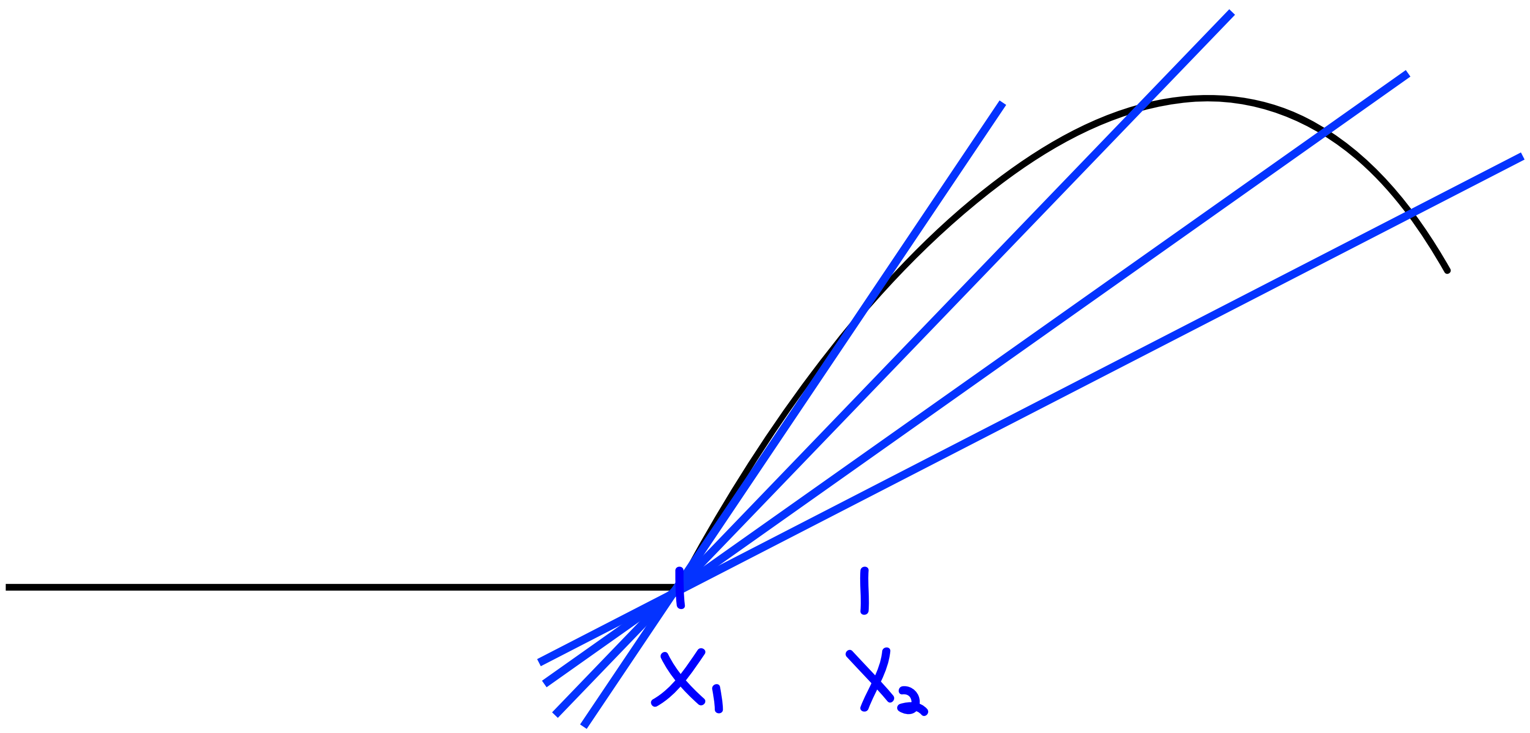

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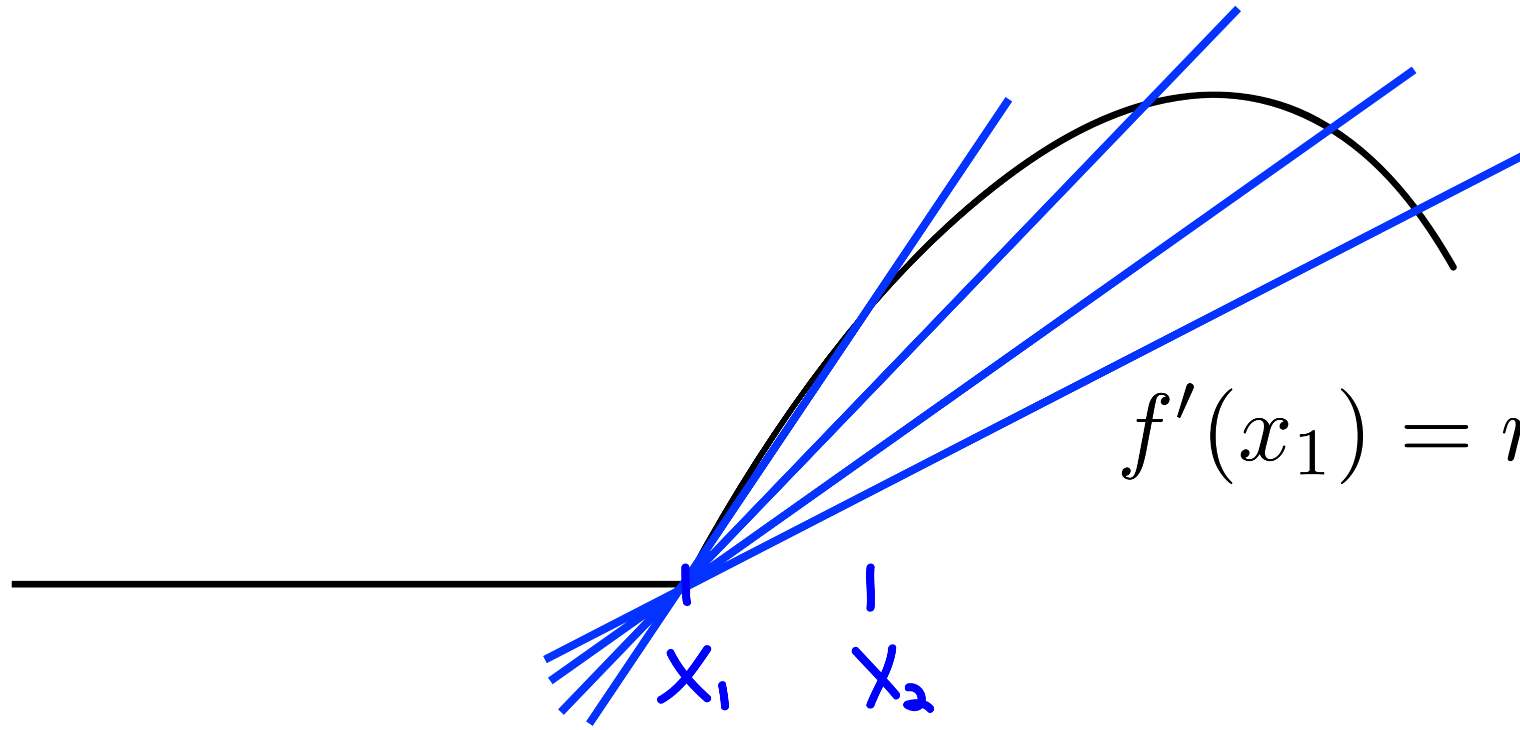

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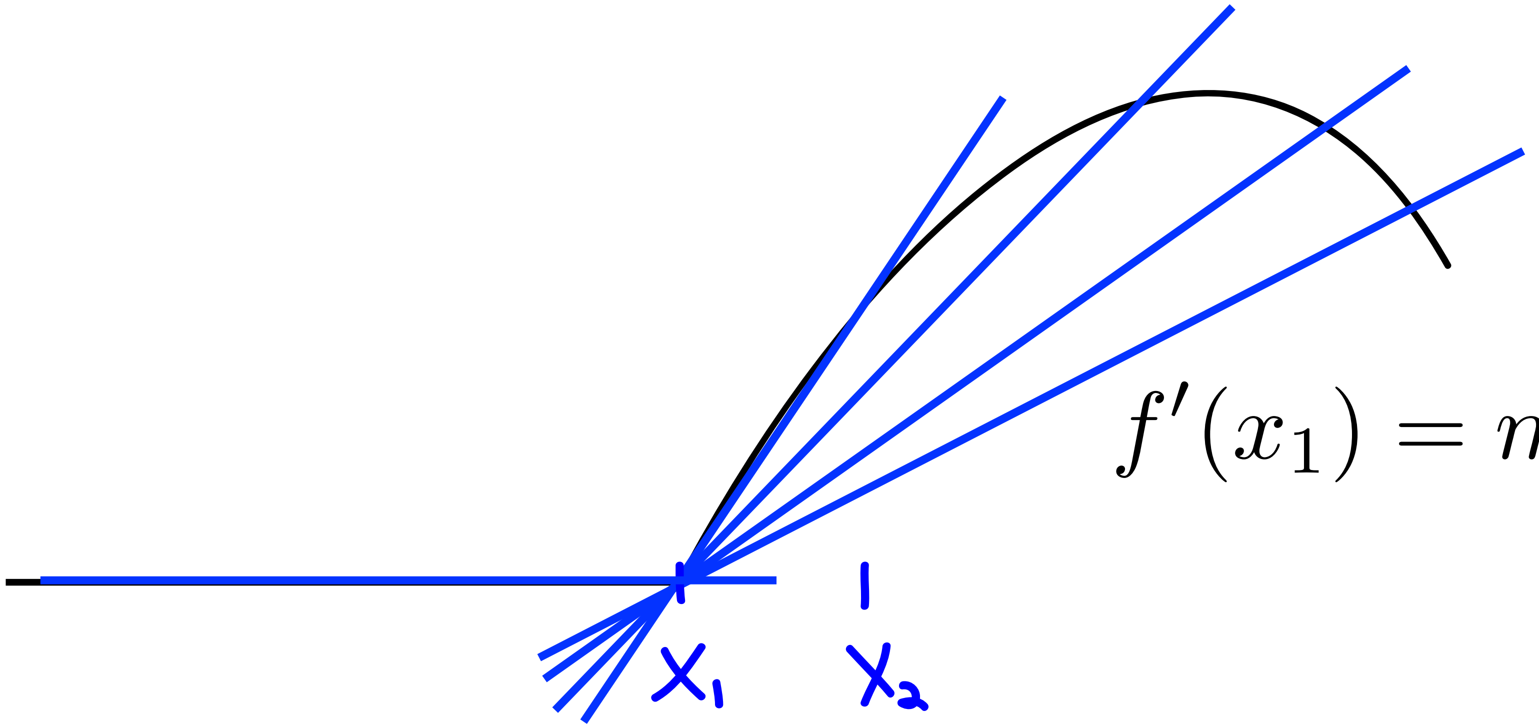

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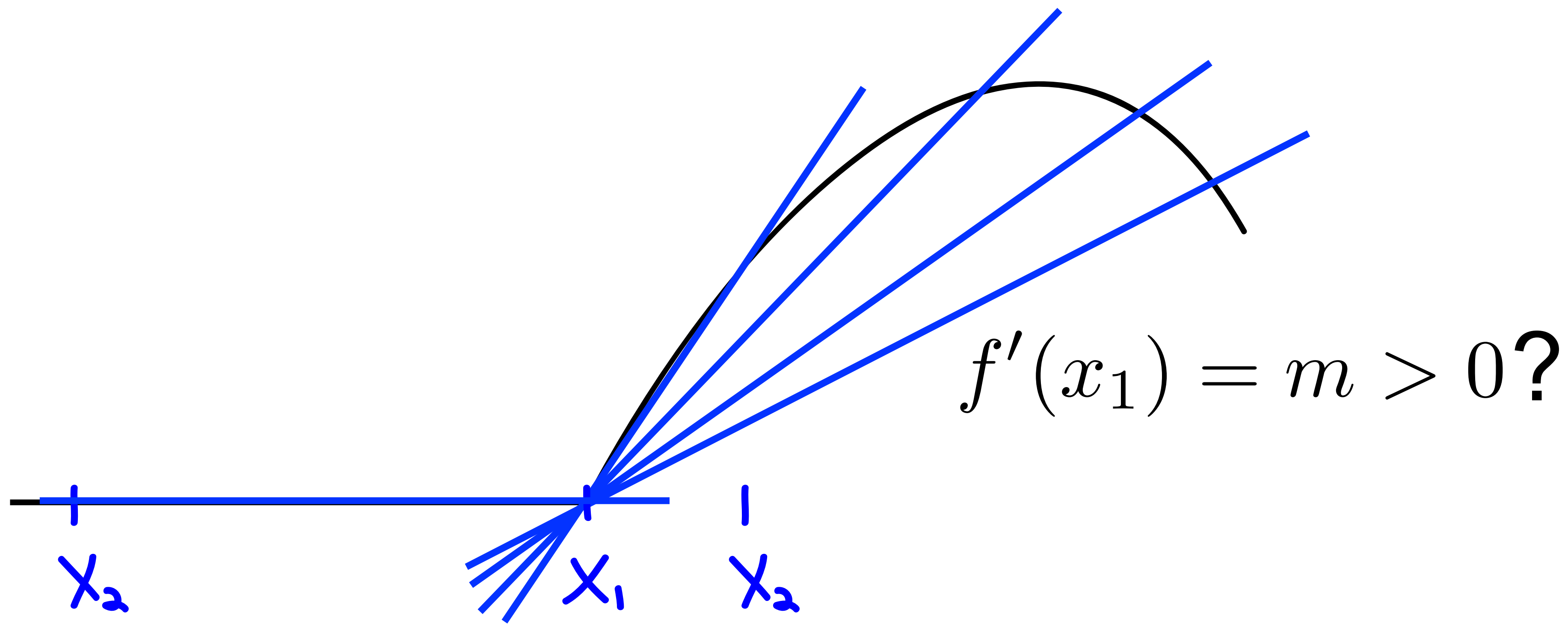

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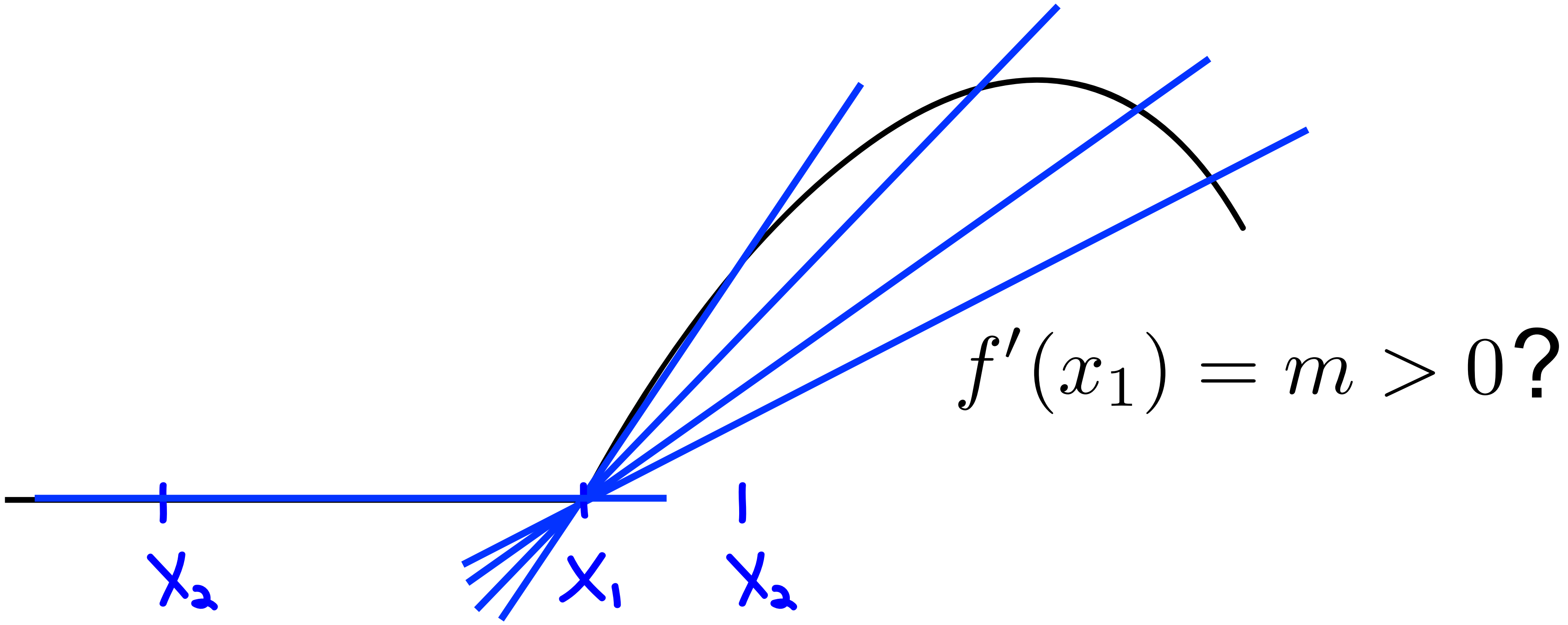

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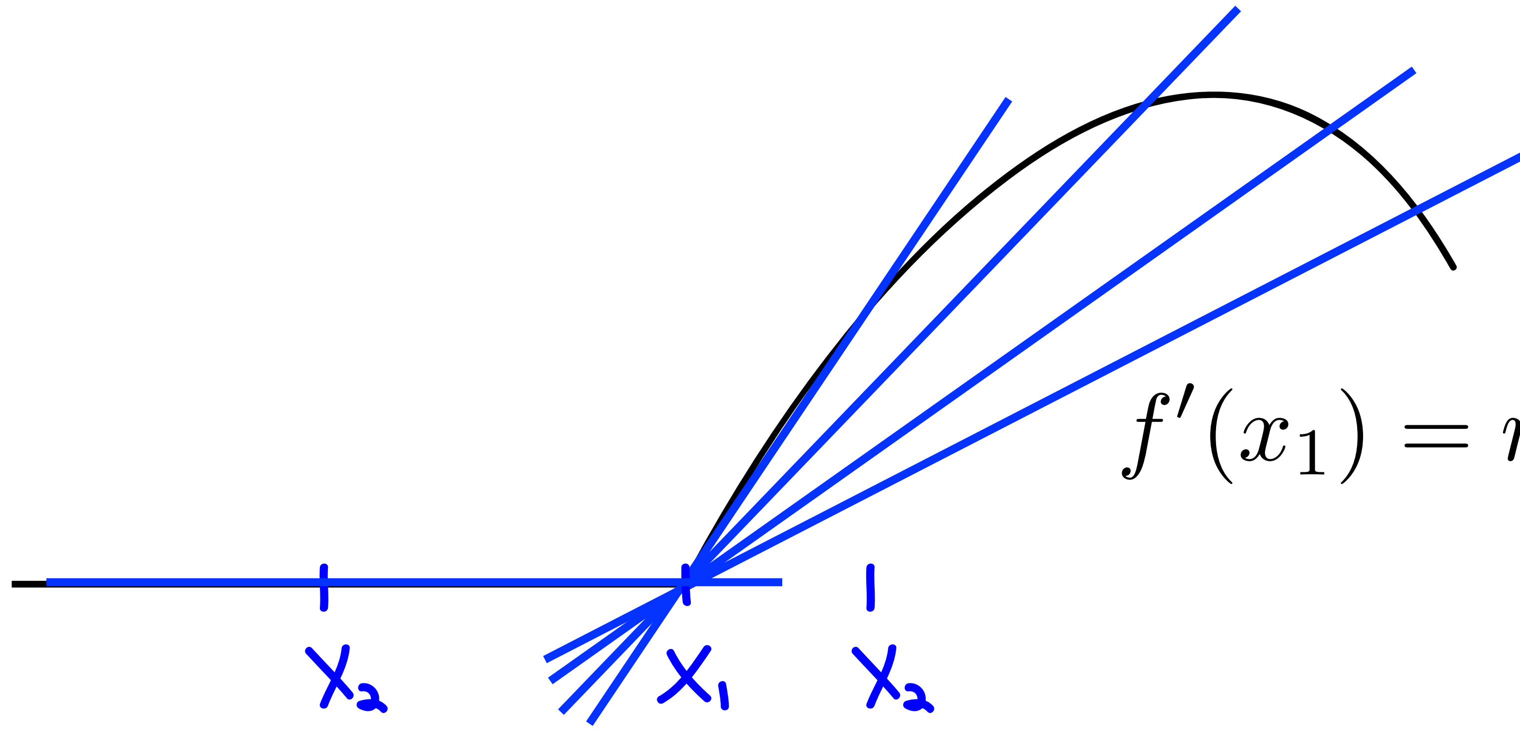

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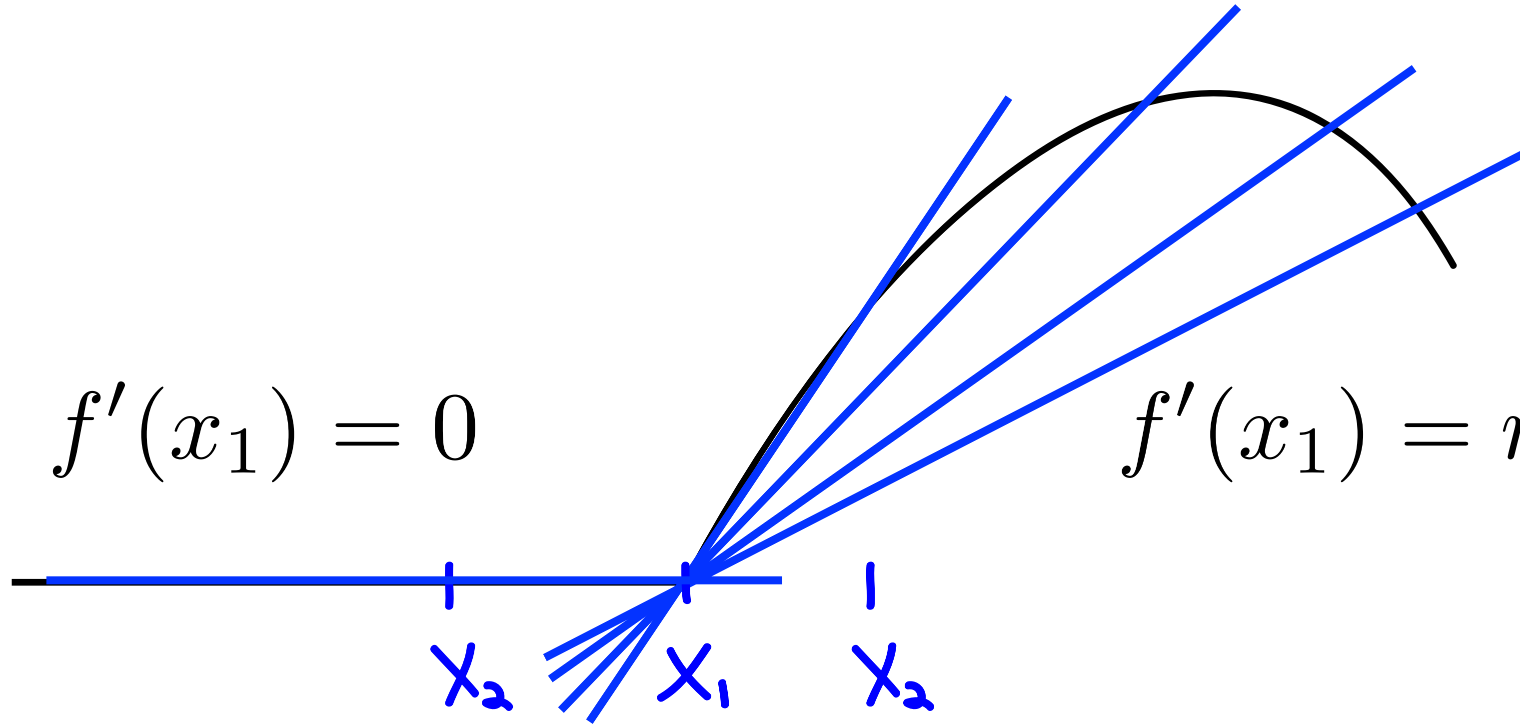

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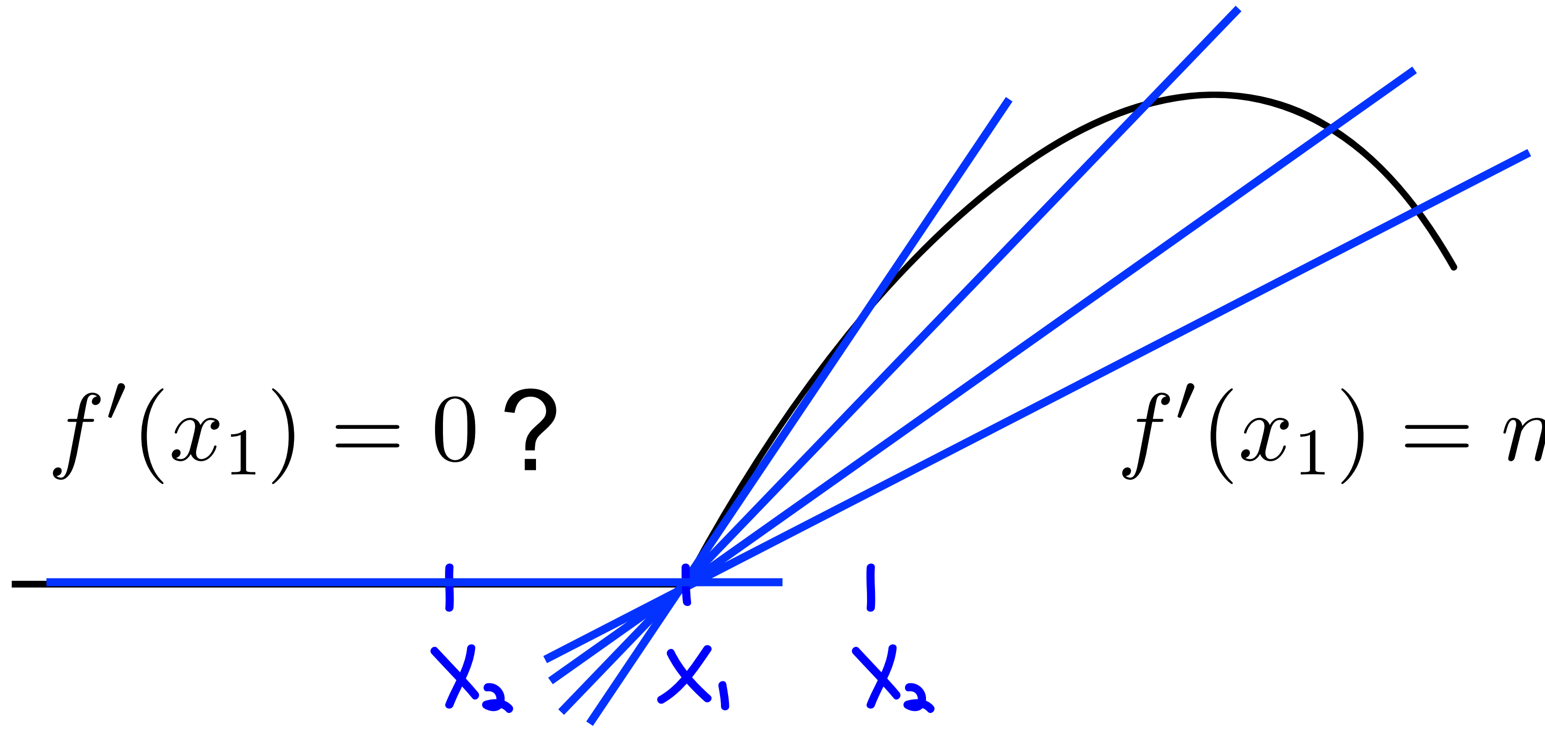

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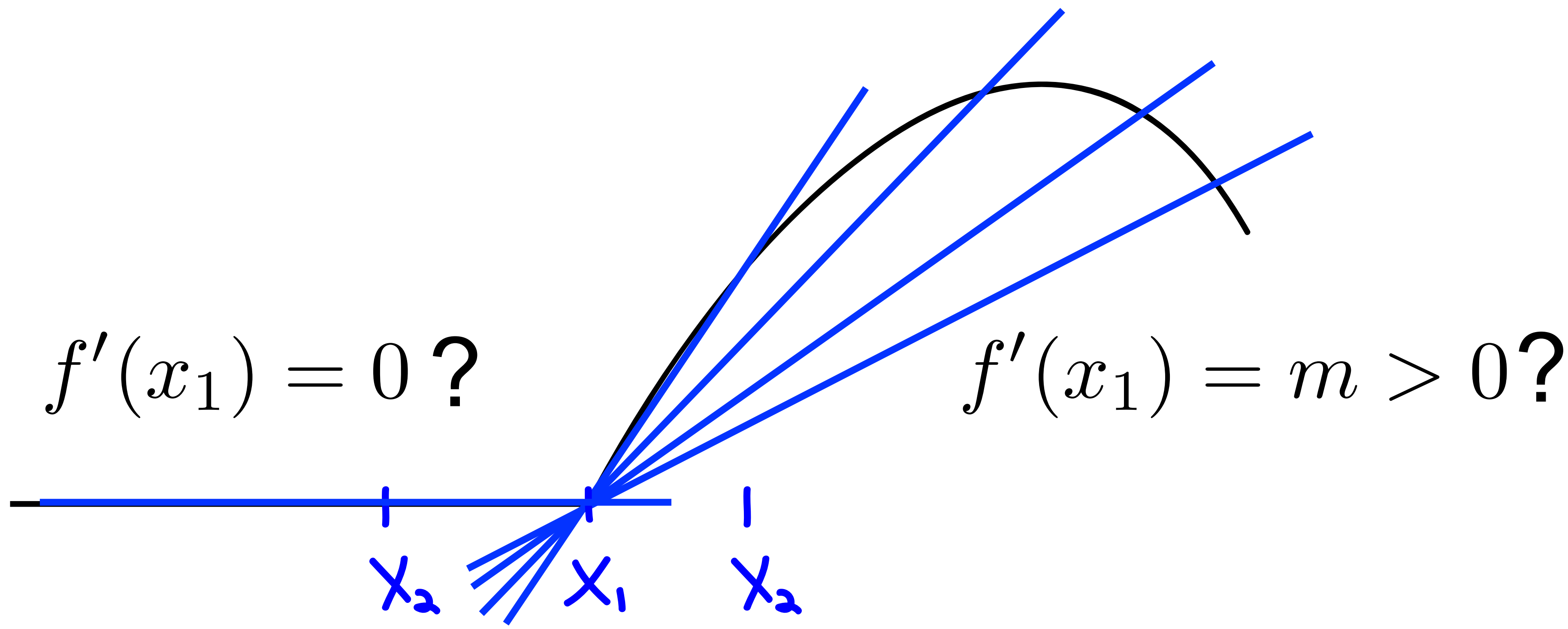

(A) $\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = m > 0$

(B) $\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = 0$

(C) Both (A) and (B)

(D) The limit does not exist.

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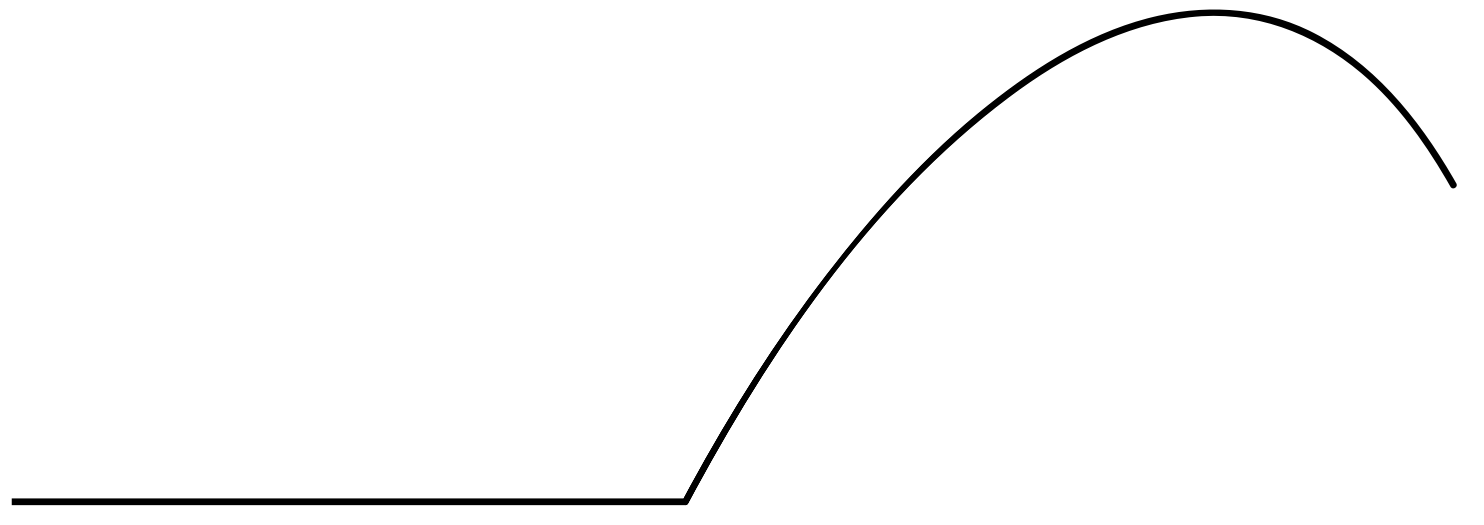

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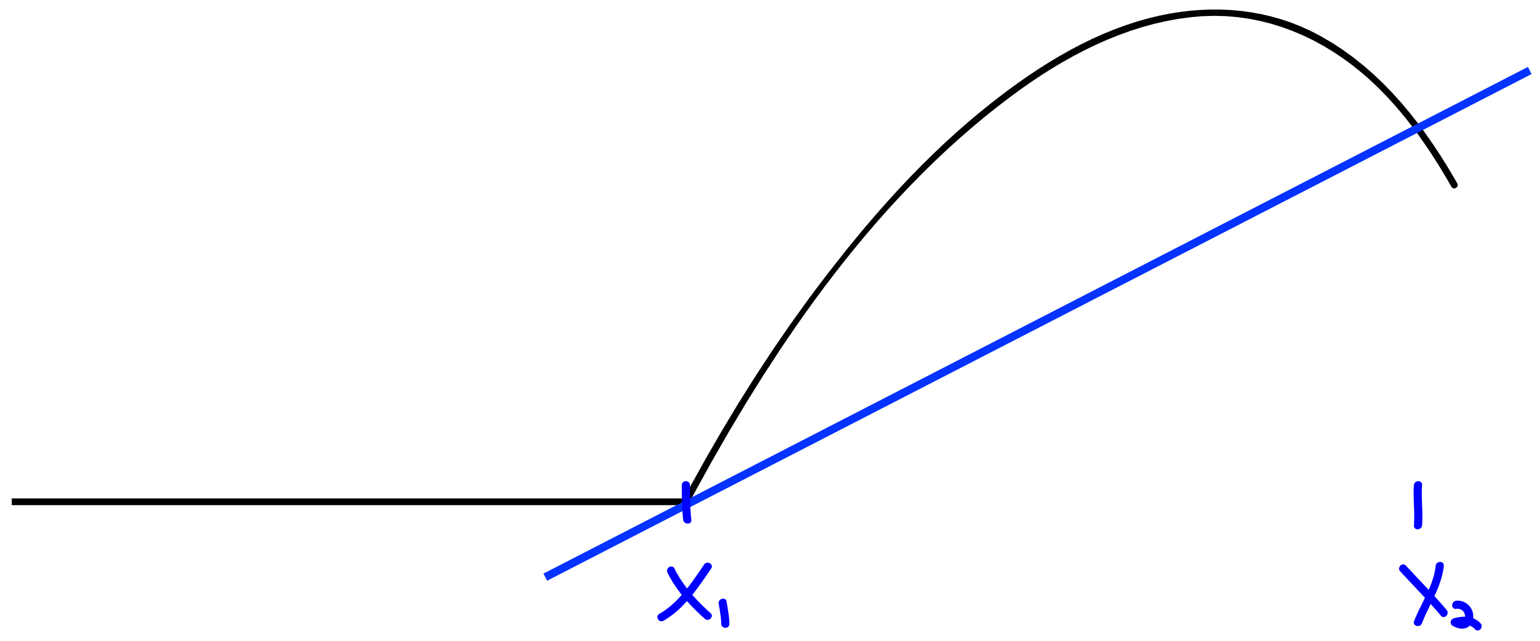

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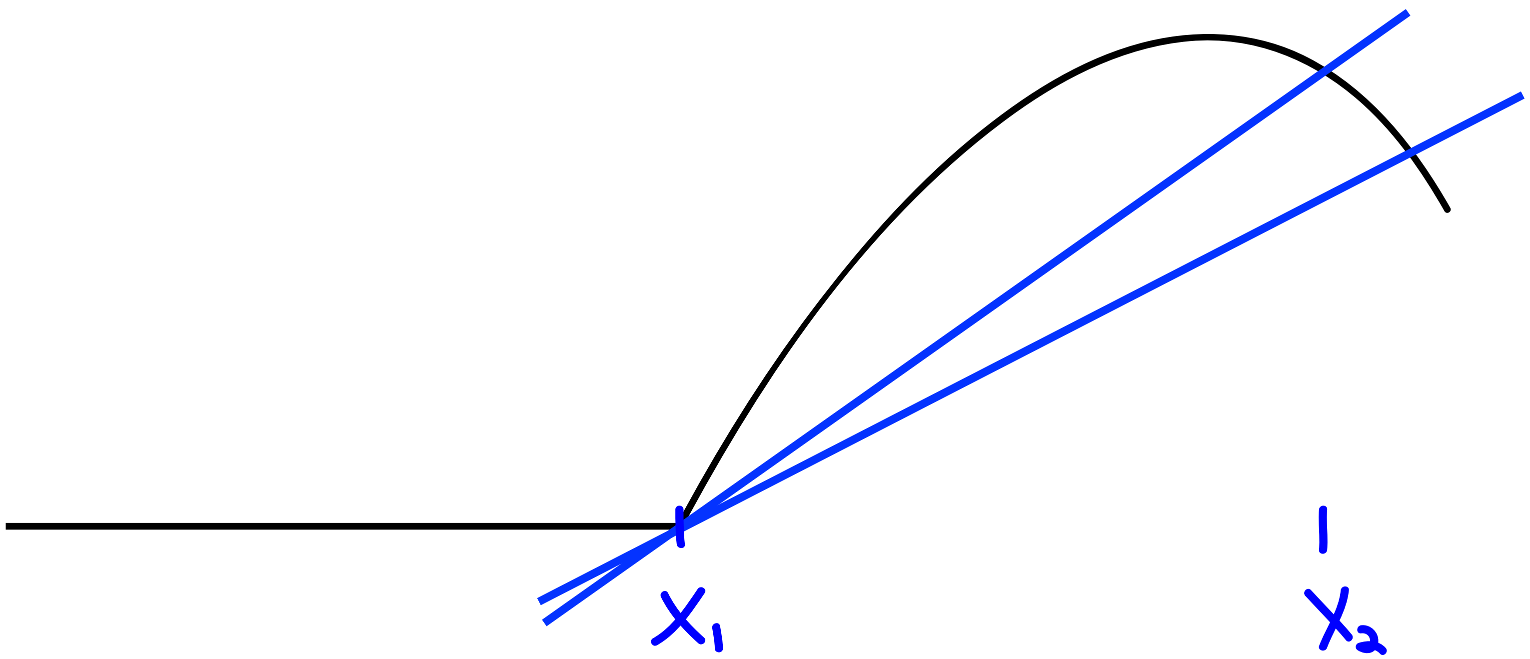

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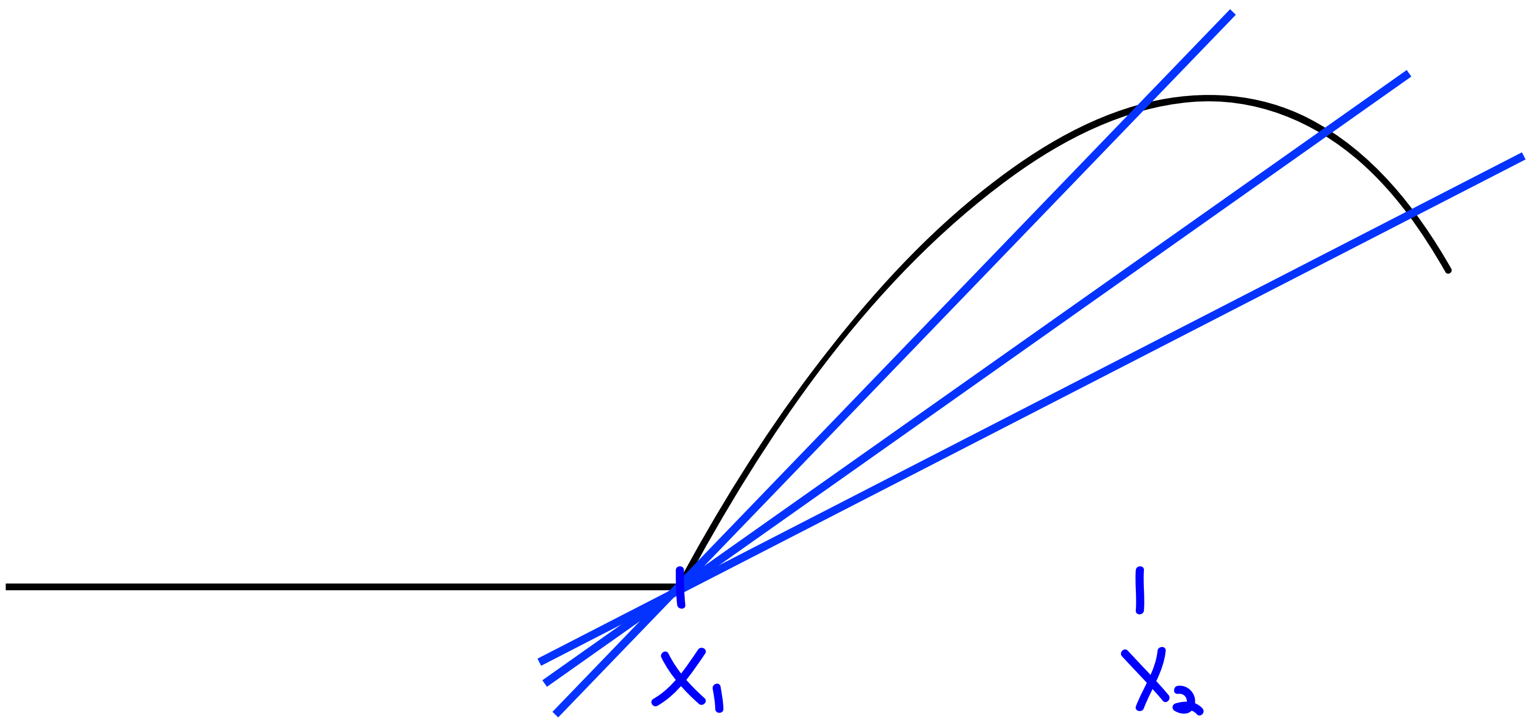

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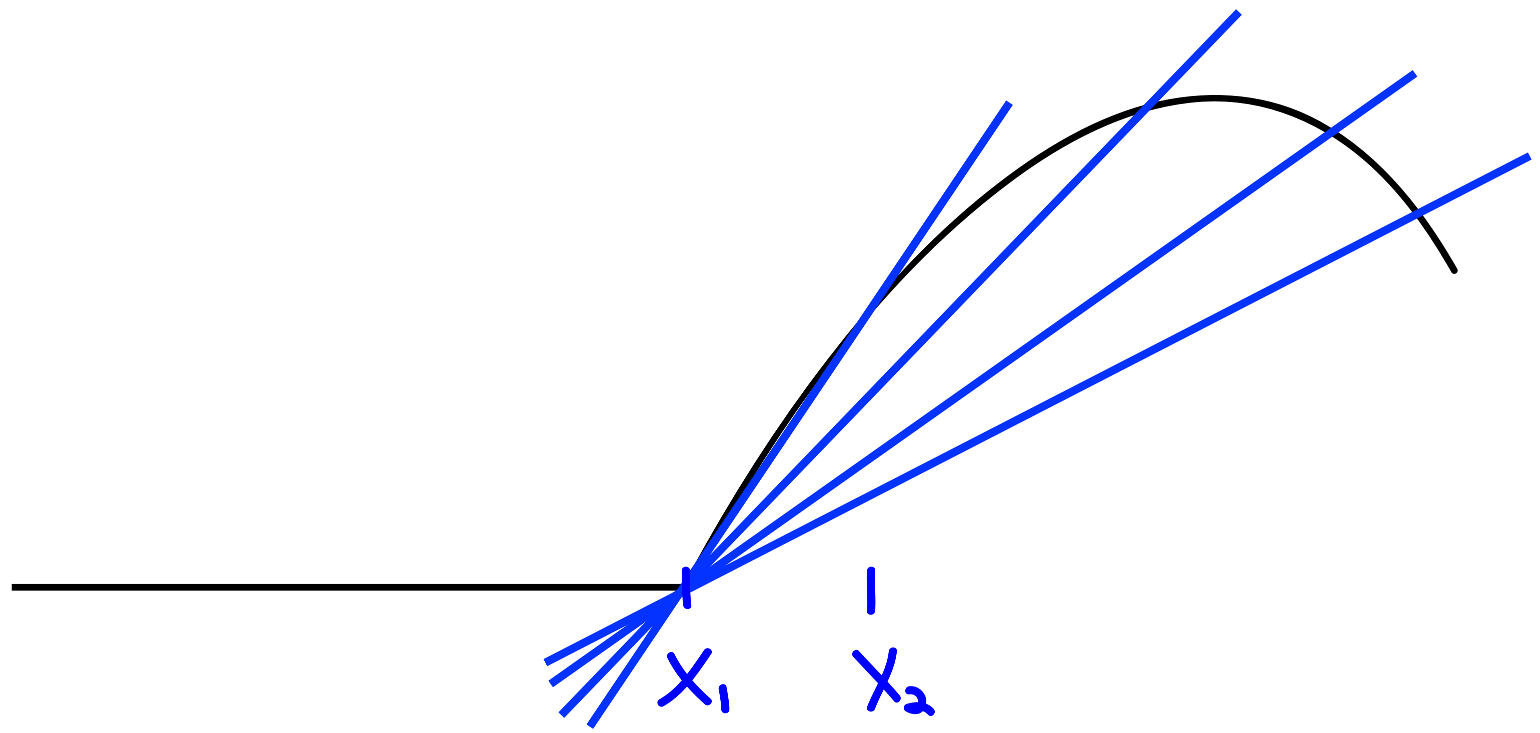

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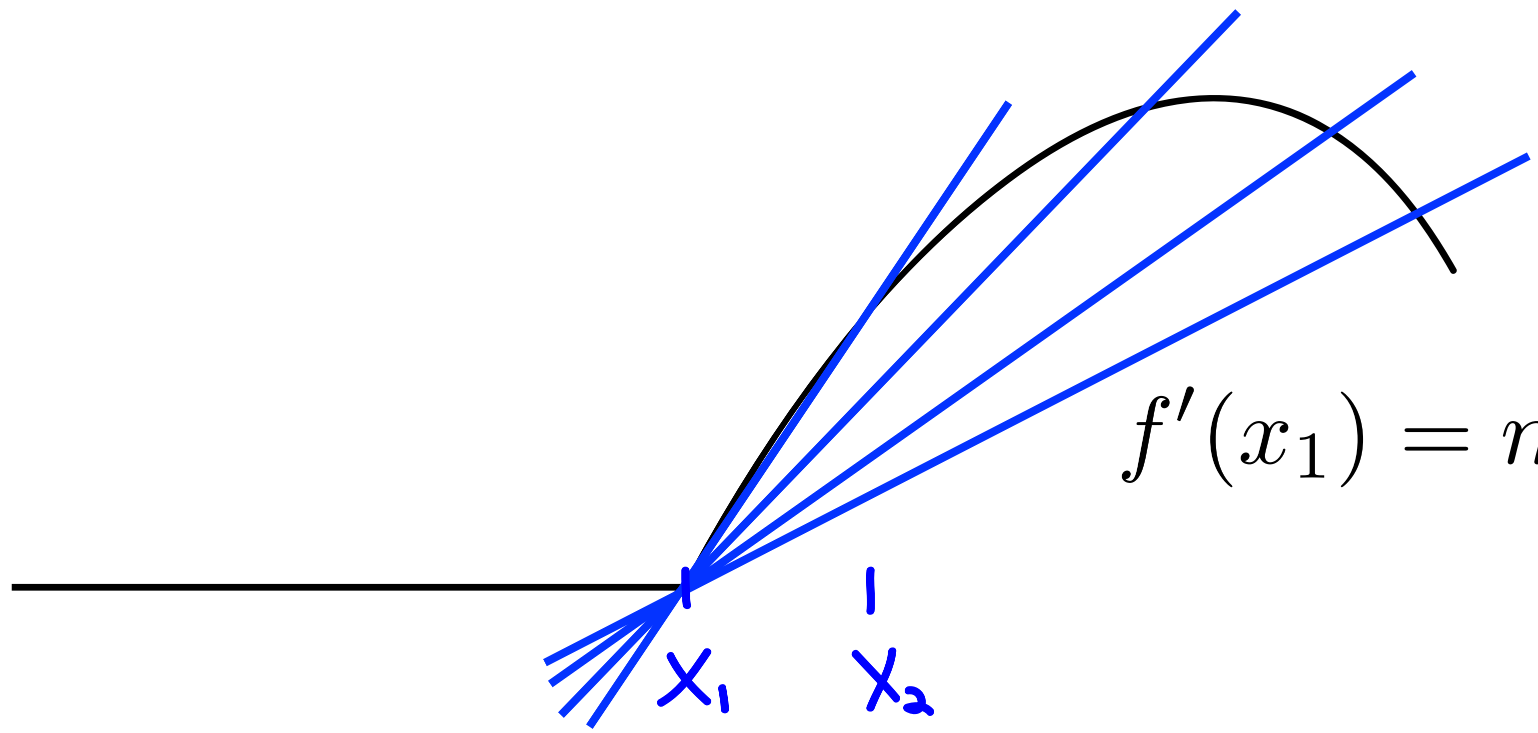

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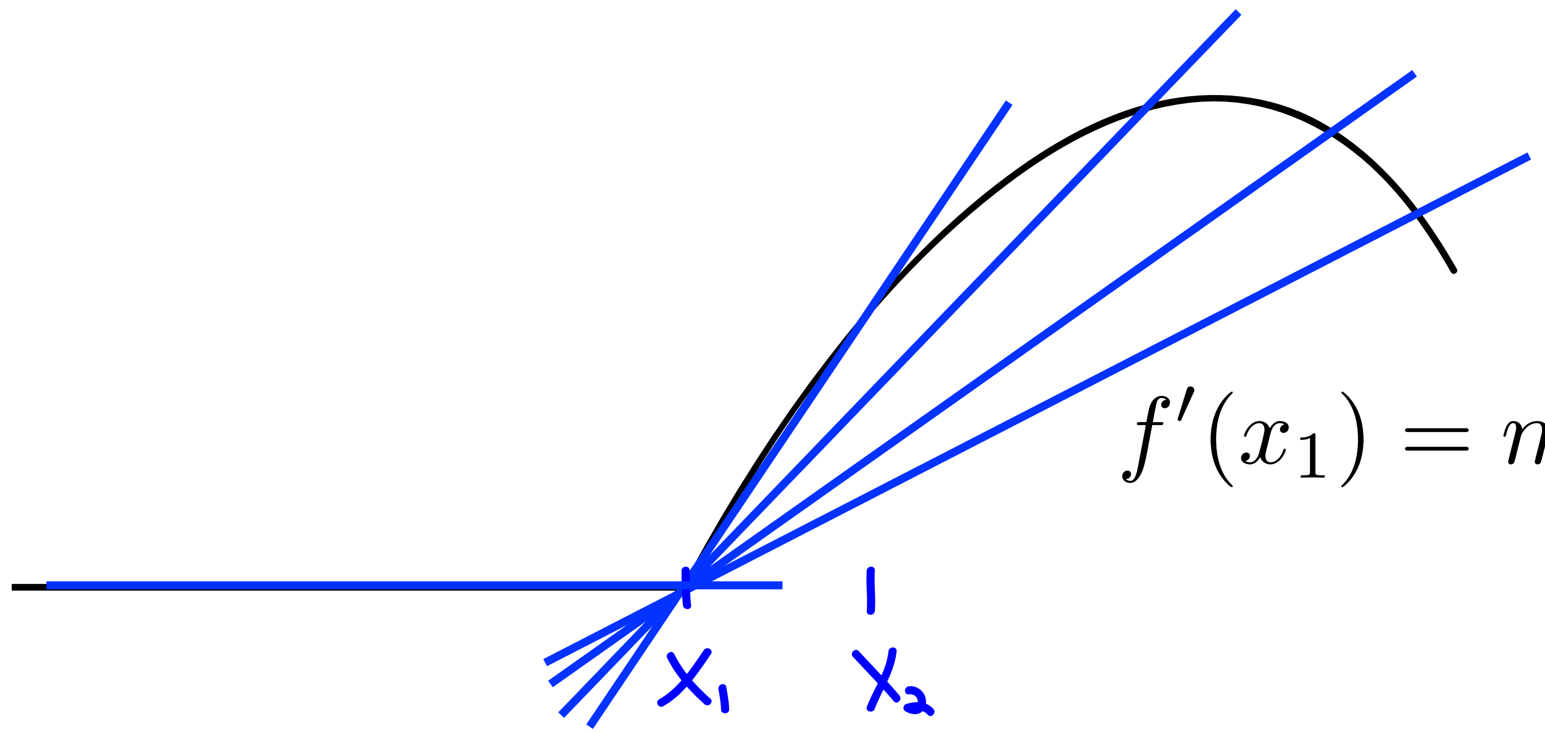

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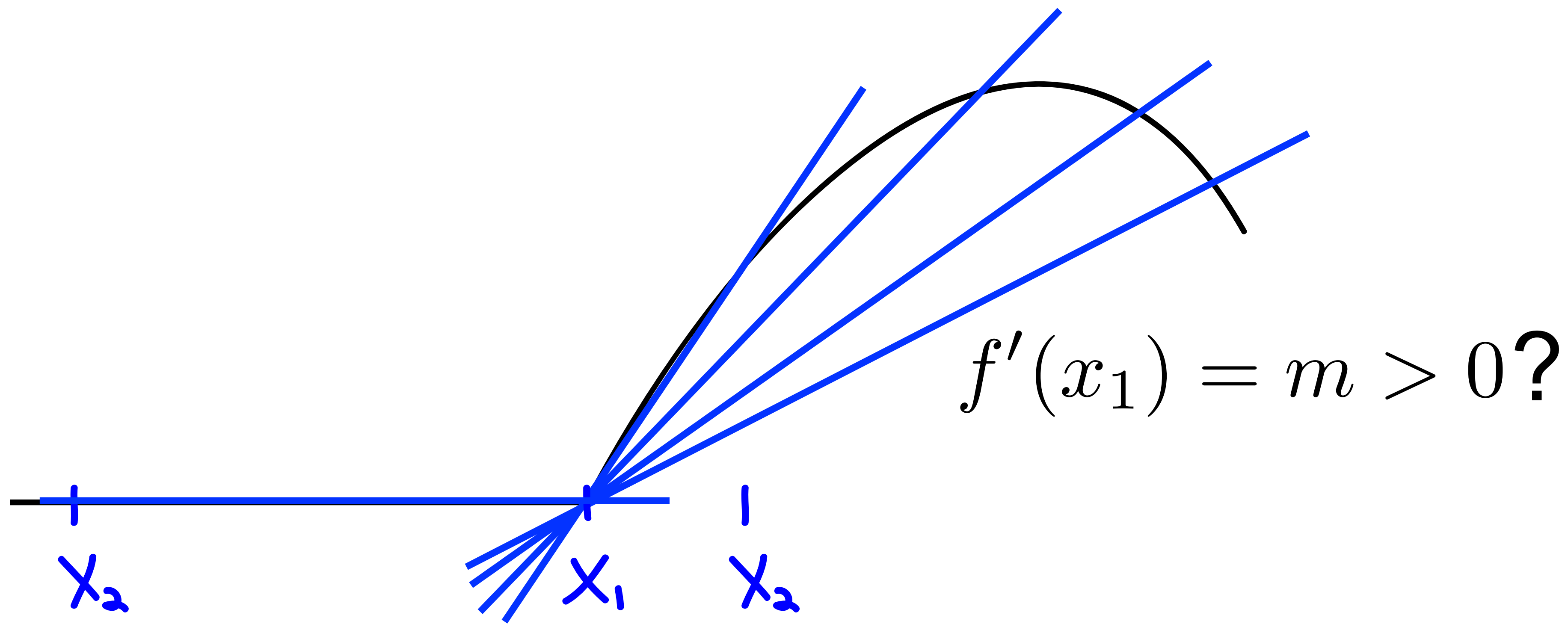

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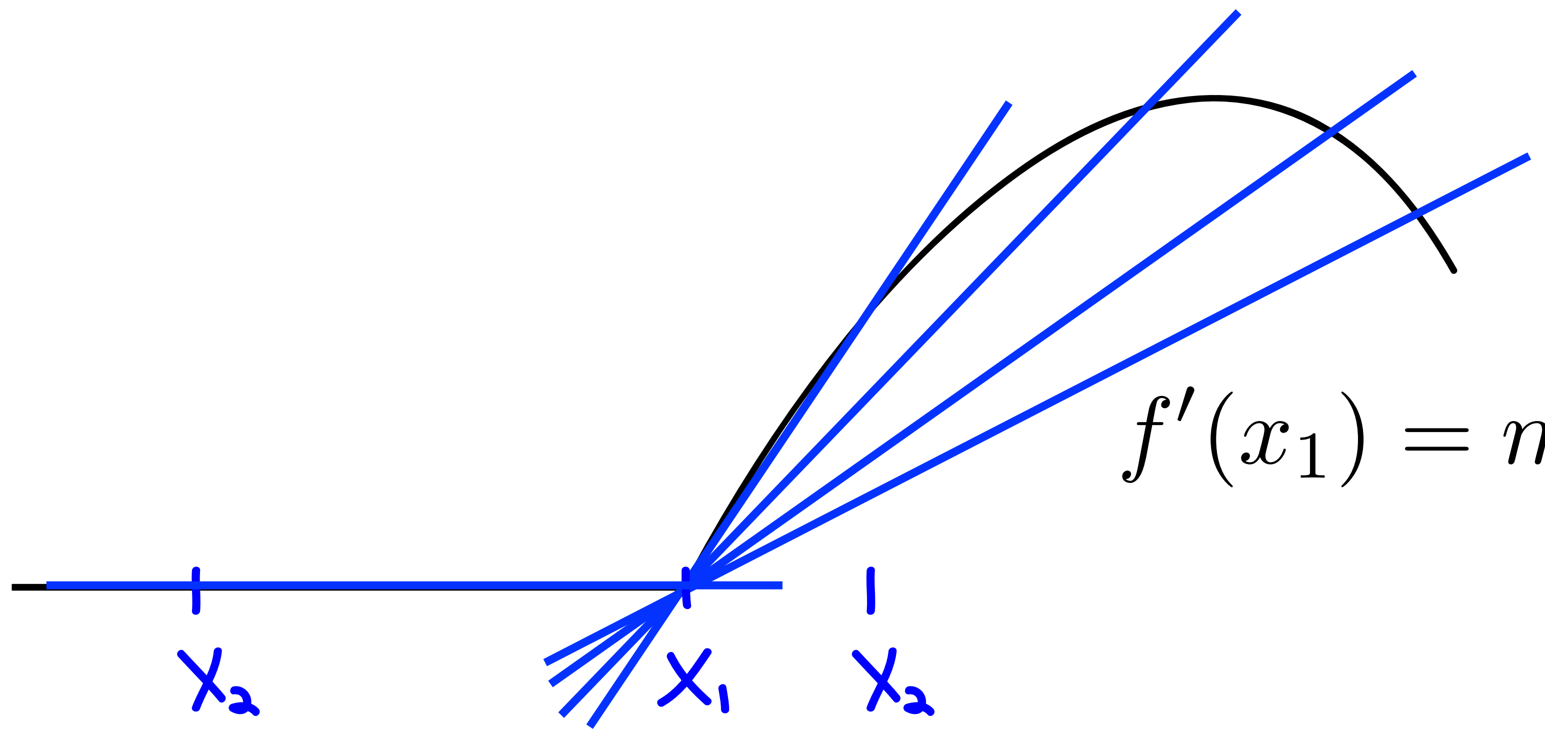

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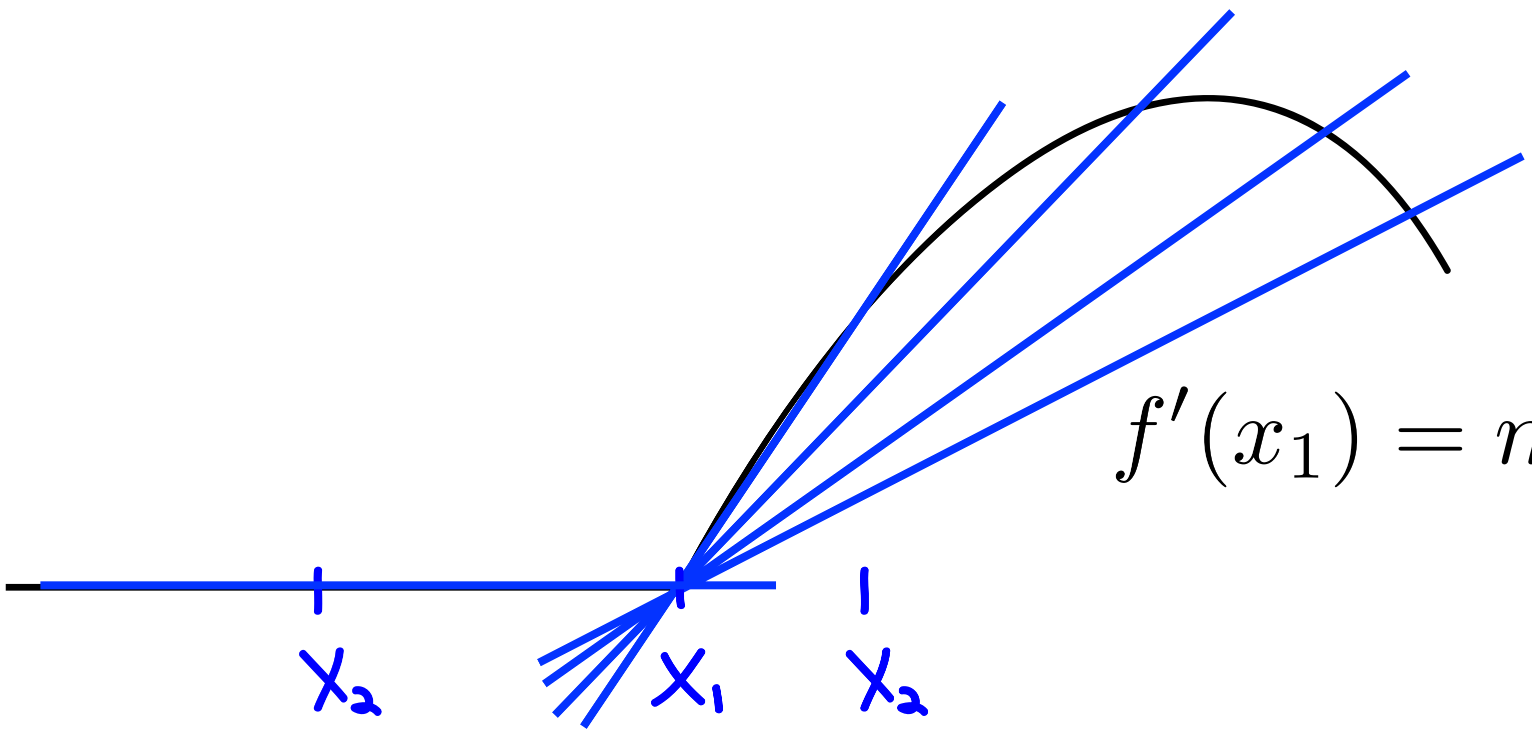

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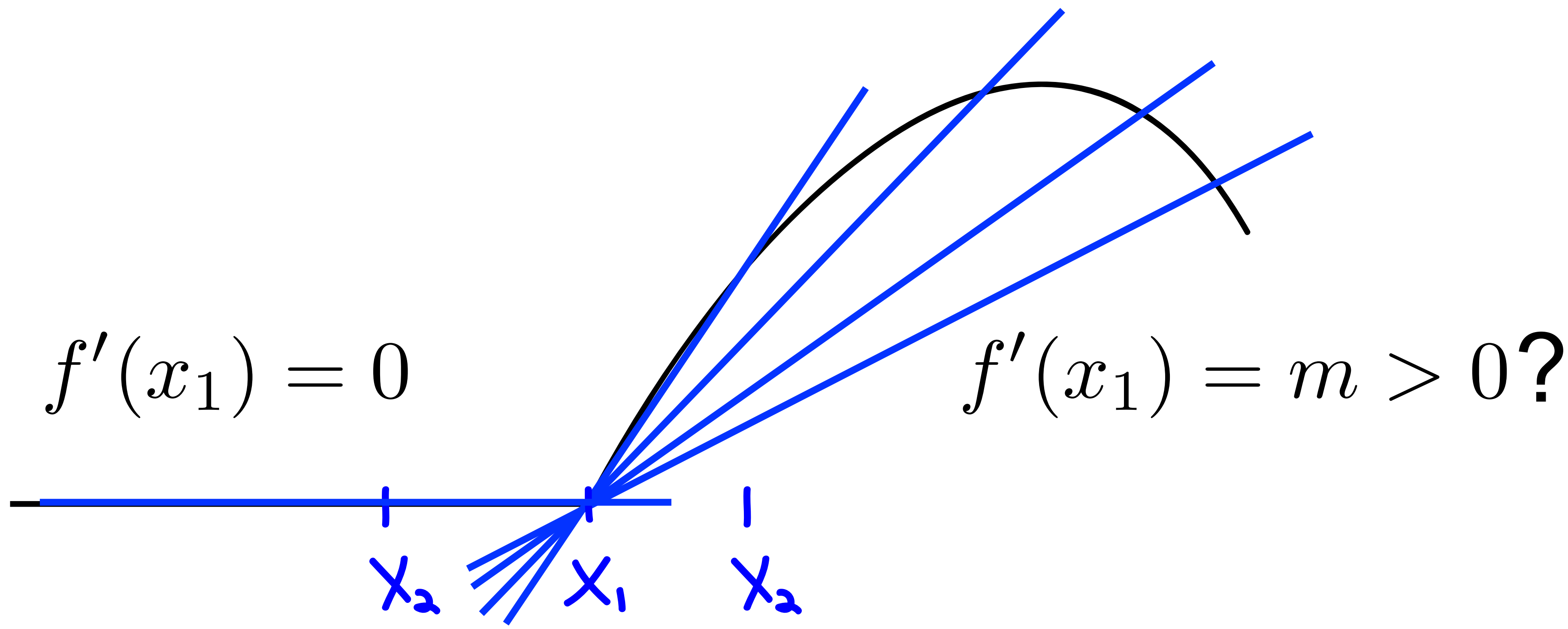

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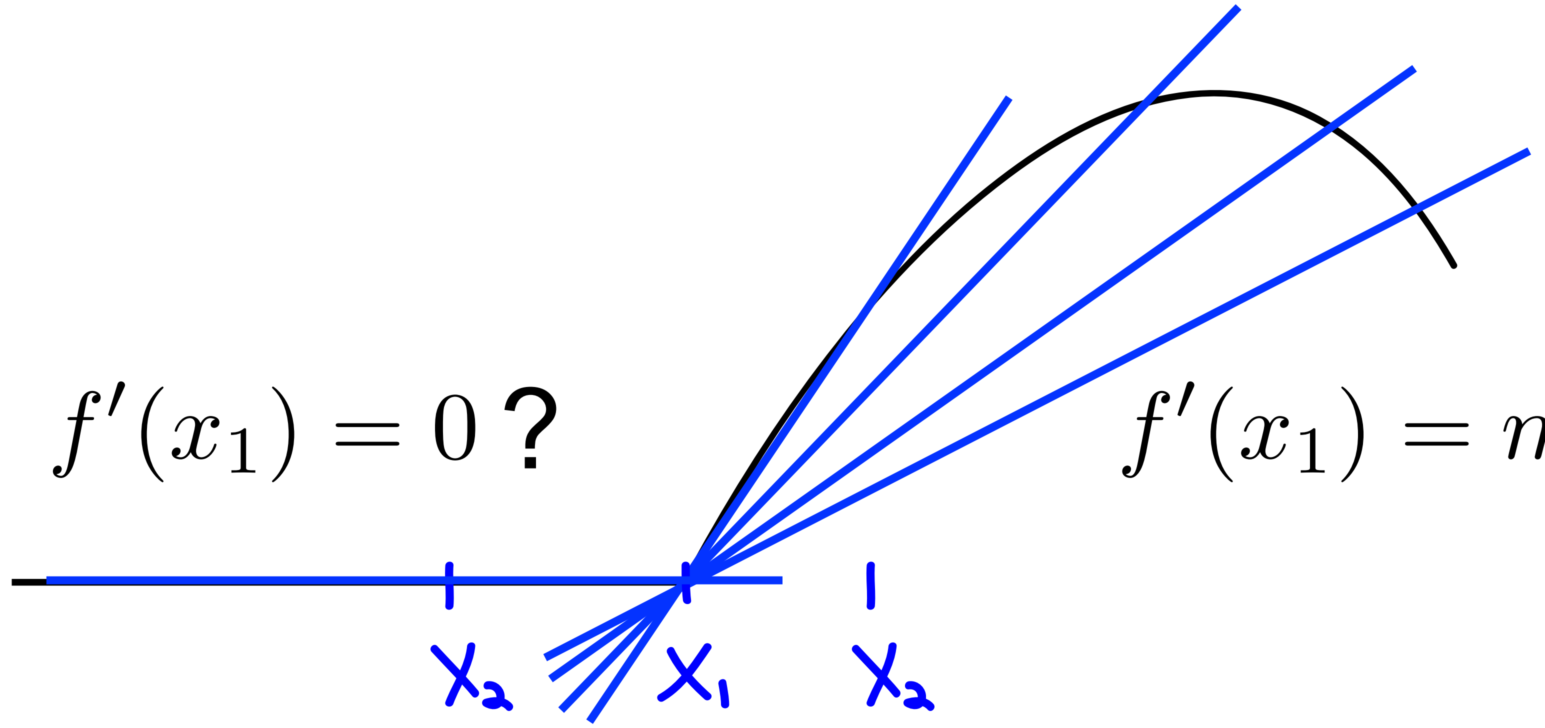

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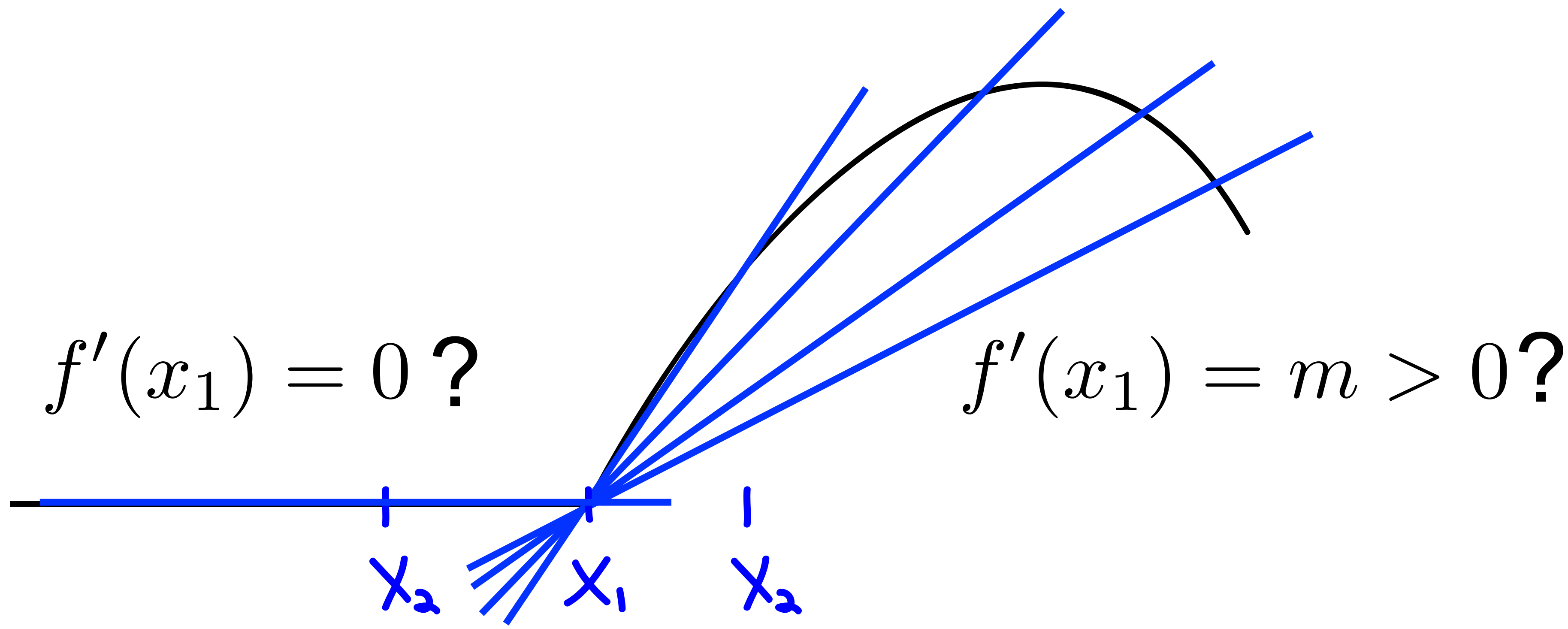

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(C) Both (A) and (B)

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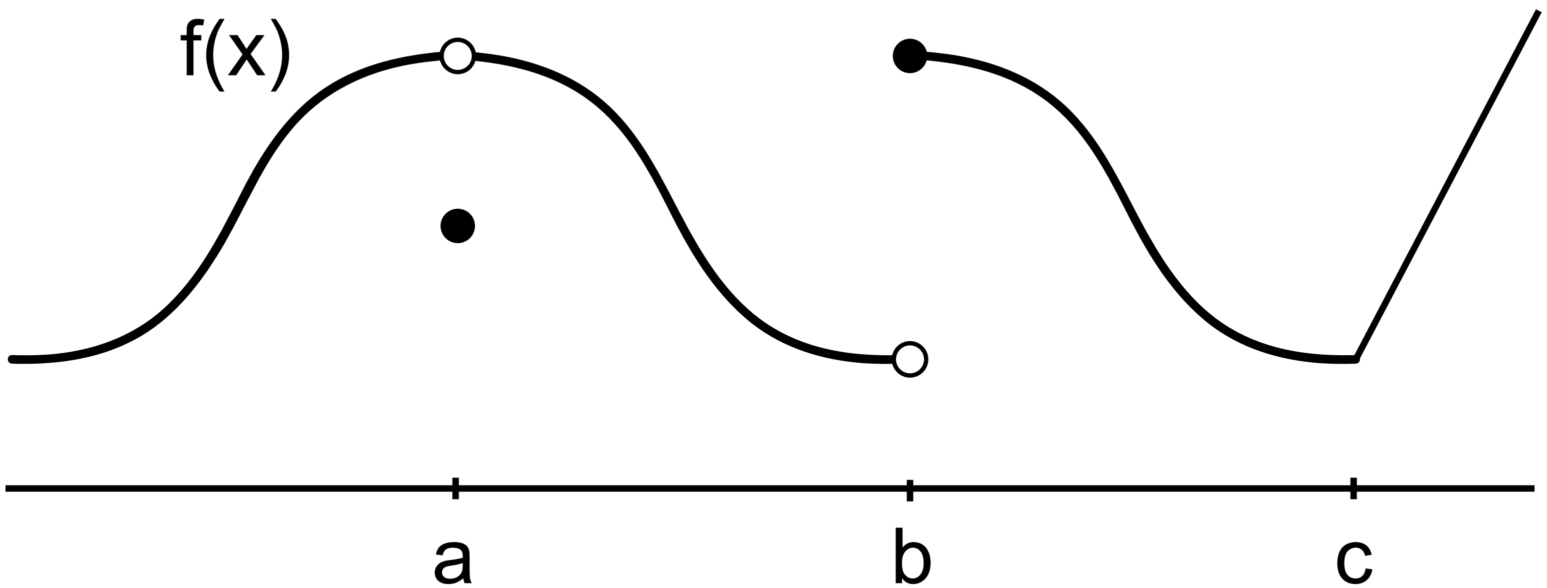
To evaluate a limit

To evaluate $\lim_{x \to a} f(x)$, plug in values closer and closer to a but you never get to a. In fact, f(a) may not even be defined. If you always get the same number no matter how you approach a, then the limit exists.

Limits



(A) 1, 4

Which of the following are true?

(B) 2, 5

1. $\lim_{x \to a} f(x) = f(a)$

4. $\lim_{x \to a} f(x)$ exists.

(C) 3

2. $\lim_{x \to b} f(x) = f(b)$

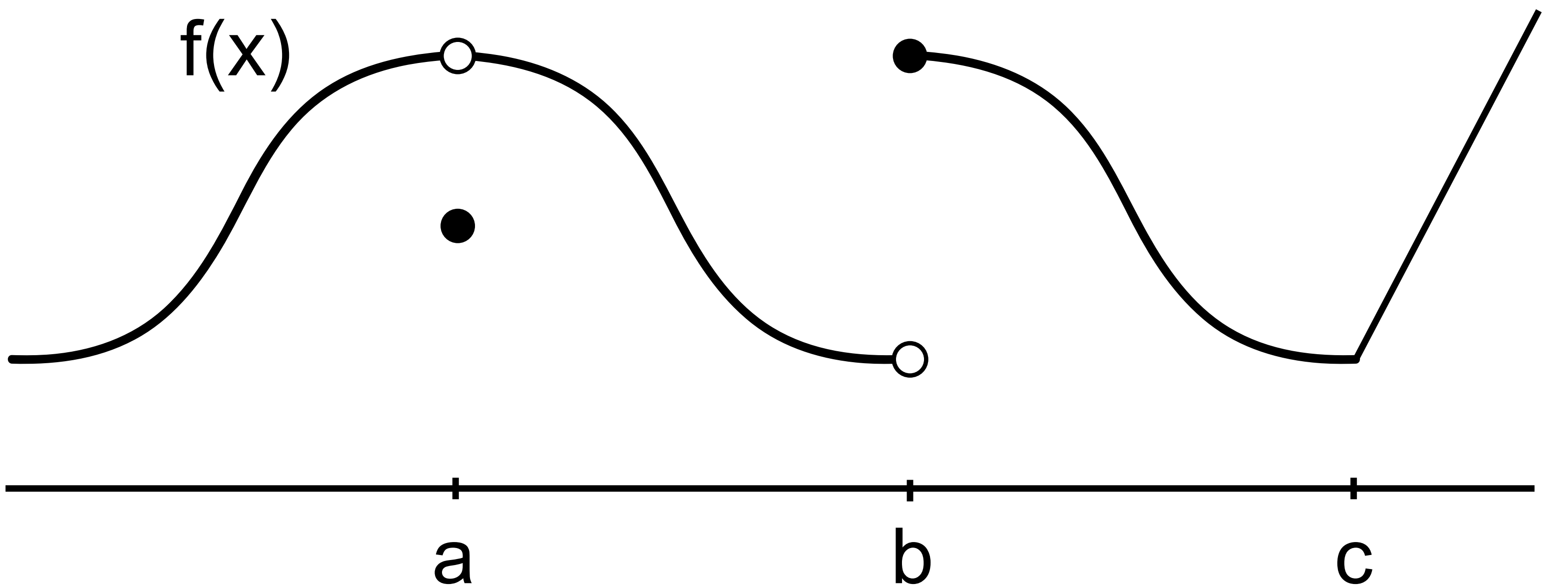
5. $\lim_{x \to b} f(x)$ exists.

(D) 4

3. $\lim_{x \to c} f(x)$ does not exist.

(E) 5

Limits



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3. $\lim_{x \to c} f(x)$ does not exist.
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(A) 1, 4

(B) 2, 5

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Left and right limits

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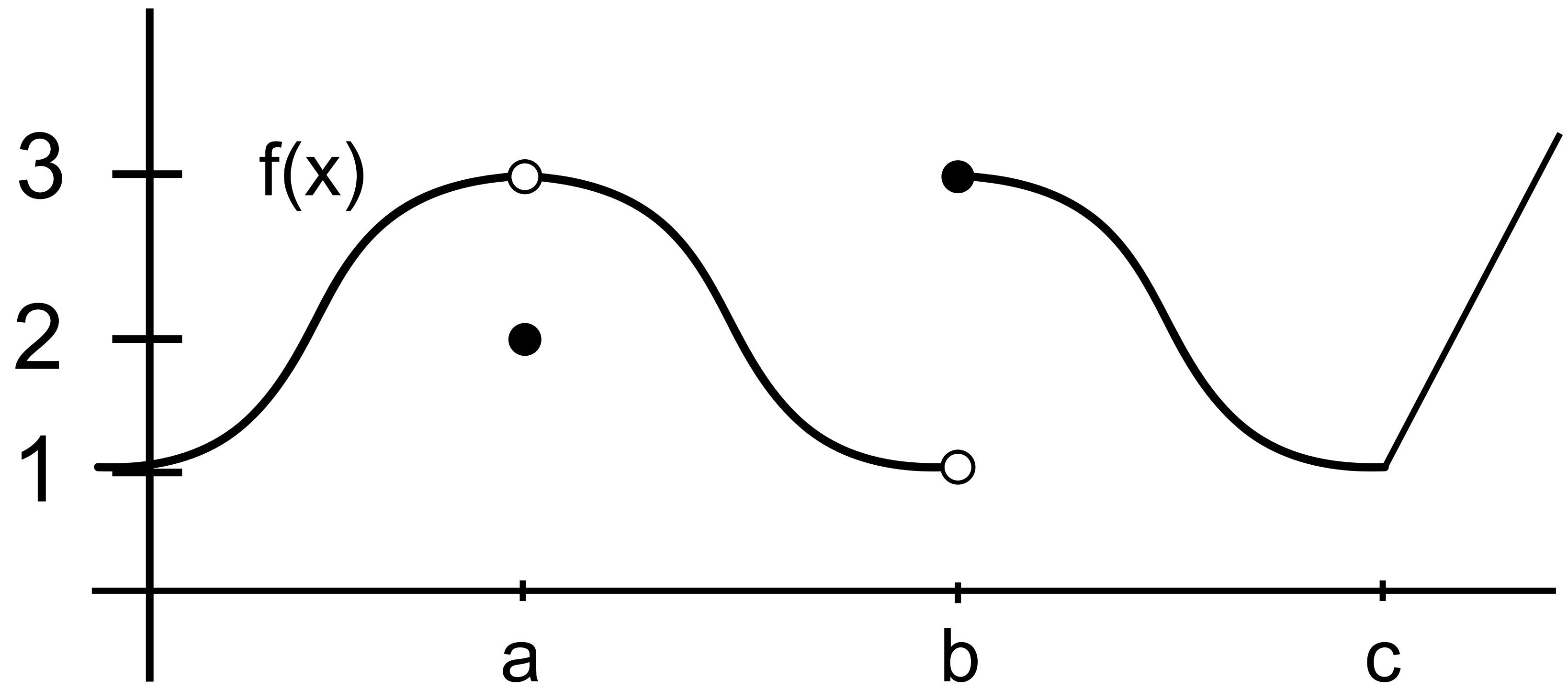
$$\lim_{x \longrightarrow a^-} f(x)$$

- When these exist and are equal, $\lim_{x \longrightarrow a} f(x)$ exists

$$\lim_{x \longrightarrow a} f(x) = \lim_{x \longrightarrow a^+} f(x) = \lim_{x \longrightarrow a^-} f(x).$$

Limits



(A) $\lim_{x \rightarrow a} f(x)$ = 2

(B) $\lim_{x \rightarrow b^-} f(x)$ = 3

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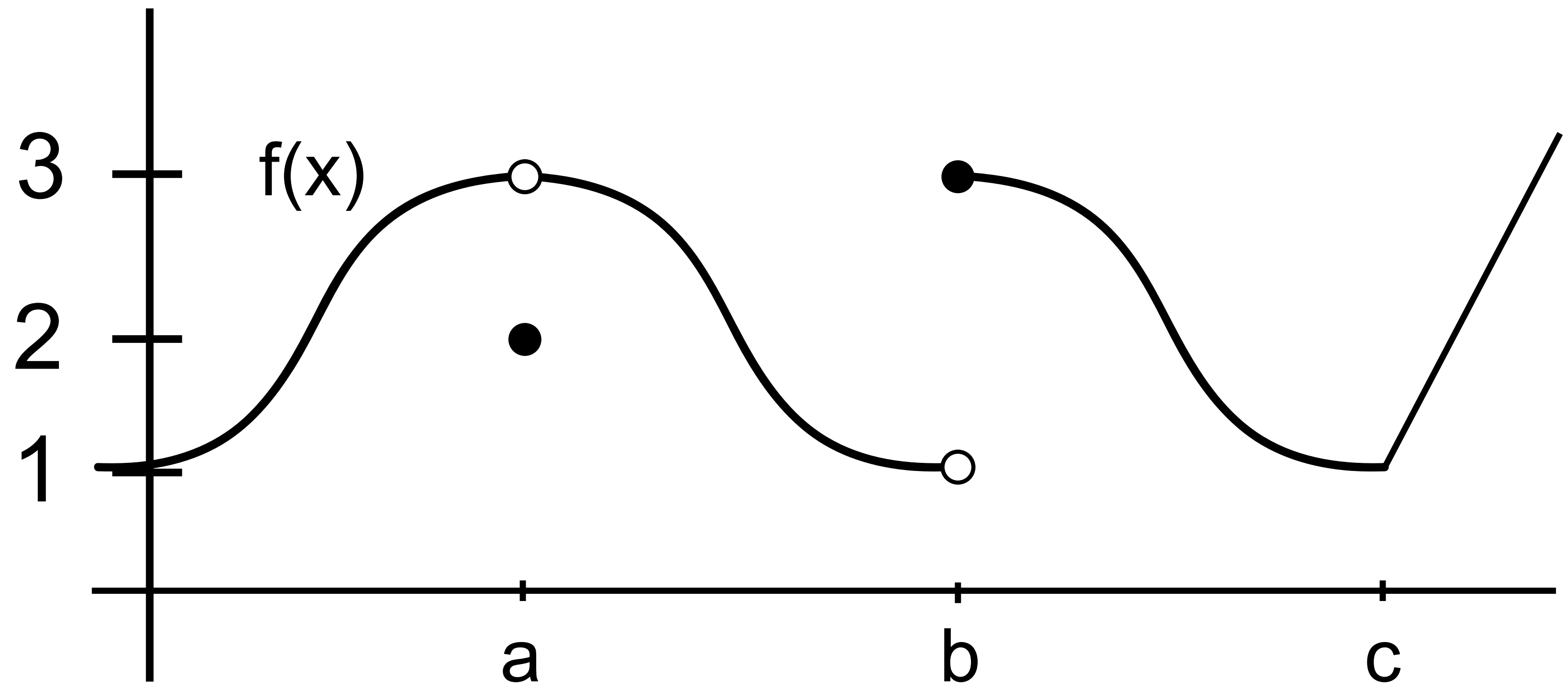

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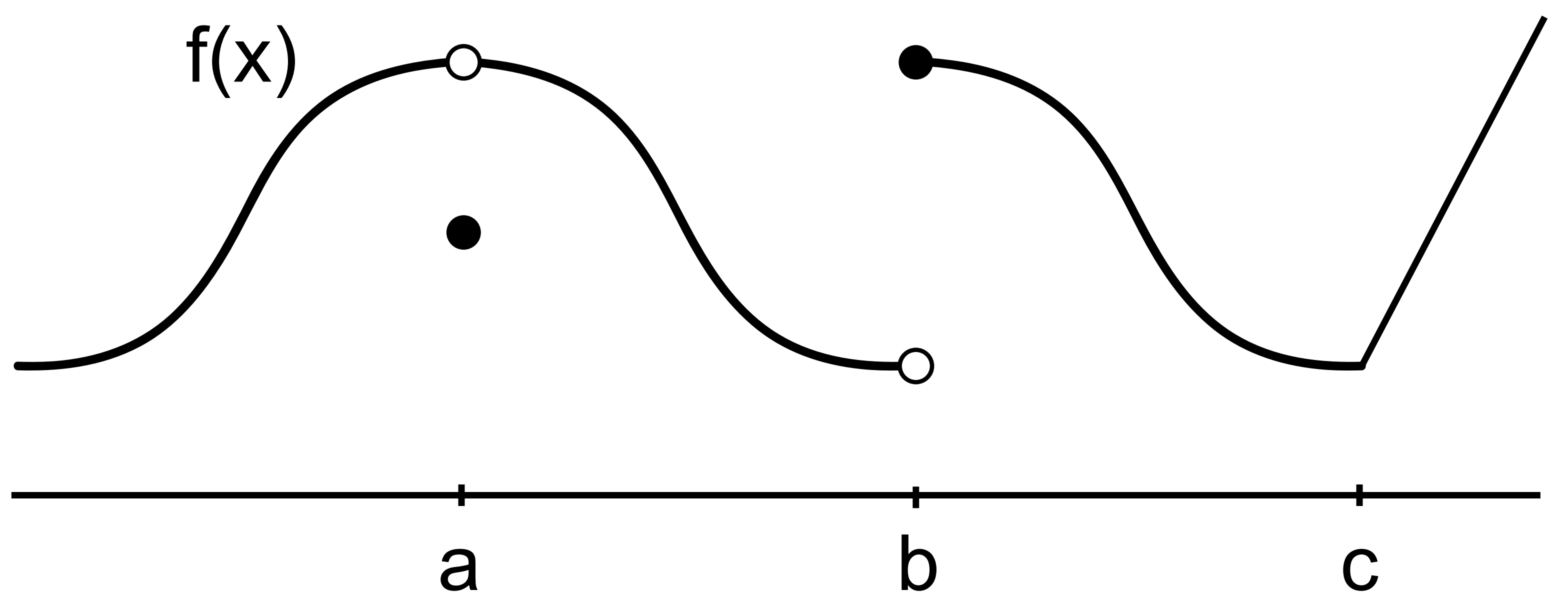

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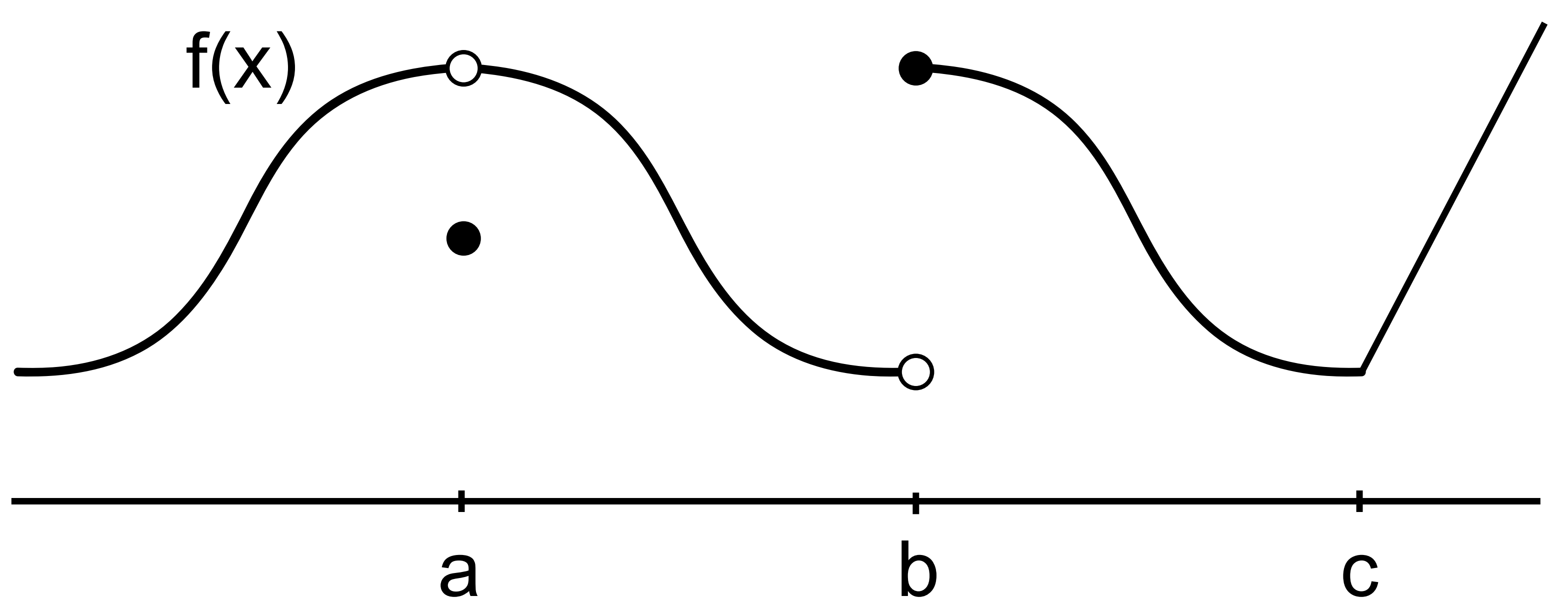

f(x) is continuous at all x except at x=a and x=b.

Types of limits we'll talk about

- Points of continuity: $\lim_{x \to a} f(x) = f(a)$
- Hole-in-the-graph (like derivative limit)
- Limits at $\pm\infty$ (asymptotes)
- Left/right, jumps
- Vertical asymptotes

Hole-in-the-graph and derivatives

Suppose you have a function $f(x)$.

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On the board: draw g(h) in "good" case.

Which of the following is a "point-of-continuity" limit?

(A) $\lim_{x \to \infty} \dfrac{x^2 - 4}{x - 2}$

(B) $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

(C) $\lim_{x \to -2} \dfrac{x^2 - 4}{x - 2}$

Which of the following is a "point-of-continuity" limit?

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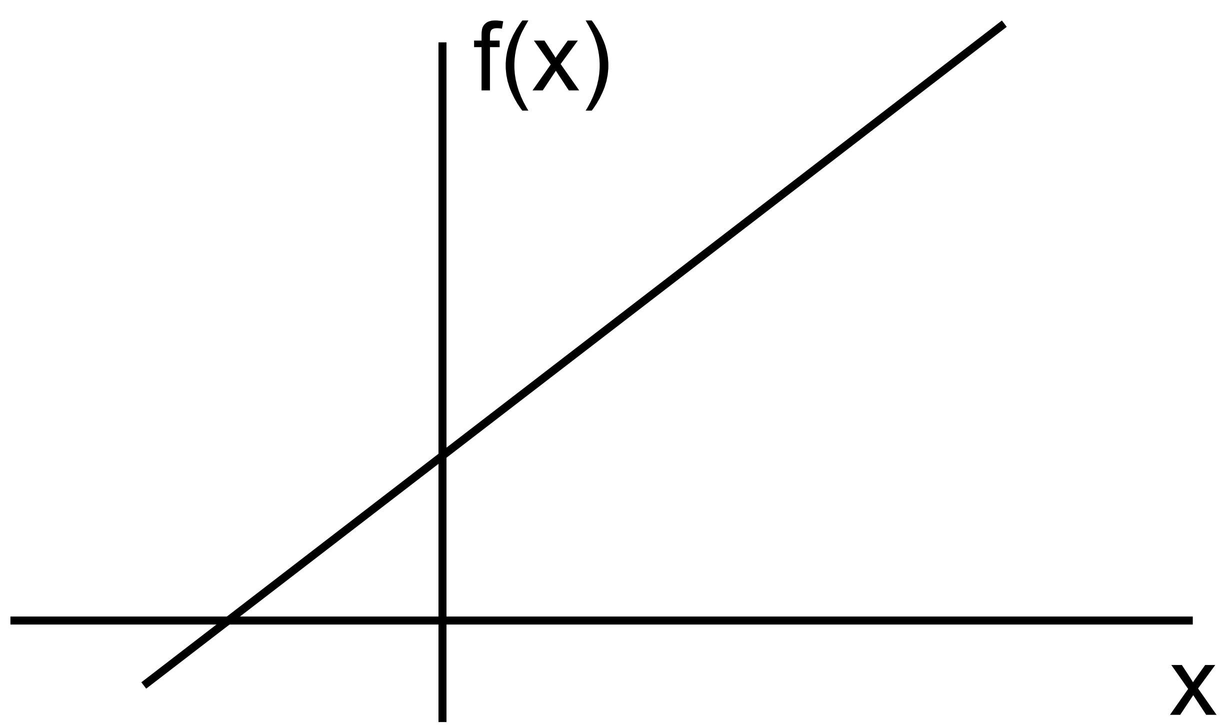

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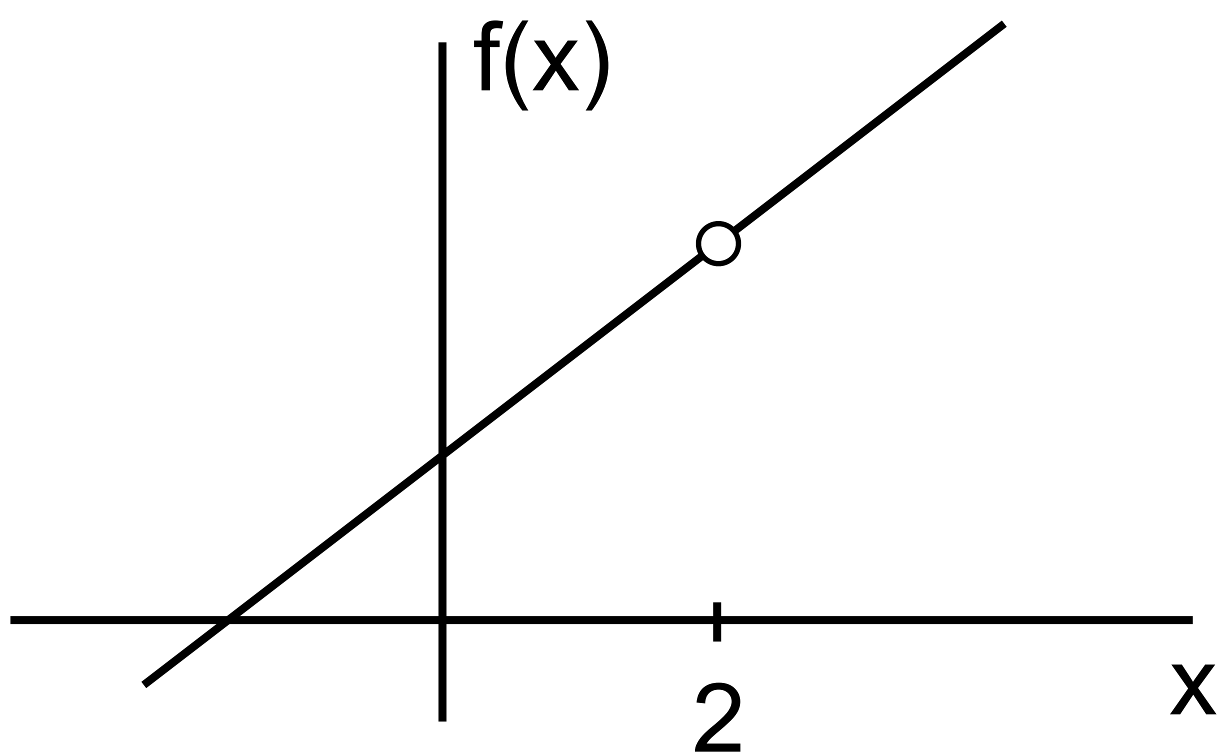

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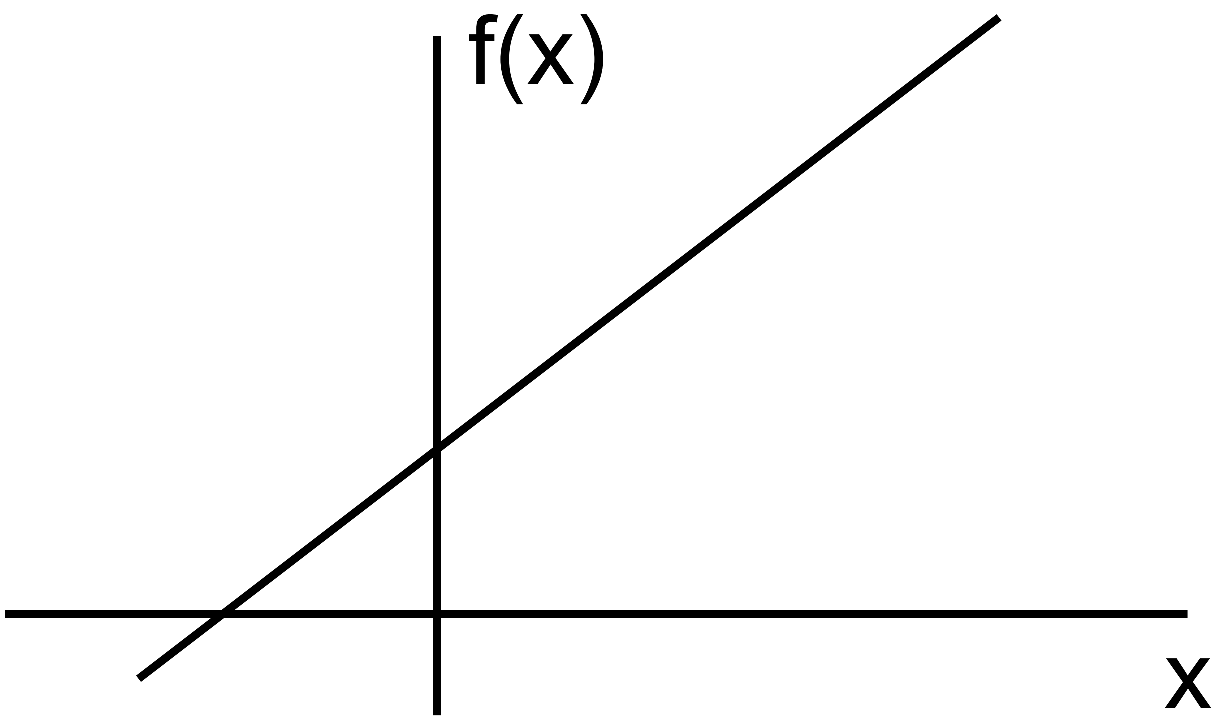

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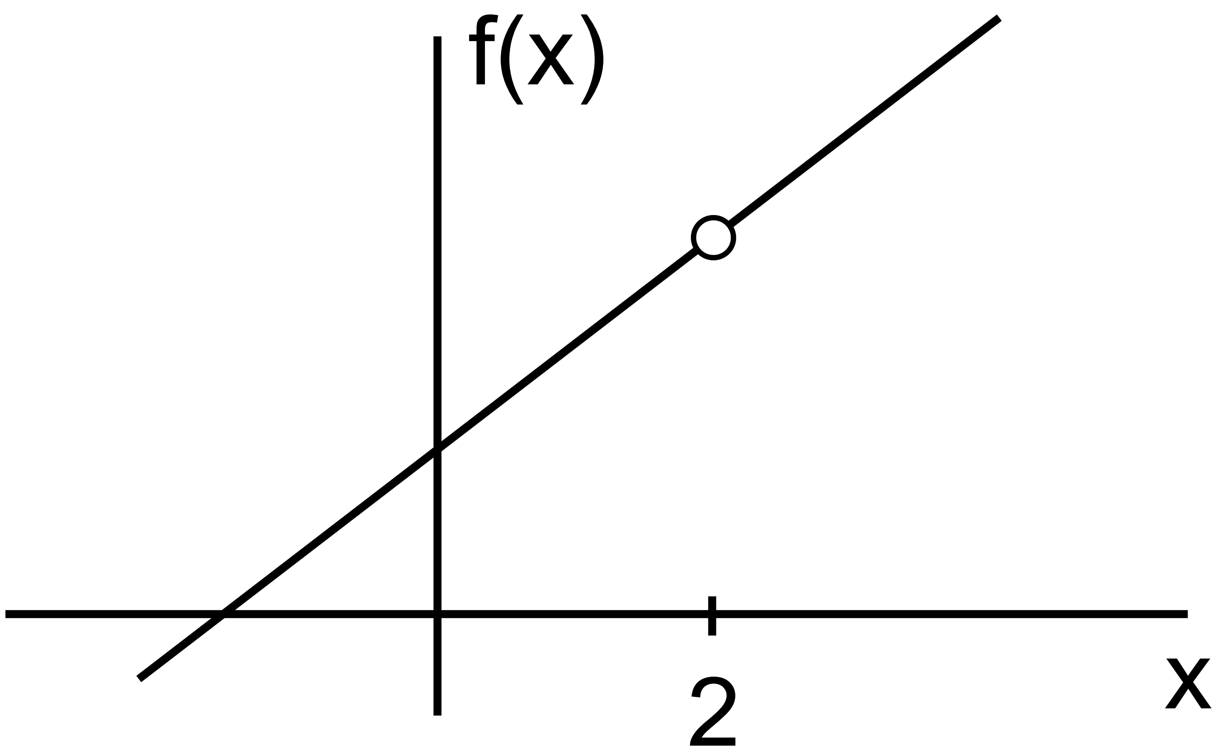
(C) $\lim_{x \to -2} \frac{x^2 - 4}{x - 2}$



Which of the following is a "point-of-continuity" limit?

(A) $\lim_{x \to \infty} \frac{x^2 - 4}{x - 2}$

(B) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

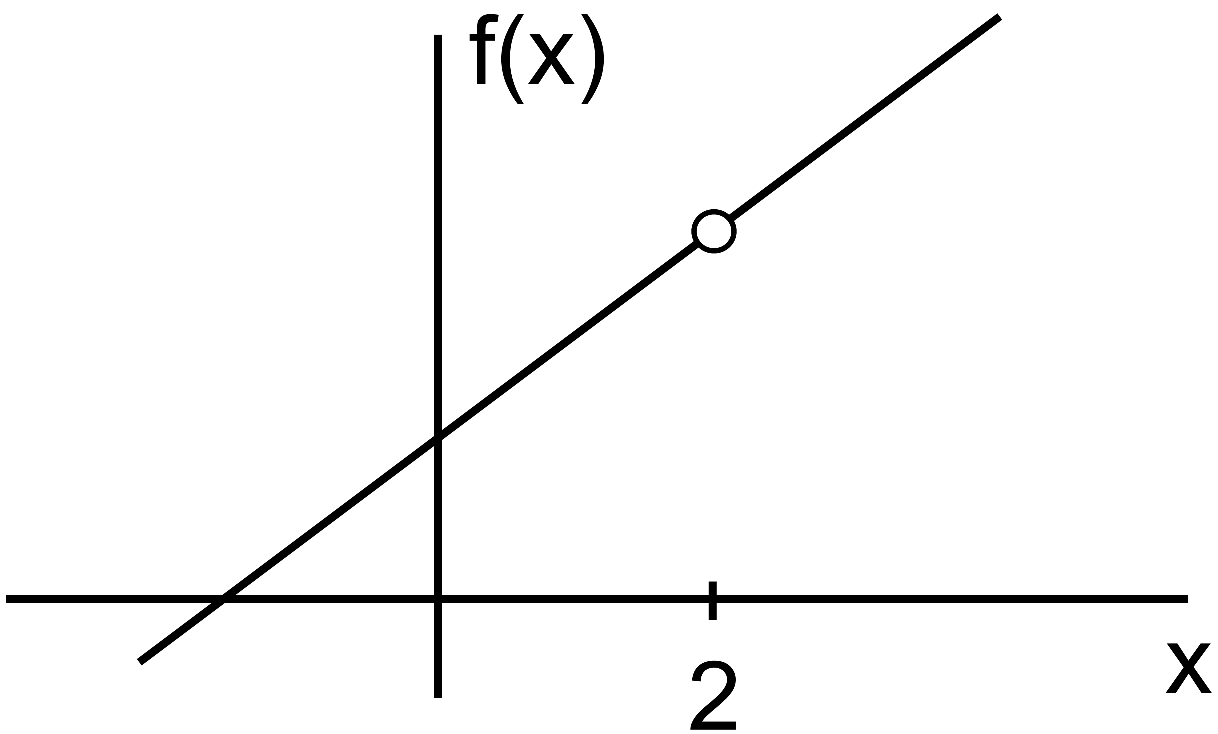
(C) $\lim_{x \to -2} \frac{x^2 - 4}{x - 2}$

Which of the following is a "hole in the graph" limit?

(A) $\displaystyle\lim_{x\to\infty} \frac{x^2-4}{x-2}$

(B) $\displaystyle\lim_{x\to 2} \frac{x^2-4}{x-2}$

(C) $\displaystyle\lim_{x\to -2} \frac{x^2-4}{x-2}$

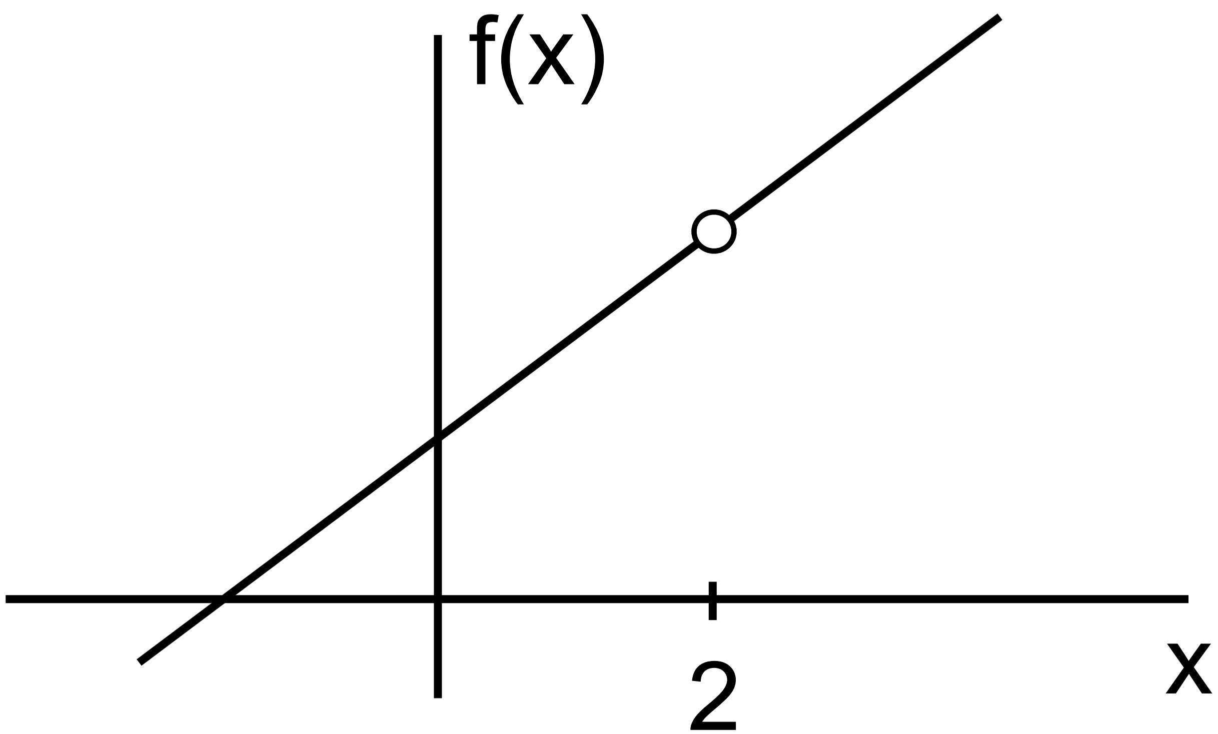


Which of the following is a "hole in the graph" limit?

(A) $\displaystyle \lim_{x \to \infty} \frac{x^2 - 4}{x - 2}$

(B) $\displaystyle \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(C) $\displaystyle \lim_{x \to -2} \frac{x^2 - 4}{x - 2}$

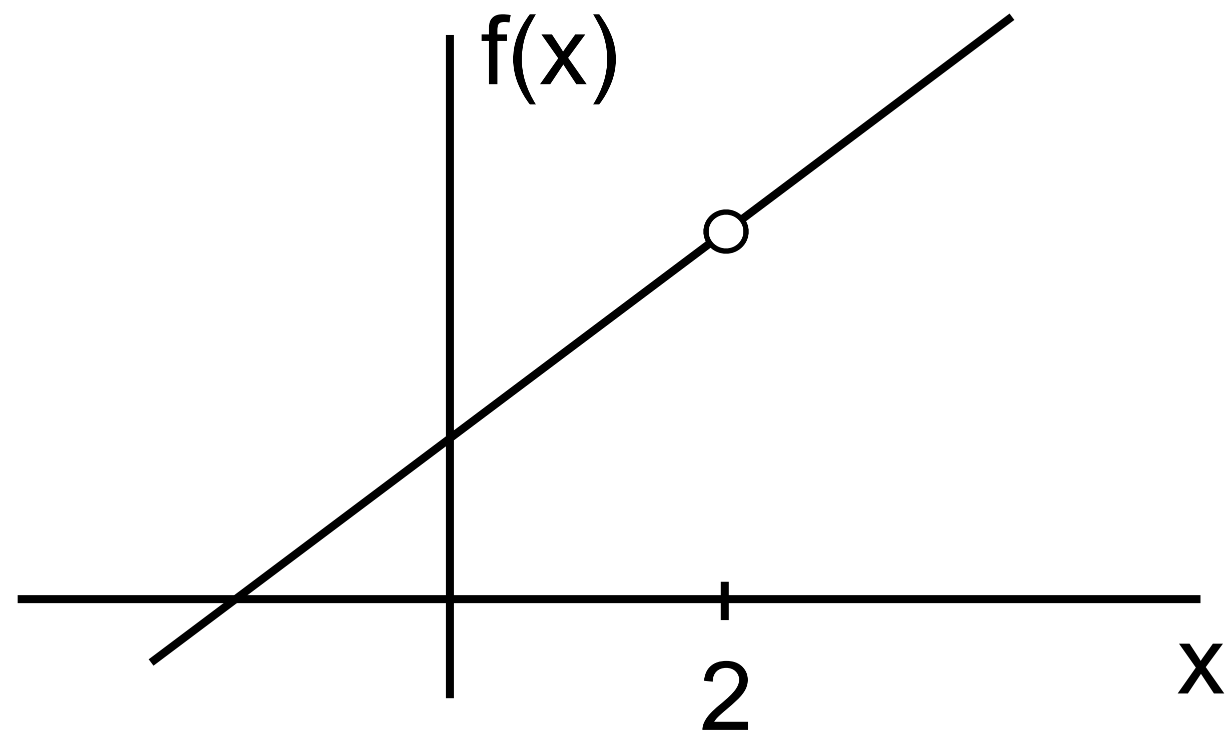


Which of the following is a "limit-at-infinity" limit?

(A) $\lim_{x \to \infty} \frac{x^2 - 4}{x - 2}$

(B) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(C) $\lim_{x \to -2} \frac{x^2 - 4}{x - 2}$

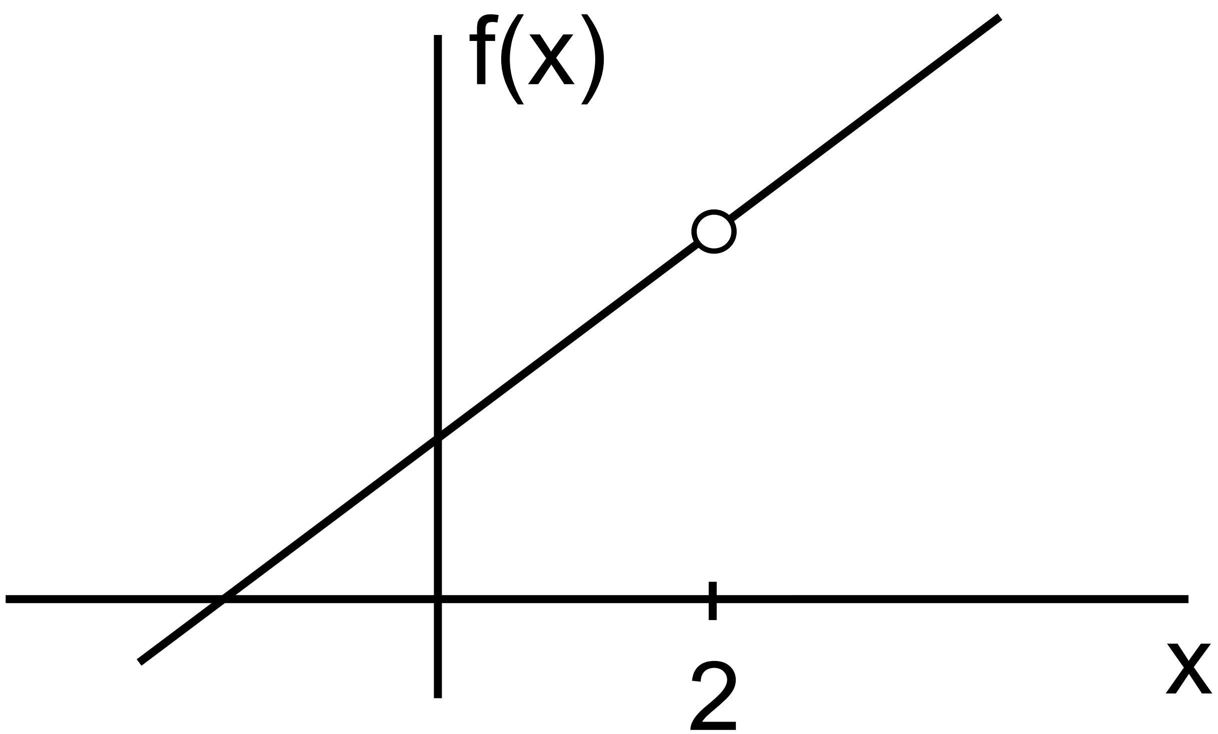


Which of the following is a “limit-at-infinity” limit?

(A) $\displaystyle\lim_{x \to \infty} \frac{x^2 - 4}{x - 2}$

(B) $\displaystyle\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(C) $\displaystyle\lim_{x \to -2} \frac{x^2 - 4}{x - 2}$



Examples in which f'(a) does not exist

On the board...