

# Today

- Reminders:

- OSH 3 on Monday, Assign. 4a on Tues 7am,

- Midterm 1 on Tues 6pm.

- S.101 – HENN 200,

- S.103 – Last name A-K: BUCH A203

- S.103 – Last name L-Z: BUCH A103

# Today

- How to choose  $x_0$  for Newton's method.
- Inc/dec, critical points and extrema.
- First and Second derivative test.
- Concavity, potential IPs and actual IPs.


# How to choose $x_0$

Desmos:


• <https://www.desmos.com/calculator/hf5ll3di1l>


# Increasing/decreasing

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# Local extrema (min/max)

- A point  $a$  is a **local minimum** of a function  $f(x)$  provided that  $f(x) > f(a)$  for all  $x$  on an interval around  $a$  (excluding  $a$ , of course).
- A point  $a$  is a **local maximum** of a function  $f(x)$  provided that  $f(x) < f(a)$  for all  $x$  on an interval around  $a$  (excluding  $a$ , of course).

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- If the function is differentiable at the minimum, then it must look like (A).

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  - If  $f'(x)$  changes sign at a CP, then the CP is an extremum (min/max) of  $f(x)$ .
  - If  $f'(x)$  does not change sign at a CP, then the CP is not an extremum of  $f(x)$ !

# Critical points – examples

Desmos

- <https://www.desmos.com/calculator/nyquvpptn5>

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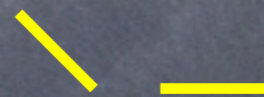
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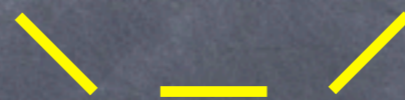
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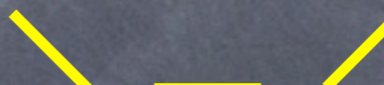


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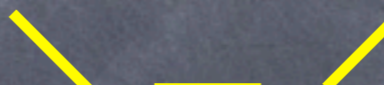

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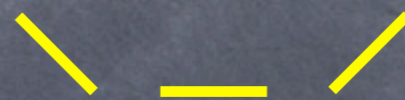
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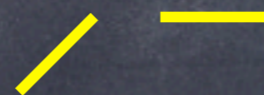
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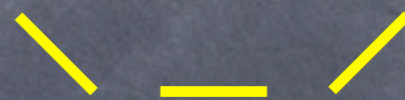
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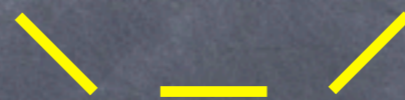
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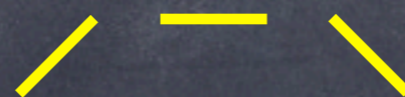
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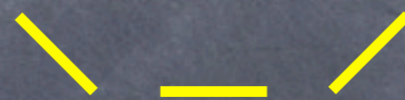
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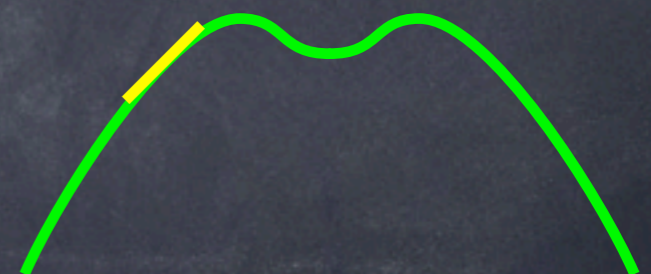
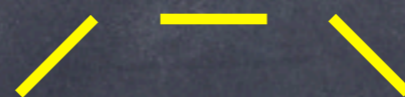
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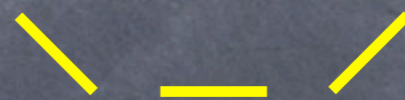
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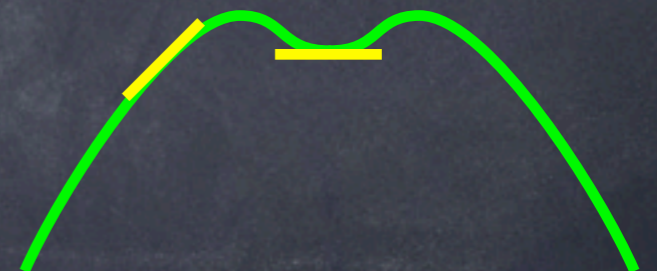
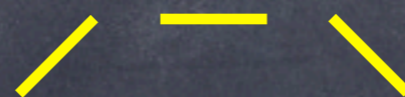
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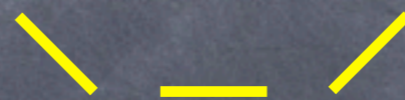




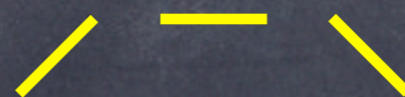
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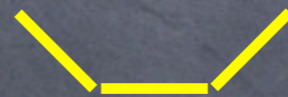
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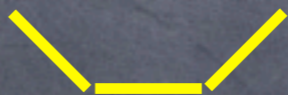


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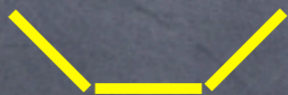



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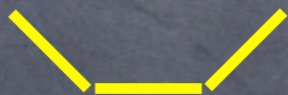

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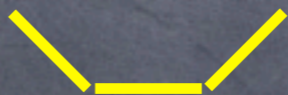

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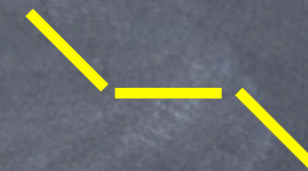
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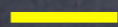
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- Ex.2:  $g(x)=x^5 \rightarrow g''(0)=0$  but  $x=0$  is neither a min nor a max.

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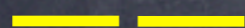
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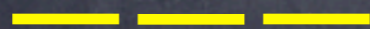
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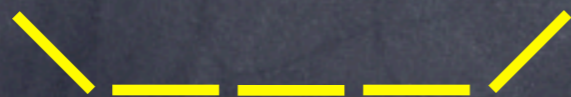
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(ii) SDT:  $g''(x) = 20x^3 - 6x = 2x(10x^2 - 3)$  so  $g''(0)=0 \rightarrow$  mystery!

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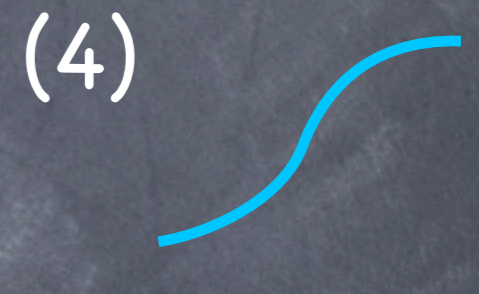
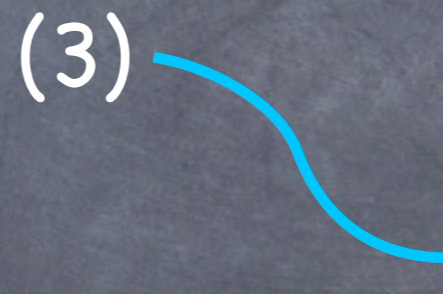
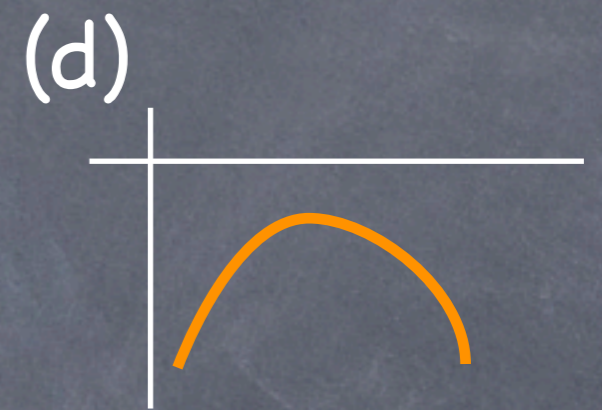
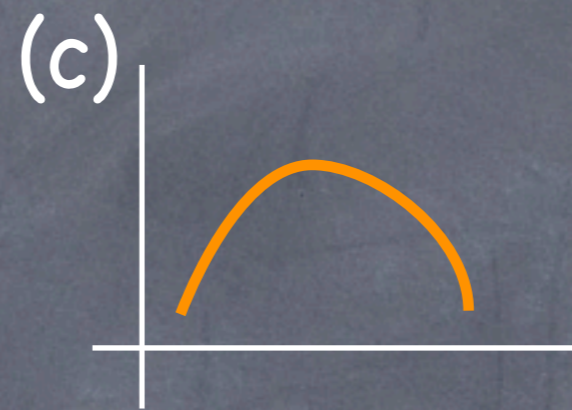
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better!!



# Match $f'(x)$ to $f(x)$



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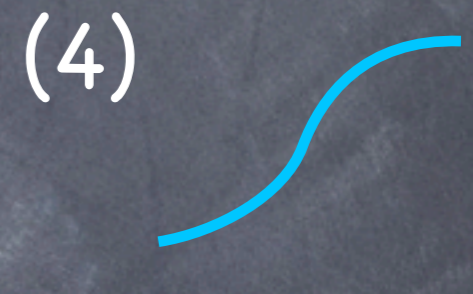
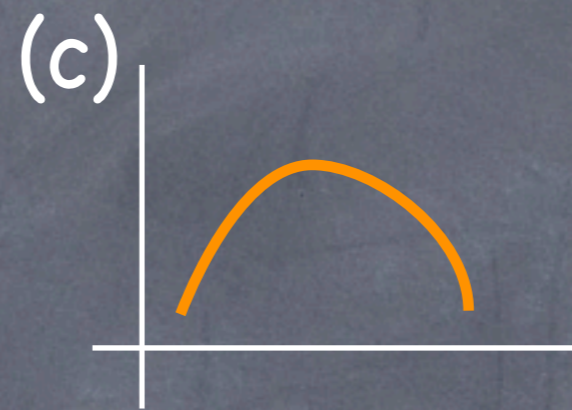
(C) 1b, 2d, 3c, 4a

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(D) 1c, 2a, 3d, 4b

(E) Don't know.

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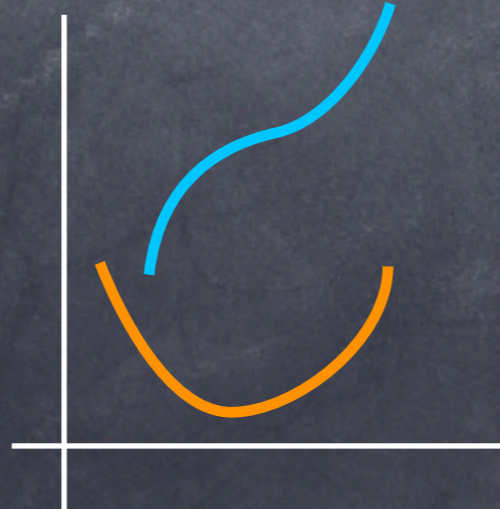
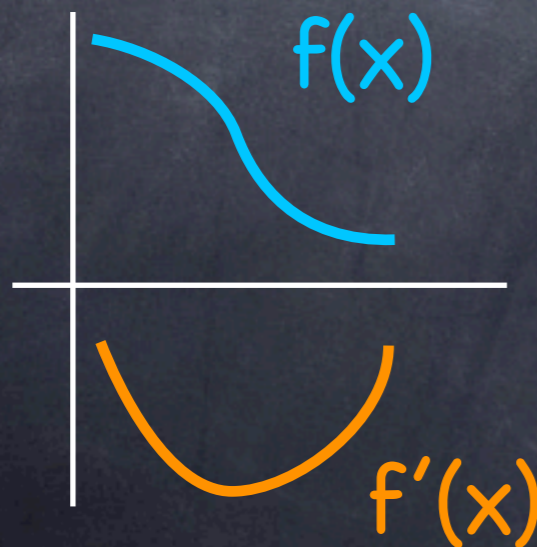
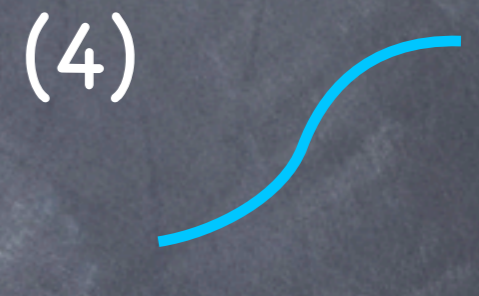
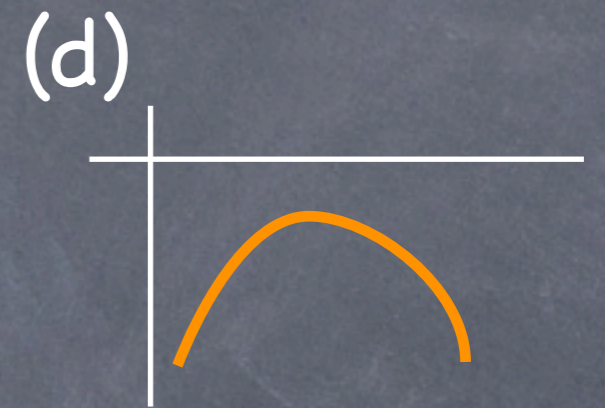
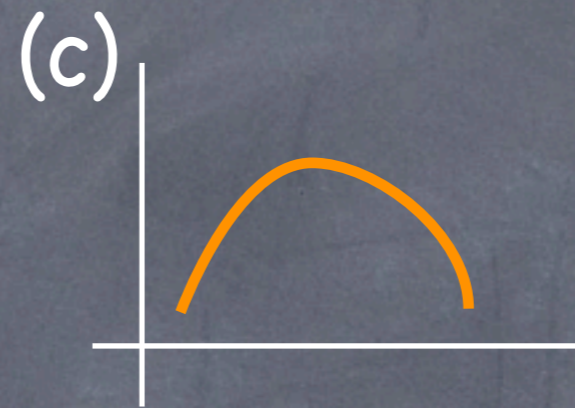
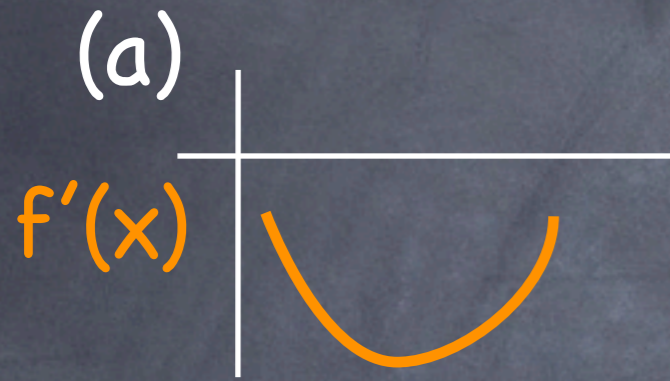
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This is "SDT" where the function considered is  $f'$  instead of  $f$ ! Would usually use "FDT".