The radius of a spherical tumor grows at a constant rate \( k \). Determine the rate of growth of the volume of the tumor when the radius is 1 cm.

\[
V(r(t)) = \frac{4}{3} \pi (r(t))^3
\]

\[
\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}
\]

We know that \( \frac{dr}{dt} = k \), so

\[
\frac{dV}{dt} = 4\pi r^2 k.
\]

Thus, the rate of growth of the volume is

\[
\frac{dV}{dt} \bigg|_{r=1} = 4\pi k
\]

when the tumor is 1 cm in radius.
Water is leaking out of a conical cup of height $H$ and radius $R$. Find the rate of change of the height of water in the cup when the cup is full, if the volume of water is decreasing at a constant rate $k$.

* Ultra challenging problem, IMHO.

Let $h(t)$ be the height of water at time $t$. Let $h(t)$ decrease slowly at first, then speed up!

The volume of water is:

$$V = \frac{1}{3} \pi r^2 h,$$

So

$$V = \frac{1}{3} \pi \frac{R^2}{H^2} h^3 \quad \leftarrow \text{Yay! A function of only } h.$$

$$R^2, H^2 \text{ are known constants}.$$

How?

$\begin{align*}
\frac{n}{h} &= \frac{R}{H} \\
&\Rightarrow r = h \frac{R}{H}
\end{align*}$

$\overline{\text{Similar triangles}}$

$\frac{\frac{R}{H}}{H} = \frac{r}{h} \Rightarrow r = \frac{hR}{H}$
We know that
\[
\frac{dv}{dt} = -K.
\]

Thus,
\[-K = \pi \frac{R^2}{H^2} h^2 \frac{dh}{dt}.
\]

\[
\Rightarrow \quad \frac{dh}{dt} = -\frac{K}{\pi \frac{R^2}{H^2} h^2}.
\]

The rate of change of height when full is
\[
\left. \frac{dh}{dt} \right|_{h=H} = -\frac{K}{\pi R^2}.
\]

\[\Rightarrow\] matches intuition!
For large \(h\), \(\frac{dh}{dt} \ll 1\),
For small \(h\), \(\frac{dh}{dt} \gg 1\).